

Orlando Gomes

Lisbon School of Accounting and Administration
of the Lisbon Polytechnic Institute
&
Business Research Unit
of the Lisbon University Institute

- Research highlights, prepared for the

Journal of Applied & Computational Mathematics

July, 2014

Economy, indeed, being concerned with quantities, has always of necessity been mathematical in its subject, but the strict and general statement, and the easy comprehension of its quantitative laws has been prevented by a neglect of those powerful methods of expression which have been applied to most other sciences with so much success. It is not to be supposed, however, that because economy becomes mathematical in form, it will, therefore, become a matter of rigorous calculation. Its mathematical principles may become formal and certain, while its individual data remain as inexact as ever.

William Stanley Jevons,
'A Brief Account of a General Mathematical Theory of Political Economy.'
published in the *Journal of the Royal Statistical Society*,
June 1866.

- These research highlights originate on four editorial papers published in the *Journal of Applied and Computational Mathematics*:
 - Gomes, O. (2012). “Applied Mathematics and Economics: Tools for Addressing Rationality, Expectations and Related Phenomena.” *Journal of Applied and Computational Mathematics*, volume 1, issue 4, doi: 10.4172/2168-9679.1000e111.
 - Gomes, O. (2012). “Spatiotemporal Modeling in Economics.” *Journal of Applied and Computational Mathematics*, vol. 2, issue 2, doi: 10.4172/2168-9679.1000e128.
 - Gomes, O. (2014). “Complex Networks in Macroeconomics: a New Research Frontier.” *Journal of Applied and Computational Mathematics*, volume 3, issue 3, doi: 10.4172/2168-9679.1000e138.
 - Gomes, O. (2014). “Scale-Free Networks in Economics.” *Journal of Applied and Computational Mathematics*, volume 3, issue 4, doi: 10.4172/2168-9679.1000e139.

- Economics is a fascinating field of knowledge;
- Economics apply rigorous notions and tools from the exact sciences to the understanding of human behavior and social relations;
- The economic science employs concepts that are unique to its domain and that require a formal design that only mathematics can provide;
- One of these tools, e.g., is the notion of **rational expectations**.

- Benchmark model of economic analysis:

$$\text{Max}_c \sum_{t=0}^{\infty} \beta^t u(c_t)$$

- A representative agent maximizes utility, u ;
- The agent draws utility from consumption, c ;
- This is an inter-temporal problem: the agent establishes a consumption plan, starting in the current period, $t=0$, and assuming an infinite horizon;
- $\beta \in (0,1)$: discount factor (the future is discounted at a constant rate).

- Utility maximization is a constrained problem.
 - The representative agent chooses how much to consume at each time period – consumption is the control variable – but subject to a constraint;
- All economic problems involve constraints → this is what makes them economic problems!
 - In this particular case, the constraint is a difference equation of accumulation of financial resources,

$$a_{t+1} = w_t + (1 + r)a_t - c_t, a_0 \text{ given}$$

- a_t : financial wealth;
- w_t : labor income;
- r : interest rate.

- The presented problem reveals the true nature of economics: *economics is a forward-looking science*,
 - Agents take decisions today that have implications over a probably long horizon;
 - The past is irrelevant for current decisions;
 - Expectations become central to the analysis – uncertainty can be mitigated resorting to the powerful notion of **rational expectations**.
 - Under rational expectations agents are endowed with the ability to avoid incurring in systematic mistakes.
 - See Muth (1961), Lucas (1972) and Sargent (1973), concerning the rational expectations revolution in economics.

- The notion of rational expectations is a paradigmatic example of how the rigor of the exact sciences may assist economists in understanding observable phenomena.
- But are agents truly rational in the way they behave and forecast future events?
- Models based on the rationality assumption helped in constructing an economic theory well equipped to explain relevant economic issues, like
 - economic growth,
 - business cycles,
 - unemployment,
 - asset pricing ,
 - the market power of firms, ...
- However, it is time to move beyond the fully rational representative agent paradigm: there are many powerful tools that **applied and computational mathematics** can offer to economics to enrich its ability to explain real world events.

- Besides the **time** dimension, **space** has also a prominent role in economics.
 - International trade, capital flows or the diffusion of knowledge occur in time, but also through the physical space.
 - An effort to merge both dimensions seems a logical step in developing a more robust economic science.
 - Example of a model that integrates space and time: Lucas (2009) and Comin *et al.* (2012) on the diffusion of ideas.

