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A Class of Weibull Mixtured Distributions

Mian Arif Shams Adnan* and Humayun Kiser

Department Of Statistics, Jahangirnagar University, Savar, Dhaka 1342, Bangladesh

Abstract

We have derived a class of mixture distributions which we call weibull mixtures of distributions. Estimation of unknown parameters along with some properties of these distributions are also prescribed.

Keywords: Mixing distribution; Mixtured distribution; Weibull distribution

Introduction

Mixture distribution was first coined in 1894. A number of authors like Pearson, Rider, Blichke, Chahine, Roy, et al., authors [1-13] defined mixtures of two distributions and studied various mixtured distributions which they called poisson mixture, binomial mixture, negative binomial mixture, Chi-square mixture, Erlang mixture, Laplace mixture, Rayleigh mixture, *F*, Dual mixture of distributions. Weibull distribution is widely being used in bio-statistics, but weibull mixture distribution has not yet been premeditated. In the present paper, we define first the weibull mixture of distributions and then weibull mixtures of normal, lognormal, gamma, exponential, beta, rectangular, erlang, chi-square, t and F distributions and studied some of their properties.

Preliminaries

A mixture distribution is a weighted average of probability distribution of positive weights that sum to one. The distributions thus mixed are called the components of the mixture. The weights themselves comprise a probability distribution called the mixing distribution. Because of this property of weights, a mixture is in particular again a probability distribution. Mixtures occur most commonly when the parameter θ of a family of distributions, given by the density by the density function $f(x, \theta)$, is itself subject to the change variation. The mixing distribution $g(x; \theta)$is then a probability distribution on the parameter of the distributions. The general formula for the finite mixture is $\sum_{i=1}^{k} f(x;\theta_i)g(\theta_i)$; the infinite analogue, in which g is a density function, is $\int f(x;\theta)g(\theta)d\theta$.

Main Results

Here in this paper, we define the weibull mixtures of some well known distributions such as normal, lognormal, gamma, exponential, beta, rectangular, erlang, chi-square, t and F distributions. Then some characteristics of these distributions such as characteristic functions, moments, and shape chaacteristics are also obtained. The main results of the paper are presented in form of definitions and theorems. Comparison of the probability density functions and the first two moments are prescribed in the tertiary section.

Definition 3.1

A random variable X is said to have a weibull mixtured distribution if its probability density function is defined as

$$f(x;a,b,\alpha)=\int_0^\infty abr^{b-1}e^{-ar^b}g(x;\alpha)dr \qquad (3.1)$$

Where $g(x,\alpha)$ is a probability density function. The name of weibull mixture distribution comes from the fact that the distribution (3.1) is the weighted average of $g(x,\alpha)$ with weights equal to the ordinates of weibull distribution.

Definition 3.2

If X follows a weibull mixture of Normal distribution with parameters a and b, then the density function is given by

$$f(x;a,b)=\int_0^\infty abr^{b-1}e^{-ar^b}\frac{e^{-\frac{1}{2}x^2}x^{2r}}{2^{r+\frac{1}{2}}\Gamma\left(r+\frac{1}{2}\right)}dr;-\infty<x<\infty \qquad (3.2)$$

with parameters a and b such that

$$\int_{-\infty}^{\infty} f(x;a,b)dx=1. \qquad (3.3)$$

The characteristic function and moments of the same distribution are presented in the theorem below.

Theorem 3.1

If X has a weibull mixture of normal distributions with parameters a and b then its characteristic function is represented as

$$\phi_x(t)=\int_0^\infty abr^{b-1}e^{-ar^b}\frac{e^{-\frac{1}{2}t^2}}{2^{r+\frac{1}{2}}\Gamma\left(r+\frac{1}{2}\right)}\sum_{m=0}^{r}\binom{2r}{2m}(it)^{2m}2^{r+\frac{1}{2}-m}\Gamma\left(r+\frac{1}{2}-m\right)dr \qquad (3.4)$$

and the $2s^{th}$ moment about origin is $\int_0^\infty abr^{b-1}e^{-ar^b}2^s\frac{\Gamma\left(r+\frac{1}{2}+s\right)}{\Gamma\left(r+\frac{1}{2}\right)}dr$

and $(2s+1)^{th}$ moment about origin is zero. Mean = 0,

$$Varinace=1+2a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right),\ \beta_1=0,\beta_2=\frac{\left[3+8a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+4a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)\right]}{\left[1+2a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right]^2}.$$

Remark: For $a = b = 0$, $\phi_x(t),\mu'_{2s},\mu'_{(2s+1)},\mu_1,\mu_2,\mu_3,\mu_4,\beta_1$ and β_2 are same for Normal distribution with mean zero and variance unity.

