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LOCOMOTIVE SCHEDULING IN FREIGHT TRANSPORT AT MPOPOMA TRAIN STATION IN BULAWAYO FOR THE SOUTHERN REGION, ZIMBABWE.

Daniel Maposa[1] and Siphosenkosi D. Swene[2]
[1]Monash South Africa (A campus of Monash University, Australia), School of Business and Economics, Private Bag X60, Roodepoort, 1725, South Africa. Email: danmaposa@gmail.com or daniel.maposa@monash.edu (Corresponding Author).
[2]National University of Science and Technology, Faculty of Commerce, Department of Insurance and Actuarial Science, P.O. Box AC939, Ascot, Bulawayo, Zimbabwe. Email: syphoed@gmail.com

ABSTRACT

In this paper the results of a locomotive scheduling problem faced by National Railways of Zimbabwe (NRZ) are presented. The study is in part inspired by similar optimization problems in public transports. This paper focuses on the planning version of the locomotive scheduling model (LSM), where there are multiple types of locomotives and a decision is made as to which set of locomotives is to be assigned to each train. The goal is to develop excellent plans along a number of dimensions for NRZ. An integrated model (Model 1) that determines the set of active and deadheaded locomotives for each train, and light travelling locomotives is presented. An important feature of this model is that consist-bustings are explicitly considered. Two other models namely Model 2 and Model 3 are also considered in this paper. A mixed integer programming (MIP) formulation of the problem that contains 20 integer variables and 32 constraints is given (Model 1). Using LINGO 10, a solution technique was developed to solve this problem within 2 minutes of computation time. When the solution was compared with the existing scenario at NRZ, savings of 38 locomotives were realised, which translates into savings of over ten million dollars annually.

Keywords: *National Railways of Zimbabwe, locomotive scheduling, locomotives, mixed integer programming, light travelling, wagons.*

1. INTRODUCTION

Delays of delivery of goods to different destinations have led to loss of customers at National Railways of Zimbabwe (NRZ) to alternative modes of transport in Zimbabwe. Perishable goods usually go bad before delivery due to these delays. Some goods get lost on the way due to vandalism and/or theft and this has negatively affected the organisation's reputation to its customers.

This paper deals with scheduling of locomotives at NRZ, Mpopoma Station, for the wagons in a way that will minimise waiting times of wagons for locomotives, that is, minimise number of idle wagons. The schedule is also aimed at minimising costs incurred with idle locomotives which are usually exorbitantly high, since there are high costs associated with late delivery of goods from customers.

The layout for the rest of the paper is that section 1 has three subsections that follow namely background, definition of terms, and delimitation of the study. Section 2 gives brief review of literature, section 3 gives the methodology, section 4 gives the presentation and analysis of the results. Finally section 5 discusses and concludes the paper.

1.1. Background

The National Railways of Zimbabwe is an integrated multi-disciplinary parastatal enterprise with its Headquarters in Bulawayo. Its main business is the movement of passengers and cargo through rail. Despite the significant successes that NRZ has had, it has been, and in some instances is still, experiencing challenges. NRZ is faced with the problem of locomotive scheduling to haul the wagons, both loaded and unloaded. This has led to the perennial delays of goods trains to the extent of cessation of some train trips. Such delays have forced the parastatal to operate below capacity.

Mpopoma station is the marshalling art of NRZ which means that all freight trains start and terminate at the station. The operations at Mpopoma Station are done sequentially. All trains go to the reception first when they enter the yard. The engine-men are notified by station-masters at the signal cabin as to which rail to move into since the reception has nine rails. At the reception, wagons are received from different points and are checked whether they are still fit to continue with the journey. If found fit, they are then marshalled into either the Departure section or Exchange section depending on destinations.

The Departure section only caters for transit traffic and consists of 16 rails. At this section trains are classified according to their destinations and according to which locomotives are required to haul the wagons. On the other hand, the Exchange section caters for terminal traffic that is going to Bulawayo. The trains at this section are classified according to their destinations namely Belmont, Kelvin North and New-Grain.

At the reception, if the trains are found to be unfit to proceed with their journey, they are marshalled to the Repair Sidings section. Disabled wagons, both loaded and unloaded are fixed at this section. There are 2 occasions where trains are marshalled to this section, that is, when a defect is found or for general service, and scheduled repair. In summary, trains are marshalled to Departure if fit and transit, to the Exchange if fit for Bulawayo, or to Repair Sidings if unfit to proceed with the journey from the reception.

The organisation tries to maintain fluidity in the locomotives. This means that as soon as enough wagons are assembled, the train is started. As a result, the starting times of trains are not aligned to some timetable; they just follow the estimated customers’ productions and demand peaks. This is done to avoid the station from reaching its maximum accommodation capabilities per time period. Therefore it is assumed that locomotives for pulling these trains are always and immediately available.