- In the mentioned framework, **spatiotemporal dynamics** are determined essentially by two parameters:
 - the frequency of meetings, given by the rate of adoption $\alpha \in (0,1)$, and a parameter translating the fall in the probability to meet when distance between agents increases, $\delta > 0$.
 - this last parameter works in the following way:
 - the probability that an agent located at point ℓ in space meets an agent at point ℓ' is $\exp(-\delta|\ell-\ell'|)$ times lower than the probability of meeting an agent located in the original point in space.

- If agents are distributed uniformly throughout the space, we can split it into a given number of locations, say N , each one containing an identical share of agents.
- Let $G(\ell, t)$ be the share of agents at location ℓ and time t that have not yet adopted the new idea.
 - the probability of not accessing the new idea in period $t+1$ conditional on not having accessed it in period t , at location ℓ' , corresponds to

$$G(\ell', t+1) = G(\ell', t) \left[\frac{\sum_{\ell=1}^N G(\ell, t) \exp(-\delta|\ell - \ell'|)}{\sum_{\ell=1}^N \exp(-\delta|\ell - \ell'|)} \right]^\alpha$$

- This difference equation allows to characterize a process of diffusion of ideas in space and time.
- Regardless of the initial state, knowledge will disseminate and, asymptotically, reach all points in space and every agent located at each point.

- The fundamental information the structure of analysis offers relates the speed of adjustment towards the steady-state of 100% adopters.
 - As it should be obvious, the more frequent are the meetings among agents (larger α) and the less localized the diffusion process is (smaller δ), the faster will be the adjustment process.
- The presented model is one viable way of taking together the impact of **time** and **space** when addressing the behavior of economic agents. Obviously, there are many other ways of addressing economics as a spatiotemporal science.
- A successful adaptation of modeling techniques originating on other sciences to economics requires the capacity to understand the specificity of economic relations, relatively to other processes of interaction one observes in society and in nature.

- The economist Friedrich Von Hayek, to whom it was awarded the Nobel memorial prize in Economic Sciences in 1974, viewed and interpreted the economic system as an entity governed by a *spontaneous order*.
 - Such term designates the potential of the market relations to be self-organized, thus not requiring any centralized coordination.
 - Self-interested agents, pursuing their own goals, will form a spontaneous, hence not planned, network of relations that scientists need to carefully analyze in order to acquire a panoramic and solid understanding on how the economy as a whole truly works.

- Despite the advancements on other sciences, e.g, biology or physics, concerning the study of network relations, economics has resisted to adopt **network analysis** as a central instrument for its research.
- Economists seem to be fully satisfied with the explanatory power of the representative agent benchmark model, and use it to the exhaustion to address every possible issue.
- Under fully rational **representative agent models**, the macro economy could be characterized taking the behavior of a single average agent, who consequently would be a central planner.
- In such a worldview, the difference between micro and macro analysis would be just a matter of scale.
- A **fallacy of composition** emerges from the above argument. In most scenarios, the whole is far from being just the sum of its constituent parts, and it is precisely this simple observation that is leading to a gradual but firm paradigm shift in economics.

- Tesfatsion (2006), Delli Gatti *et al.* (2010), Kirman (2012), Bargigli and Tedeschi (2014): **the macro economy is a complex adaptive network**, where the same micro units might generate different macro outcomes in response to different patterns of interaction.
- Economic relations are no longer seen as being mechanical; instead, they are the result of strategic interaction by agents who meet locally, leading to **unrepeatable complex outcomes and out-of-equilibrium dynamics**.
- A complexity approach allows for replacing a strict view of rationality by a series of behavioral characteristics one encounters in the real world, namely **deliberate experimentation, learning from experience** or the **ability to adapt to existing social interaction patterns and norms**.

- The literature on **complex networks** apparently provides a meaningful setting to study patterns of collective behavior as the ones economic relations involve.
- Economic networks are truly complex:
 1. links may acquire many different shapes, e.g., they can be undirected or directed, they might represent strong or weak ties between two units, and the strength of the connections they represent is likely to change over time.
 2. some peculiar and well known complex network forms are well suited to address economic issues, namely, those that relate to **small-world networks** and **scale-free networks**.
 3. economic networks are, in their essence, dynamic, in the sense they involve relations between agents that adapt their behavior, learn and form expectations on future events.
 4. economic agents interact locally, i.e., they seldom have an overall and integrated view of the whole of the relevant economic relations; furthermore, their actions are inherently strategic.

