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A Concrete Design Model for Members Subjected to Bending Moments

Tahar Latrache*

Doctorate Student in Civil Engineering, B.P. 129 Salem Lalmi, 40003 Khenchela, Constantine University, Algeria

Abstract

Several designers know that there's been a lack in the modeling of concrete by a rectangle or a parabola-rectangle by the using actual codes for the dimensionality of RC members at ultimate limit state. In this approach we present an accurate model of concrete which holds well with the experimental results, and we develop the formulas for the dimensionality of RC members subjected to a bending moment. The results were compared with results of the British BS and the French BAEL codes. The comparison shows that the proposed model is more economic and present minus shrinking of concrete and more stretching than the uneconomic limit in the steel bars than the tow largely used codes indicated.

Keywords: Concrete model; Reinforced concrete design; Design of beams; Bending moment

Introduction

More than three decades ago for the born of the theory of limit states and the model of concrete being modeled by a rectangle in all of the design codes of the world. This model in fact, represent a simplification of a real model of concrete all the world knows its allure and curve [1]. This simplification give, as we know a good results and in general in the sense of the security and safety for the users of reinforced concrete constructions; but in all cases, where the simplification adopted present advantages or inconveniences, not present a real model of concrete which represent a reality of the behavior of concrete under compression [2]. All the references indicated below; present a method of the resistance ultimate limit state for that the behavior of concrete at this case was modeled by a rectangle or a parabola-rectangle. The proposed model of concrete being with good agreement with the experimental results, made to determine the envelopes of concrete in compression, and we adopt the same limits of the contraction of concrete and of the dilatation of steel bars adopted by the design codes [3]. The integration of the function present the envelope of the concrete model is difficult; but we proposed a good agreement function which holds well with the numerical integration results [4]. After that, we developed the expressions of a reduce moment, the depth of contraction concrete, the dilation of steel bars, and finally, the sections of the tension and compression bars which present in tables for simplicity use by the designers or the engineers [5]. All these expressions indicated are compared with the expressions of the British BS code and the French BAEL code. The results of the comparison, shows that the proposed model is more resistant to contraction and by conclusion is more economic than the indicated codes, which in fact present that for the greet projects and for the more greet bending moments [6,7].

The Proposed Concrete Model

The proposed model for concrete function of the fraction $\chi=(\varepsilon_c/\varepsilon_{cc})$ is expressed as follows:

$$\frac{f_c}{f_{cc}} = ((7\tanh(1.1\chi))/(4.598+\chi^3)) - 0.00098\chi^4 \tag{2.1}$$

Function for that the fraction (f_c/f_{cc}) represents the variable strength of concrete to the characteristic strength of concrete for ε_{cc}=0.002 or for χ= 1. The proposed model is being shown in figure 1.1.

In fact that the function proposed for the model of concrete has no primitive and we cannot determine theoretically its center of gravity reported to the top of the beam which we have to design; but we searched tow functions which holds well with the numerical data of the surface between the curve of this function and the χ axis, and the center of gravity of the surface of the function. These tow functions are expressed respectively as follows and represented with the numerical data in figure 1.2.

$$\gamma_1(\varepsilon_c)=((3(500\varepsilon_c)^{1.025}/(3.5+1.075(500\varepsilon_c)2.178))-0.0001(500\varepsilon_c)0.5 \tag{2.2}$$

$$\gamma_2(\varepsilon_c)=(1/3)+(0.05533((500\varepsilon_c)^{1.3})(\tanh(500\varepsilon_c+0.001))^{1.8}) \tag{2.3}$$

The Expressions Formulae the Present Model

In the ultimate limit state of resistance for members subjected to bending moment only, one have to talk about the diagram of tow pivots. The Pivot A, which represent the maximal dilatation of the bottom steel bars (ε_s=0‰) and which pivots from ε_c=0‰ to ε_c=3.5‰. The Pivot B, which represent the maximal contraction of concrete for the top of the bending beam (ε_c=3.5‰), and pivots around this point to make