***Corresponding author:** Mian Arif Shams Adnan, Department Of Statistics, Jahangirnagar University, Savar, Dhaka 1342, Bangladesh, E-mail: julias284@yahoo.com

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Definition 3.3

If a random variable X has the density function

$$f(x;a,b)=\int_0^\infty abr^{b-1}e^{-ar^b}\frac{e^{-\frac{1}{2}(\log x)^2}(\log x)^{2r}}{x2^{r+\frac{1}{2}}\Gamma\left(r+\frac{1}{2}\right)}dr;x>0 \quad (3.5)$$

then it is said to have a weibull mixture of Lognormal distribution with parameters a,b since

$$\int_0^\infty f(x;a,b)dx=1 \quad (3.6)$$

Various moments of the distribution are given in the next theorem.

Theorem 3.2

If X is a weibull mixture of lognormal variable with parameters a,b then its characteristic function is given by

$$\phi_x(t)=\int_0^\infty abr^{b-1}e^{-ar^b}\frac{1}{2^{r+\frac{1}{2}}\Gamma\left(r+\frac{1}{2}\right)}\sum_{k=0}^\infty\frac{(it)^k}{k!}e^{\frac{1}{2}k^2}$$

$$\sum_{m=0}^r\binom{2r}{2m}k^{2r-2m}2^{m+\frac{1}{2}}\Gamma\left(m+\frac{1}{2}\right)dr \quad (3.7)$$

and the s^{th} moment about origin is

$$\int_0^\infty abr^{b-1}e^{-ar^b}\frac{e^{\frac{1}{2}s^2}}{2^{r-m}\Gamma\left(r+\frac{1}{2}\right)}\sum_{m=0}^r\binom{2r}{2m}s^{2r-2m}\Gamma\left(m+\frac{1}{2}\right)dr.$$

Definition 3.4

A random variable X having the density function

$$f(x;a,b,\alpha,\beta)=\int_0^\infty abr^{b-1}e^{-ar^b}\frac{\beta^{\alpha+r}e^{-\beta x}x^{\alpha+r-1}}{\Gamma(\alpha+r)}dr;x>0 \quad (3.8)$$

is defined a weibull mixture of Gamma distribution with parameters a,b,α and β whereas

$$\int_0^\infty f(x;a,b,\alpha,\beta)dx=1. \quad (3.9)$$

The characteristic function and moments are followed by the next theorem.

Theorem 3.3

If X denotes a weibull mixture of gamma variate with parameters a,b,α and β then its characteristic function is obtain as

$$\phi_x(t)=ab\left(1-\frac{it}{\beta}\right)^{-\alpha}\int_0^\infty r^{b-1}e^{-ar^b-r\ln\left(1-\frac{it}{\beta}\right)}dr \quad (3.10)$$

$$Mean=\frac{1}{\beta}\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right],$$

$$Variance=\frac{1}{\beta^2}\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)-\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\right],$$

$$\beta_1=\frac{\left[\begin{array}{l}2\alpha+2a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+3a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)+a^{-\frac{3}{b}}\Gamma\left(1+\frac{3}{b}\right)-3\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2-\\ 3\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}\left\{a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)\right\}+2\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^3\end{array}\right]^2}{\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)-\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\right]^3},$$

$$\beta_2=\frac{\left[\begin{array}{l}3\alpha^2+6\alpha+(6\alpha+6)a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+(6\alpha+11)a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)+6a^{-\frac{3}{b}}\Gamma\left(1+\frac{3}{b}\right)\\ +a^{-\frac{4}{b}}\Gamma\left(1+\frac{4}{b}\right)-(6\alpha+8)\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2-12\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}\left\{a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)\right\}\\ -4\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}\left\{a^{-\frac{3}{b}}\Gamma\left(1+\frac{3}{b}\right)\right\}+6\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^3\\ +6\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\left\{a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)\right\}-3\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^4\end{array}\right]}{\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)-\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\right]^2}.$$

Remark: $\phi x(t),\mu_1,\mu_2,\mu_3,\mu_4,\beta_1$ and β_2 are true for Gamma distribution with parameters α and β when $a=b=0$. For $\alpha=1$, weibull mixture of Gamma distribution should be equivalent to weibull mixture of Exponential distribution. As such we also derived the weibull mixture of Exponential distribution.