- So far, the models on complex networks have approached essentially two topics:
 - (i) Financial contagion. Financial networks allow for studying credit markets and asset markets. The nodes will correspond to the investors and the links will represent credit-debit relations;
 - (ii) The organization of decentralized markets of goods and services. In these, nodes represent buyers and sellers and the market structure evolves endogenously given the specific links that the interaction in the network allows for.
- In synthesis, one might say that recent literature on the structure and dynamics of complex networks, both the theoretical contributions and the applications to fields that range from engineering to medicine, are paving the way for a new kind of science, less centered on optimal or efficient decisions and more focused on concrete and observable patterns of interaction.
- A better understanding of interaction processes is particularly vital in macroeconomics.

- Regarding **complex networks**, an important discovery was made by Barabási and Albert (1999).
 - These authors claimed that many observable networks display power-law shaped degree distributions and, consequently, they can be designated **scale-free networks**.
 - When the degree of a network follows a **power-law distribution**, a restrict number of nodes is strongly connected to the rest of the network and a large percentage of nodes is poorly connected, i.e., they exhibit few links to other points in the network.
- According to Barabási (2009), the emergence of scale-free networks is essentially the outcome of two features that one often encounters in socio-economic relations: **incremental growth** and **preferential attachment**.
 1. Incremental growth relates to the idea that networks are not static structures; they evolve with the systematic addition of new nodes.
 2. Preferential attachment signifies that the new nodes that enter the network prefer to attach to the nodes that display a higher degree of connectivity; this is often described as a 'rich-gets-richer' process.

- Those who are acquainted with how business relations are organized in a decentralized economy, will encounter in the above description of scale-free networks some familiar features.
- A market for a given good, the financial system or the world economy, all display characteristics of a **complex network** and, more specifically, of a **scale-free network**.
 - They are all frameworks involving thousands or millions of individual entities that have different degrees of connectivity inside the network;
 - typically, a few economic agents have a dominant position, which might translate in a high degree of connectivity within the network, whereas the large majority of the agents are linked only with a small group of other agents.
- Scale-free networks offer a substantive tool that the economic science can resort to in order to explain most observable phenomena.
- Their analytical tractability, associated to the fact that they accurately translate many aspects of the economic life, as the organization of markets, the functioning of the financial system, the distribution of wealth or the correlation of forces in the global economy, make them an indispensable tool to approach economic issues.

References

- **Barabási, A.L. and R. Albert** (1999). “Emergence of Scaling in Random Networks.” *Science*, vol. 286, pp. 509-512.
- **Barabási, A.L.** (2009). “Scale-Free Networks: a Decade and Beyond.” *Science*, vol. 325, pp. 412-413.
- **Bargigli, L. and G. Tedeschi** (2014). ‘Interaction in Agent-Based Economics: a Survey on the Network Approach.’ *Physica A*, vol. 399, pp. 1-15.
- **Comin, D.A.; M. Dmitriev and E. Rossi-Hansberg** (2012). “The Spatial Diffusion of Technology.” *NBER working paper* n° 18534.
- **Delli Gatti, D.; E. Gaffeo and M. Gallegati** (2010). ‘Complex Agent-Based Macroeconomics: a Manifesto for a New Paradigm.’ *Journal of Economic Interaction and Coordination*, vol. 5. pp. 111-135.
- **Kirman, A.P.** (2012). ‘Can Artificial Economies Help us Understand Real Economies?’ *Revue de l’OFCE*, n° 124, pp. 15-41.
- **Lucas, R.E.** (1972). ‘Expectations and the Neutrality of Money.’ *Journal of Economic Theory*, vol. 4, pp. 103-124.
- **Lucas, R.E.** (2009). “Ideas and Growth.” *Economica*, vol.76, pp.1-19.
- **Muth, J.F.** (1961). ‘Rational Expectations and the Theory of Price Movements.’ *Econometrica*, vol. 29, pp. 315-335.
- **Sargent, T.J.** (1973). ‘Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment.’ *Brookings Papers on Economic Activity*, vol. 4, pp. 429-480.
- **Tesfatsion, L.** (2006). ‘Agent-based Computational Economics: a Constructive Approach to Economic Theory.’ Tesfatsion, L. and K.L. Judd (eds.), *Handbook of Computational Economics*, vol. 2, pp. 831-880 (chapter 16). Amsterdam: Elsevier.