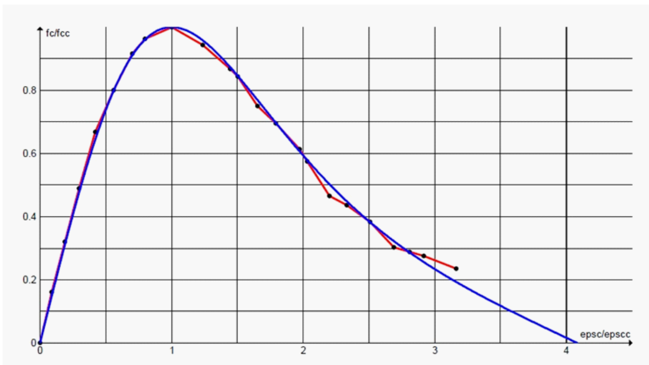

Figure 1.1: The proposed model for concrete compared with the experimental results.

***Corresponding author:** Tahar Latrache, Doctorate Student in Civil Engineering, B.P. 129 Salem Lalmi, 40003 Khenchela, Constantine University, Algeria, Tel: (00213)778936882; E-mail: latrache.tahar@yahoo.ca

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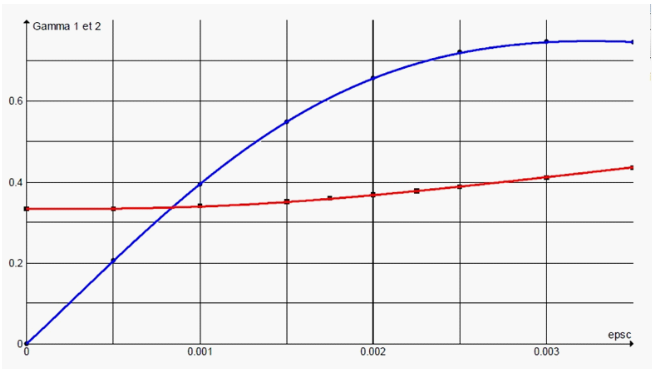

Figure 1.2: γ_1 and γ_2 function of ε_c.

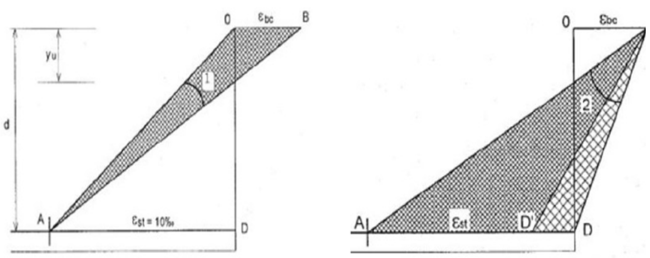

Figure 2.1: The pivots A and B diagram.

the dilatation, of the bottom steel bars, decrease from their maximal dilatation to $\varepsilon_c=f_e/E_s$ which equals for f_e=400MPa and E_s=200000MPa, ε_e=2‰. For values of $\varepsilon_s<\varepsilon_e$, we have to take compression steel bars to reinforce the dilative steel bars at the bottom for not exceed this minimum value, because if not, the dilative steel bars not exceed their limit state, and their section becoming not economic (Figure 2.1).

Pivot A

The bending moment in this case is small, and its expression is:

$$M=\sigma_{cc}bx\gamma_1(\varepsilon_c)(d-\gamma_2(\varepsilon_c)x) \tag{3.1}$$

Expression with $\sigma_{cc}=f_{cc}/\gamma_c$ and x, represent the depth of the compression region of the concrete. Suppose that $x=\alpha d$, such that d is the utile depth, the last expression becomes:

$$M=\sigma_{cc}bd^2\gamma_1(\varepsilon_c)\alpha(1-\gamma_2(\varepsilon_c)\alpha) \tag{3.2}$$

Define that the reduce moment as:

$$\mu=M/\sigma_{cc}bd^2=\gamma_1(\varepsilon_c)\alpha(1-\gamma_2(\varepsilon_c)\alpha) \tag{3.3}$$