Estimates of parameters by the method of moments: Let X_1, X_2, X_3,……..,X_m be a random sample from the distribution (3.8). We assume that parameters a,b and β are known. Then the distribution contains only one unknown parameter α. We have $\mu_1'=\frac{1}{\beta}\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right]$, and $m_1'=\frac{\sum x_i}{m}=\bar{X}$. Hence by the method of moments, we get, $\frac{1}{\beta}\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right]=\bar{X}$. Therefore, $\hat{\alpha}=\bar{X}\beta-a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)$. (3.11)

Definition 3.5

A random variable X having the density function

$$f(x;a,b,\alpha)=\int_0^\infty abr^{b-1}e^{-ar^b}\frac{\alpha^{r+1}e^{-\alpha x}x^r}{\Gamma(r+1)}dr;x>0 \quad (3.12)$$

is said to have a weibull mixture of Exponential distribution with parameters a,b α, and

$$\int_0^\infty f(x;a,b,\alpha)dx=1 \quad (3.13)$$

Theorem 3.4

If X follows weibull mixture of exponential distributions with parameters a,b and α then its characteristic function is given by

$$\phi_x(t)=ab\left(1-\frac{it}{\alpha}\right)^{-1}\int_0^\infty r^{b-1}e^{-ar^b-r\ln\left(1-\frac{it}{\alpha}\right)}dr, \quad (3.14)$$

$$Mean=\frac{1}{\alpha}\left[1+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right],$$

$$Variance=\frac{1}{\alpha^2}\left[1+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)-\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\right],$$

$$\beta_1=\frac{\left[\begin{array}{l}2+2a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+3a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)+a^{-\frac{3}{b}}\Gamma\left(1+\frac{3}{b}\right)-3\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\\ -3\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}\left\{a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)\right\}+2\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^3\end{array}\right]^2}{\left[1+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)-\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\right]^3},$$

$$\beta_2=\frac{\left[\begin{array}{l}9+12a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+17a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)+6a^{-\frac{3}{b}}\Gamma\left(1+\frac{3}{b}\right)\backslash\\ +a^{-\frac{4}{b}}\Gamma\left(1+\frac{4}{b}\right)-14\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2-12\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}\left\{a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)\right\}\\ -4\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}\left\{a^{-\frac{3}{b}}\Gamma\left(1+\frac{3}{b}\right)\right\}+6\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^3\\ +6\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\left\{a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)\right\}-3\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^4\end{array}\right]}{\left[1+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)-\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\right]^2}.$$

Remark: When $a = b = 0$, then all of $\Pi_x(t),\mu_1,\mu_2,\mu_3,\mu_4,\beta_1$ and β_2 are similar to these of Exponential distribution with parameter α.

Parameter estimation: If $X_1, X_2, X_3,\ldots\ldots,X_m$ be a random sample drawn from the distribution (3.12) and parameters a,b are assumed known, then the distribution contains only one unknown parameter α. So, $\mu_1'=\frac{1}{\alpha}\left[1+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right]$, and $m_1'=\frac{\sum x_i}{m}=\bar{X}$. Therefore, $\frac{1}{\alpha}\left[1+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right]=\bar{X}$. Hence,

$$\hat{\alpha}=\frac{\left[1+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right]}{\bar{X}} \tag{3.15}$$

Definition 3.6

If a random variable X has the density function

$$f(x;a,b,\alpha,\beta)=\int_0^\infty abr^{b-1}e^{-ar^b}\frac{(\alpha\beta)^{\alpha+r}e^{-\alpha\beta x}x^{\alpha+r-1}}{\Gamma(\alpha+r)}dr;x>0 \tag{3.16}$$

then it is said to have a weibull mixture of Erlang distribution with parameters a,b,α and β since

$$\int_0^\infty f(x;a,b,\alpha,\beta)dx=1 \tag{3.17}$$

The characteristic function as well as the moments is stated in the following theorem.