We know that,

$$\alpha = \varepsilon_c/(\varepsilon_c+10‰) \tag{3.4}$$

Replace α by its expression (Eqn. 3.4) and γ_1 and γ_2 from Eqn. 2.2 and Eqn. 2.3 respectively, in the expression of μ (Eqn. 3.3), we may find an expression of only ε_c. We know the value of μ such we know the dimensions of the section of the beam, the maximal strength of the concrete and the exterior bending moment. Then we have to solve the equation (3.3) for ε_c. The expression is more complicated to solve theatrically; but with a chart or a table we can solve it numerically. The maximal value of μ which agreed with ε_c = 3.5‰ is μ = 0.1715; but for the BS code is 0.138 and for the BAEL is 0.158, which concludes that the model proposed make the concrete resist good to the contraction due to same bending moment than the tow indicate codes, except that for a very low bending moments. The curve of μ vs. ε_c is being shown by the figure 2.2 and the variations by the table 1 for the reason of the designing use. The section of the dilated steel bars is as known expressed as:

$$A_{st}=M/z\sigma_e \tag{3.5}$$

Expression such that $\sigma_e=f_e/\gamma_s$ and, $z = d(1-\gamma_2(\varepsilon_c)\alpha(\varepsilon_c))$ (3.6)

We can define $\rho(\varepsilon_c)$ as:

$$\rho(\varepsilon_c)=A_{st}\sigma/bd\sigma cc=M/bd^2\sigma cc(1-\gamma 2(\varepsilon_c)\alpha(\varepsilon_c))$$

Then we can express it as:

$$\rho(\varepsilon_c)=\gamma_1(\varepsilon_c)\alpha(\varepsilon_c) \tag{3.7}$$

This last expression is being shown by the figure 2.3 and the variations of ρ function of ε_c are numerically listed in table 1.

We can then calculate μ firstly, as we know the values of b, d, M and σ_{cc} and we compare it with 0.1715. If $\mu \le 0.1715$ then the diagram pivots around A and $\varepsilon_c \le 0.0035$, and then we can determine (ε_c) from table 1 using linear interpolation, and then we can compute A_{st}. The function of linear interpolation, such that μ falls between tow values, and then ρ, $\mu_{min} \le \mu \le \mu_{max}$ and then $\rho_{min} \le \rho \le \rho_{max}$, is expressing as:

$$\rho=\rho_{min}+((\mu-\mu_{min})(\rho_{max}-\rho_{min})/(\mu_{max}-\mu_{min})) \tag{3.8}$$

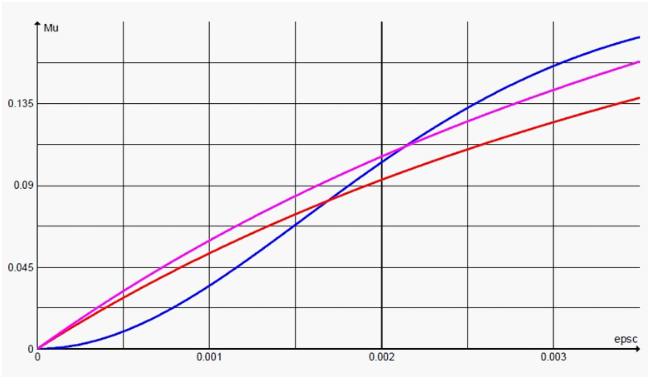

Figure 2.2: The variations of μ vs. ε_c (Pivot A) in comparison with the BS and BAEL.

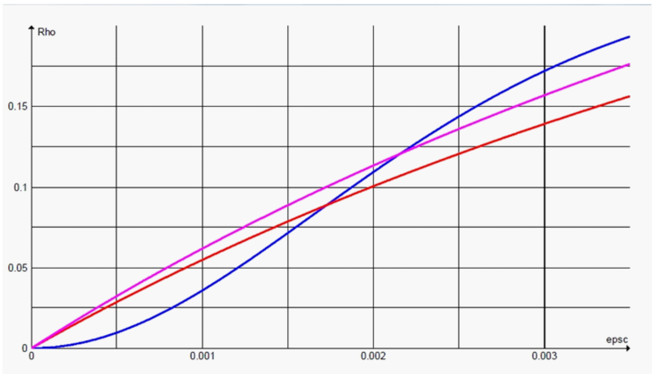

Figure 2.3: The variations of ρ vs. ε_c (Pivot A) in comparison with the codes.

μ	ε_c	ρ	μ	ε_c	ρ	μ	ε_c	ρ
0,0004	0,0001	0,0004	0,0544	0,0013	0,0566	0,1325	0,0025	0,1437
0,0016	0,0002	0,0016	0,0613	0,0014	0,0640	0,1377	0,0026	0,1499
0,0035	0,0003	0,0036	0,0682	0,0015	0,0715	0,1426	0,0027	0,1558
0,0062	0,0004	0,0063	0,0752	0,0016	0,0791	0,1473	0,0028	0,1615
0,0096	0,0005	0,0097	0,0822	0,0017	0,0867	0,1516	0,0029	0,1669
0,0136	0,0006	0,0138	0,0891	0,0018	0,0943	0,1556	0,0030	0,1720
0,0181	0,0007	0,0185	0,0959	0,0019	0,1018	0,1594	0,0031	0,1768
0,0232	0,0008	0,0238	0,1026	0,0020	0,1093	0,1628	0,0032	0,1813
0,0288	0,0009	0,0296	0,1090	0,0021	0,1166	0,1660	0,0033	0,1856
0,0347	0,0010	0,0359	0,1153	0,0022	0,1237	0,1689	0,0034	0,1896
0,0410	0,0011	0,0425	0,1213	0,0023	0,1306	0,1715	0,0035	0,1934
0,0476	0,0012	0,0494	0,1271	0,0024	0,1373			

Table 1: Variations of ε_c and ρ function of μ (Pivot A).

μ	ε_{st}	ρ	μ	ε_{st}	ρ	μ	ε_{st}	ρ
0,3429	0,0020	0,4746	0,2567	0,0048	0,3145	0,2028	0,0076	0,2352
0,3353	0,0022	0,4579	0,2519	0,0050	0,3071	0,1998	0,0078	0,2310
0,3280	0,0024	0,4424	0,2474	0,0052	0,3000	0,1969	0,0080	0,2270
0,3208	0,0026	0,4279	0,2430	0,0054	0,2933	0,1940	0,0082	0,2231
0,3139	0,0028	0,4143	0,2387	0,0056	0,2868	0,1912	0,0084	0,2194
0,3073	0,0030	0,4016	0,2346	0,0058	0,2807	0,1885	0,0086	0,2157
0,3008	0,0032	0,3896	0,2306	0,0060	0,2748	0,1859	0,0088	0,2122
0,2946	0,0034	0,3783	0,2268	0,0062	0,2691	0,1833	0,0090	0,2088
0,2886	0,0036	0,3676	0,2230	0,0064	0,2637	0,1808	0,0092	0,2055
0,2828	0,0038	0,3576	0,2194	0,0066	0,2584	0,1784	0,0094	0,2023
0,2772	0,0040	0,3480	0,2159	0,0068	0,2534	0,1760	0,0096	0,1993
0,2718	0,0042	0,3390	0,2125	0,0070	0,2486	0,1737	0,0098	0,1963
0,2666	0,0044	0,3304	0,2092	0,0072	0,2440	0,1715	0,0100	0,1934
0,2615	0,0046	0,3223	0,2059	0,0074	0,2395			

Table 2: Variations of ε_{st} and ρ function of μ (Pivot B).

And then we can compute the section of the tensile steel bars using the expression:

$$A_{st}=\rho b d\sigma_{cc}/\sigma_e \tag{3.9}$$

For μ=0.138, which is the value correspond to ε_c=0.0035 as indicated by the BS and suppose that the width of the beam is *b*, we have to reduce this width by a factor of 14.5%b to exceed ε_c=0.0035 according to BAEL, and 24.3%*b* according to the present model. The section of tensile steel bars are reduced too, by this factors reported to the section computed by the BS.