Theorem 3.5

If X has weibull mixture of erlang distributions with parameters a,b, α and β then its characteristic function is given by

$$\phi_x(t)=ab\left(1-\frac{it}{\alpha\beta}\right)^{-\alpha}\int_0^\infty r^{b-1}e^{-ar^b-r\ln\left(1-\frac{it}{\alpha\beta}\right)}dr \tag{3.18}$$

$$Mean=\frac{1}{\alpha\beta}\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right],$$

$$Variance=\frac{1}{(\alpha\beta)^2}\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)-\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\right],$$

$$\beta_1=\frac{\left[\begin{array}{l}2\alpha+2a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+3a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)+a^{-\frac{3}{b}}\Gamma\left(1+\frac{3}{b}\right)-3\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\\ -3\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}\left\{a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)\right\}+2\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^3\end{array}\right]^2}{\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)-\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\right]^3},$$

$$\beta_2=\frac{\left[\begin{array}{l}3\alpha^2+6\alpha+(6\alpha+6)a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+(6\alpha+11)a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)\\ +6a^{-\frac{3}{b}}\Gamma\left(1+\frac{3}{b}\right)+a^{-\frac{4}{b}}\Gamma\left(1+\frac{4}{b}\right)-(6\alpha+8)\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\\ -12\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}\left\{a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)\right\}-4\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}\left\{a^{-\frac{3}{b}}\Gamma\left(1+\frac{3}{b}\right)\right\}\\ +6\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^3+6\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\left\{a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)\right\}-3\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^4\end{array}\right]}{\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)-\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\right]^2}.$$

Remark: $a = b = 0$ provides all the values of $\phi x(t),\mu_1,\mu_2,\mu_3,\mu_4,\beta_1$ and β_2 to be true for Erlang distribution with parameters α and β.

Estimating parameters: For a random sample $X_1, X_2, X_3,\ldots\ldots,X_m$ from the distribution (3.16), we assume that parameters a,b and β are known and α unknown parameter. Here, $\mu_1'=\frac{1}{\alpha\beta}\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right]$, and $m_1'=\frac{\sum x_i}{m}=\bar{X}$. We obtain $\frac{1}{\alpha\beta}\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right]=\bar{X}$. Therefore,

$$\hat{\alpha}=\frac{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)}{(\bar{X}\beta-1)} \tag{3.19}$$

Definition 3.7

A random variable X having the density function

$$f(x;a,b,m)=\int_0^\infty abr^{b-1}e^{-ar^b}\frac{(r+1)x^r}{m^{r+1}}dr;0<x<m \tag{3.20}$$

is said as weibull mixture of Rectangular distribution with parameters a,b and m satisfying

$$\int_0^m f(x;a,b,m)dx=1. \tag{3.21}$$

Different moments of the above mentioned distribution are expressed below.

Theorem 3.6

If X follows a weibull mixture of rectangular distribution with parameters a,b and m then its characteristic function is obtained as

$$\phi_x(t)=\int_0^\infty abr^{b-1}e^{-ar^b}\sum_{k=0}^\infty\frac{(it)^k(r+1)m^{r+k+1}}{k!m^{r+1}(r+k+1)}dr \tag{3.22}$$

and the s^{th} moment about origin is $m^s\int_0^\infty abr^{b-1}e^{-ar^b}\frac{r+1}{r+s+1}dr$.

Remark: If $a = b = 0$ then all the values of $\phi_x(t),\mu_1,\mu_2,\mu_3,\mu_4,\beta_1$ are true for Rectangular distribution with parameter m.

Definition 3.8

A random variable X having the density function

$$f(x;a,b,\alpha,\beta)=\int_0^\infty abr^{b-1}e^{-ar^b}\frac{x^{\alpha+r-1}(1-x)^{\beta-1}}{B(\alpha+r,\beta)}dr;0<x<1 \tag{3.23}$$

is said to have a weibull mixture of Beta distribution of 1st kind with parameters a,b, α and β. Here we have

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$$\int_0^1 f(x;a,b,\alpha,\beta)dx = 1 \tag{3.24}$$

Theorem 3.7

If X follows weibull mixture of beta distributions of first kind with parameters *a,b,* α and β, then its s^{th} moment about origin is given by

$$\phi_x(t) = \int_0^\infty abr^{b-1}e^{-ar^b}\frac{B(\alpha+s+r,\beta)}{B(\alpha+r,\beta)}dr \tag{3.25}$$

Remark: For $a = b = 0$, all the values of μ_s', μ_1', μ_2' and μ_2 are true for Beta distribution of 1st kind with parameters α and β.