Pivot B

In this section we talk about the maximal shrinking of the concrete (i.e., when ε_c=3.5‰ and μ>0.1715) and the steel dilatation pivots from 10‰ to f_e/E_s (for the FeE400, f_e/E_s=2‰). The bending reduces moment is expressed as:

$$\mu=M/\sigma_{cc}bd^2=\gamma_1(3.5‰)\alpha(1-\gamma_2(3.5‰)\alpha) \tag{3.10}$$

Where α now is a function of the steel bars dilatation ε_{st}:

$$\alpha(\varepsilon_{st})= 3.5‰/(3.5‰+ \varepsilon_{st}) \tag{3.11}$$

$$\gamma_1 3.5‰=0.7458$$

$$\gamma_2 3.5‰=0.4361$$

Then the expression of μ (Eqn. 3.10) becomes:

$$\mu=0.7458(3.5‰/(3.5‰+\varepsilon_{st}))(1-0.4361(3.5‰/(3.5‰+\varepsilon_{st}))) \tag{3.12}$$

We know the value of μ, then we can solve the equation (3.12) to find ε_{st} as expressed:

$$\varepsilon_{st}=3.5‰[0.7458-2\mu + ((2\mu - 0.7458)^2 + 4\mu(0.42055662 - \mu))1/2]/2\mu \tag{3.13}$$

The variations μ of function of ε_{st} are shown in figure 2.4 in comparison with the indicated codes and listed in table 2. We have to compare before all the value of μ with the value:

$$\mu_e = 0.7458(3.5‰/(3.5‰+\varepsilon_e))(1-0.4361(3.5‰/(3.5‰+\varepsilon_e))) \tag{3.14}$$

Such that ε_e=fe/Es for FeE400, ε_e=2‰, and the value μ_e to compare with is 0.343).

- If $\mu \le \mu_e$, then:

$$\rho(\varepsilon_{st})=\gamma_1(3.5‰)\alpha(\varepsilon_{st})=A_{st}\sigma_e/bd\sigma_{cc}=0.7458(3.5‰/(3.5‰+\varepsilon_{st})) \tag{3.15}$$

The variations of (ε_{st}) are shown in figures 2.5, 2.6 and table 2 using the interpolation formula of equation 3.8. We know ρ, then we can compute the tensile steel bars:

$$A_{st}=\rho bd\sigma_{cc} \tag{3.16}$$

- If $\mu > \mu_e$ then, $\sigma < \sigma_e$,

and with consequence, the tensile steel bars are not economic. We have in this case reinforced the compression concrete by compression steel bars [8,9]. The compression steel bars have to resist to the difference between M and $Me=\mu_e bd^2\sigma_{cc}$, then the compression steel bars are computed as:

$$A's=(\mu-\mu_e)bd^2\sigma_{cc}/(d-d')\sigma_e \tag{3.17}$$

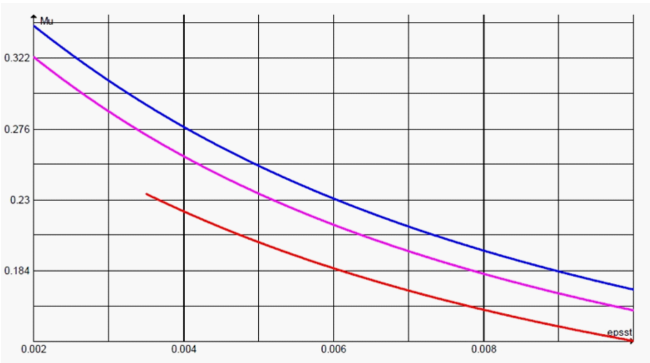

Figure 2.4: The variations of μ vs. ε_{st} (Pivot B) in comparison with the codes.

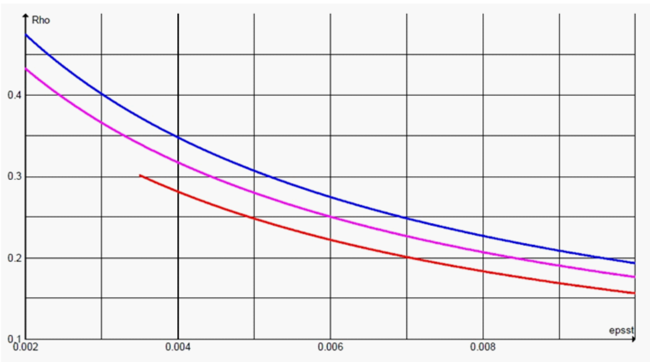

Figure 2.5: The variations of ρ vs. ε_{st} (Pivot B) in comparison with the codes.

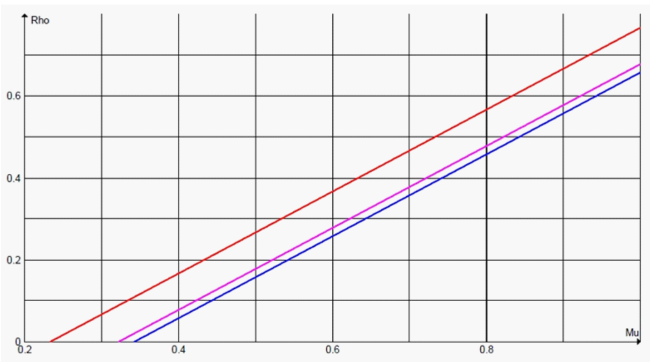

Figure 2.6: The variations of $\mu - \mu_e$ vs. ε_{st} (Pivot B) in comparison with the codes.

Such that $\alpha(\varepsilon_e)=3.5‰/(3.5‰+\varepsilon_e)$.

As a comparison between the values of μ_e given by the present model and the BS and the BAEL codes, for this model μ_e=0.343, for the BS 0.2738 and for the BAEL 0.3226 (for FeE400). We can observe that the present model is more resistant alone to compression than the indicated codes. For the same characteristics of concrete and steel, we suppose that μ=0.343; according to the present, the section needs only tensile steel bars computed by the following formula $A_s=\mu_e(bd^2\sigma_{cc}/(1-0.4361\alpha(\varepsilon_e))\sigma_e)$ but according to BS, $A'_s=0.0692bd^2\sigma_{cc}/(d-d')\sigma_e$ and according to BAEL, $A'_s=0.0204bd^2\sigma_{cc}/(d-d')\sigma_e$ and we have to add these sections to the tensile steel bars.

Conclusion

For the same bending moment, the present model is economic than the British BS and the French BAEL codes; but the good news is that this model is more resistant to shrinking and to compression alone which make it economic in concrete section as well as in steel. For the Pivot A, we see its real curve which different to the simplifications of a rectangle or parabola-rectangle adopted by the different codes. Its behavior is a real behavior of concrete to compression, experienced throughout the entire world. Its increase phase (hardening) of resistance function of the contraction is more applicable with the experiments than the model of a parabola; and its decrease phase (softening) too is adopted under different experiments, which make it in the sense of security. As a future work with this model, we try to develop it for the section subjected to a bending moment and a concentrated force, such the case of course of the columns. Moreover, the model is very simple to use by the engineer or a reinforced concrete designer, we can compute the sections of steel bars by a one step, using the tables proposed [10-12].

References

1. Betonvereniging: The Concrete Society and Deutscher Benton-Verein DB (2005) Design of Concrete Structures: Design Aids for Eurocode 2., E & FN Spon, UK.
2. Bond AJ, Brooker O, Harris AJ, Harrison T, Moss RM, et al. (2006) How to design concrete structures using Eurocode 2. The concrete centre, A Cement and Concrete Industry publication, UK.
3. Chana P (2005) Eurocode 2 worked examples. European concrete Platform ASBL, Brussels, Belgium.
4. CD-DTU V2 (2007) Technical design rules and calculation works and reinforced concrete structures following the limit state method. Règles BAEL 91 révisées 99 (150thedn) GTCC Works Section 1: reinforced concrete) + Amendment A1, CSTB, France.
5. Genest N (2004) Reinforced Concrete. Transports Québec, Canada.
6. Kong FK, Evans RH (1996) Reinforced and Prestressed Concrete. (3rdedn), Chapman and Hall, London, UK.
7. Legrand P, Nana JMT (2009) During the following concrete-BAEL changes rules 91 and 99. International Institute for Water and Environmental Engineering, France.
8. Mosley WH, Bungey GH, Hulse R (1990) Reinforced concrete design. (5thedn), MacMillan, UK.
9. Multon S (2012) Béton armé. Internet Document.
10. Ray SS (1995) Reinforced concrete: analysis and design (1stedn), Blackwell Science, UK.
11. Reynolds CE, Steedman JC (1999) Reinforced concrete designer's handbook. (10thedn) E and FN Spon, UK.
12. Walraven JC (2008) Eurocode 2: Design of concrete structures. Delft University of Technology, Stevinweg, Delft, The Netherlands.