Definition 3.9

A random variable X having the density function

$$f(x;a,b,\alpha,\beta) = \int_0^\infty abr^{b-1}e^{-ar^b}\frac{x^{\alpha+r-1}}{B(\alpha+r,\beta)(1+x)^{\alpha+\beta+r}}dr; x>0 \tag{3.26}$$

is called a weibull mixture of Beta distribution of 2nd kind with parameters *a,b,* α and β. Moreover,

$$\int_0^\infty f(x;a,b,\alpha,\beta)dx = 1 \tag{3.27}$$

Next theorem presents some properties of the same distribution.

Theorem 3.8

If *X follows weibull mixture of beta distribution of second kind with parameters a,b,* α and β *then its* s^{th} *moment about origin is given by* $\int_0^\infty abr^{b-1}e^{-ar^b}\frac{B(\alpha+s+r,\beta-s)}{B(\alpha+r,\beta)}dr$.

Remark: Putting $a = b = 0$ then all the values of μ_s', μ_1', μ_2' and μ_2 are true for Beta distribution of 2nd kind with parameters α and β.

Definition 3.10

A random variable X^2 with the density function

$$f(\chi^2;a,b,n) = \int_0^\infty abr^{b-1}e^{-ar^b}\frac{e^{-\frac{1}{2}\chi^2}(\chi^2)^{\frac{n}{2}+r-1}}{2^{\frac{n}{2}+r}\Gamma\left(\frac{n}{2}+r\right)}dr; \chi^2>0 \tag{3.28}$$

is said to have a weibull mixture of Chi-square distribution having the parameters *a,b* and *n* since

$$\int_0^\infty f(\chi^2;a,b,n)d\chi^2 = 1. \tag{3.29}$$

Theorem 3.9

If X^2 has weibull mixture chi-square distribution with parameters *a,b* and *n* then its characteristic function is expressed as

$$\phi_x(t) = ab(1-2it)^{-\frac{n}{2}}\int_0^\infty abr^{b-1}e^{-ar^b - r\ln(1-2it)}dr \tag{3.30}$$

$$Mean = n + 2a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right),$$

Remark: Setting a=b=0 we find that all the values of $\phi_x(t), \mu_1, \mu_2, \mu_3, \mu_4, \beta_1$ and β_2 are true for Chi-square distribution with parameters *n*.

Parameter estimation: Let $X_1, X_2, X_3, \ldots\ldots, X_m$ be a random sample from the distribution (3.28). We assume that parameters *a* and *b* are known and n is unknown. Now, $\mu_1' = n + 2a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)$, and $m_1' = \frac{\sum x_i}{m} = \bar{X}$. As such, $n + 2a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right) = \bar{X}$. Therefore, $\hat{n} = \bar{X} - 2a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)$ (3.31)

Sl.	Name of the distribution	Probability density function *f*(*x*)	Support	Parameters
1	Weibull mixtured Normal	$\int_0^\infty abr^{b-1}e^{-ar^b}\frac{e^{-\frac{1}{2}x^2}x^{2r}}{2^{r+\frac{1}{2}}\Gamma\left(r+\frac{1}{2}\right)}dr$	$-\infty < x < \infty$	*a,b*
2	Weibull mixtured Lognormal	$\int_0^\infty abr^{b-1}e^{-ar^b}\frac{e^{-\frac{1}{2}(\log x)^2}(\log x)^{2r}}{x2^{r+\frac{1}{2}}\Gamma\left(r+\frac{1}{2}\right)}dr$	$x > 0$	*a,b*
3	Weibull mixtured Gamma	$\int_0^\infty abr^{b-1}e^{-ar^b}\frac{\beta^{\alpha+r}e^{-\beta x}x^{\alpha+r-1}}{\Gamma(\alpha+r)}dr$	$x > 0$	*a,b,α,β*
4	Weibull mixtured Exponential	$\int_0^\infty abr^{b-1}e^{-ar^b}\frac{\alpha^{r+1}e^{-\alpha x}x^r}{\Gamma(r+1)}dr$	$x > 0$	*a,b,α*
5	Weibull mixtured Erlang	$\int_0^\infty abr^{b-1}e^{-ar^b}\frac{(\alpha\beta)^{\alpha+r}e^{-\alpha\beta x}x^{\alpha+r-1}}{\Gamma(\alpha+r)}dr$	$x > 0$	*a,b,α,β*
6	Weibull mixtured Rectangular	$\int_0^\infty abr^{b-1}e^{-ar^b}\frac{(r+1)x^r}{m^{r+1}}dr$	$0 < x < m$	*a,b,m*
7	Weibull mixtured Beta 1st kind	$\int_0^\infty abr^{b-1}e^{-ar^b}\frac{x^{\alpha+r-1}(1-x)^{\beta-1}}{B(\alpha+r,\beta)}dr$	$0 < x < 1$	*a,b,α,β*
8	Weibull mixtured Beta 2nd kind	$\int_0^\infty abr^{b-1}e^{-ar^b}\frac{x^{\alpha+r-1}}{B(\alpha+r,\beta)(1+x)^{\alpha+\beta+r}}dr$	$x > 0$	*a,b,α,β*
9	Weibull mixtured Chi-square	$\int_0^\infty abr^{b-1}e^{-ar^b}\frac{e^{-\frac{1}{2}\chi^2}(\chi^2)^{\frac{n}{2}+r-1}}{2^{\frac{n}{2}+r}\Gamma\left(\frac{n}{2}+r\right)}dr$	$\chi^2 > 0$	*a,b,n*
10	Weibull mixtured *t*	$\int_0^\infty abr^{b-1}e^{-ar^b}\frac{t^{2r}}{n^{\frac{1}{2}+r}B\left(\frac{1}{2}+r,\frac{n}{2}\right)\left(1+\frac{t^2}{n}\right)^{\frac{n+1}{2}+r}}dr$	$-\infty < t < \infty$	*a,b,n*
11	Weibull mixtured *F*	$\int_0^\infty abr^{b-1}e^{-ar^b}\frac{\left(\frac{n_1}{n_2}\right)^{\frac{n_1}{2}+r}F^{\frac{n_1}{2}+r-1}}{B\left(\frac{n_1}{2}+r,\frac{n_2}{2}\right)\left(1+\frac{n_1}{n_2}F\right)^{\frac{n_1+n_2}{2}+r}}dr$	$F > 0$	a,b,n^1,n^2

Table 1: Comparison of density functions of different Weibull mixture distributions. $\chi^2>0$.

Sl.	Name of the distribution	Mean	Variance
1	Weibull mixtured Normal	0	$1+2a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)$
2	Weibull mixtured lognormal	can be obtained from equation 3.7	can be obtained from equation 3.7
3	Weibull mixtured Gamma	$\frac{1}{\beta}\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right]$	$\frac{1}{\beta^2}\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)-\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\right]$
4	Weibull mixtured Exponential	$\frac{1}{\alpha}\left[1+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right]$	$\frac{1}{\alpha^2}\left[1+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)-\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\right]$
5	Weibull mixtured Erlang	$\frac{1}{\alpha\beta}\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right]$	$\frac{1}{(\alpha\beta)^2}\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)-\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\right]$
6	Weibull mixtured Rectangular	can be achieved from equation 3.22	can be achieved from equation 3.22
7	Weibull mixtured Beta 1st kind	equation 3.25 provides	equation 3.25 provides
8	Weibull mixtured Beta 2nd kind	$\frac{1}{\beta-1}\left[\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right]$	$\frac{1}{(\beta-1)(\beta-2)}\left[\alpha(\alpha+1)+(2\alpha+1)a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)\right]-\left[\frac{1}{\beta-1}\left\{\alpha+a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}\right]^2$
9	Weibull mixtured Chi-square	$n+2a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)$	$\left[2n+4a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+4a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)-4\left\{a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}^2\right]$
10	Weibull mixtured *t*	0	$\frac{n}{n-2}\left[1+2a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right]$
11	Weibull mixtured *F*	$\frac{n_2}{n_1(n_2-2)}\left[n_1+2a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right]$	$\frac{n_2^2}{n_1^2(n_2-2)(n_2-4)}\left[n_1(n_1+2)+4(n_1+1)a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+4a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)\right]-\left[\frac{n_2}{n_1(n_2-2)}\left\{n_1+2a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right\}\right]^2$

Table 2: Comparison among first two moments of different Weibull mixtured distributions.

Definition 3.11

If *t* as a random variable has the density function

$$f(t;a,b,n)=\int_0^\infty abr^{b-1}e^{-ar^b}\frac{t^{2r}}{n^{\frac{1}{2}+r}B\left(\frac{1}{2}+r,\frac{n}{2}\right)\left(1+\frac{t^2}{n}\right)^{\frac{n+1}{2}+r}}dr;-\infty<t<\infty \quad (3.32)$$

then it is said to have a weibull mixture of t distribution with parameters *a*,*b* and *n* if

$$\int_{-\infty}^{\infty} f(t;a,b,n)dt=1 \quad (3.33)$$

The following theorem expresses here some of the properties of the distribution.

Theorem 3.10

If t is weibull mixture of t distribution with parameters *a*,*b* and *n* then the $2s^{th}$ moment about origin is given by $n^s\int_0^\infty abr^{b-1}e^{-ar^b}\frac{\Gamma\left(r+s+\frac{1}{2}\right)\Gamma\left(\frac{n}{2}-s\right)}{\Gamma\left(\frac{1}{2}+r\right)\Gamma\left(\frac{n}{2}\right)}dr$ and the $(2s+1)^{th}$ moment about origin is zero, $\beta_1 = 0, \beta_2 = \frac{n-2}{n-4} \frac{\left[3+8a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)+4a^{-\frac{2}{b}}\Gamma\left(1+\frac{2}{b}\right)\right]}{\left[1+2a^{-\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)\right]^2}$.

Remark: If $a = b = 0$ then all the values of μ_{2s+1}, μ_{2s}, μ_1, μ_2, μ_3, μ_4, β_1 and β_2 are true for t distribution with parameter n.

Definition 3.12

A random variable F having the density function

$$f(F;a,b,n_1,n_2) = \int_0^\infty abr^{b-1}e^{-ar^b} \frac{\left(\frac{n_1}{n_2}\right)^{\frac{n_1}{2}+r} F^{\frac{n_1}{2}+r-1}}{B\left(\frac{n_1}{2}+r,\frac{n_2}{2}\right)\left(1+\frac{n_1}{n_2}F\right)^{\frac{n_1+n_2}{2}+r}} dr; F>0 \quad (3.34)$$

is said to have a weibull mixture of F distribution with parameters a,b, n_1 and n_2 if

$$\int_0^\infty f(F;a,b,n_1,n_2)dF = 1 \quad (3.35)$$

The following theorem presents the characteristic function and moments of this distribution.

Theorem 3.11

If F follows weibull mixture of F distribution with parameters a,b, n_1 and n_2 then its characteristic function is given by

$$\phi_x(t) = \int_0^\infty abr^{b-1}e^{-ar^b} \sum_{x=0}^{\infty} \frac{\left(it\frac{n_2}{n_1}\right)^x}{x!} \frac{\Gamma\left(\frac{n_2}{2}+r+x\right)\Gamma\left(\frac{n_2}{2}-x\right)}{\Gamma\left(\frac{n_1}{2}+r\right)\Gamma\left(\frac{n_2}{2}\right)} dr \quad (3.36)$$

and the S^{th} moment about origin is $\left(\frac{n_2}{n_1}\right)^s \int_0^\infty abr^{b-1}e^{-ar^b} \frac{\Gamma\left(\frac{n_2}{2}+r+s\right)\Gamma\left(\frac{n_2}{2}-s\right)}{\Gamma\left(\frac{n_1}{2}+r\right)\Gamma\left(\frac{n_2}{2}\right)} dr$,

Remark: For $a = b = 0$ all the values of $\phi_x(t)$, μ_1, μ_2 and μ_2 are true for F distribution with parameters n_1 and n_2.

Comparison

A Comparison among various features of the different weibull mixtured distributions is shown in the following table 1 and table 2.

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