The NRZ, however, is faced with delay problems in moving loaded and unloaded wagons. One contributing factor to this problem is that there are a limited number of locomotives to haul the wagons. The limited locomotives, on the other hand, are not optimally scheduled to curb these delays as a locomotive may haul wagons of far-fetched different destinations resulting in some goods to be delivered after several days of delay.

The principal aim of this study is to compute feasible starting and arrival times of the trains such that the wagons are transported as fast as possible from their start to the destination within the trains. At intermediate shunting stations the stopover of wagons should not exceed certain limits. The specific objectives of this study are to reduce operating expenses, to use as few locomotives as possible to pull all trains, and to schedule locomotives in such a way that the possibilities of deadheading and light travelling are reduced to a minimum.

It is hoped that the findings of this study will help the management at NRZ realise that they are not providing enough service capacity resulting in excessive waiting and all its unfortunate consequences. It is also hoped that the results of this study will enable the management to schedule locomotives in such a way that the few functioning locomotives at NRZ fully cater for all trips as this will reduce their operating costs.

1.2. Definition of Key Terms

(i) Wagons

A wagon is a rolling stock for freight transport. The wagons have to be delivered between a source and a destination point (goods station) within the network. Large customers produce and/or consume so many goods that they order whole trains. In these cases, the route of the wagons equals the route of the train. Smaller customers order individual wagons and the wagons of different customers are then assembled to trains and pulled as a whole to an intermediate destination (a shunting yard), where the trains are disaggregated and reassembled to new trains. The trains and the yards where the wagons transfer between trains are known in advance. When changing the starting time of the trains, one has to take care that these transfers still remain feasible. The wagons mostly used at this station are Covered Wagons (COV), Drop-Sided Wagons (DSI) and High-Sided Wagons (HSI).

The COV are those wagons that have their tops covered. They are ideal for goods that should not be interfered with either human or nature deeds along the way. The DSI, as the name suggests, have sides that can be dropped. They are ideal for goods that may overlap.

The HSI have high sides. They are ideal for transporting minerals and bulk material such as coal.

(ii) Trains

A freight train (also called production trip) consists of several wagons. Each train has a start and a destination point; the goods stations or railroad shunting yards are the starting and destination points. The starting times and arrival times of the trains are known. These are usually intervals, in which the start or the arrival has to take place. The trip duration is the time difference between start and arrival. The average travel speed of freight trains is not as high as in passenger transport, especially at daytime when passenger trains always have priority, such that some trips can last up to 3 days. The trains have different length and weight and thus require locomotives with sufficient driving power. A train is always pulled by a single locomotive. At the start a locomotive is attached to the train, and at the destination station it is detached (uncoupled). For both coupling processes, a certain train-dependent amount of time has to be taken into account. At this stage, technical checks and refuelling of diesel locomotives are carried out.

(iii) Locomotives

The locomotives mainly used at Mpopoma station are the DE6 and DE9 diesel locomotives. The main difference between the locomotives is the driving power of the engines. The DE6 can haul an average gross load of 1050 tonnes whilst the DE9 can haul an average gross load of 800 tonnes.

(iv) Deadheads

A locomotive is either active (i.e. pulling a train) or deadheading (i.e. driving under its own power without pulling a train from the destination station of one train to the start of another train). The duration for a deadhead trip depends on the distance between these two points and on the type of the locomotive.

(v) Light travelling

Light travel of locomotives is travelling of locomotives in a group on their own between different stations to reposition themselves.

(vi) Consist-busting

A *consist* is a set of locomotives. A consist is said to be *busted* if a set of locomotives on an inbound train is broken into subsets for reassignment to two or more outbound trains (Ahuja et al., 2005). *Consist-busting* is a normal phenomenon in railroads because the needs for outgoing locomotives at a station do not precisely match the incoming needs.

(vii) Marshalling art

In the context of trains, it is the point where all trains start and terminate.

(viii) Transit traffic

This is the traffic that is making a passage or passes across a point. The traffic will not be destined for that point.

(ix) Terminal traffic

This is traffic that has reached its destination, in this case, traffic going around Bulawayo.

(x) Station

This a named place on a railway line where an official is on duty for commercial purposes and/or to control train arrangements.

(xi) Station-master

The station-master is the senior employee in charge of a station.

(xii) Engine-master

An engine-man is the employee in charge of and responsible for the working of a locomotive or motor.

1.3. Delimitations of the Study

Since the wagons work nationwide and are not exactly allocated to particular stations, it is difficult to carry out a survey of them at Mpopoma Station only. This study will assume that wagons were allocated to different stations using proportion of the goods moved from the station and goods moved nationwide versus number of functioning wagons nationwide. The data collection is however focused on the Southern Area of the country which consists of the following regions: Victoria Falls to Somabhula, Heany Junction to West Nicholson and

Bulawayo to Botswana Border. This study concentrates on the train trips in the Southern Area only. The study also assumes that trains start their trips repeatedly every 24 hours and all rail-roads are functioning properly, i.e. no delays due to infrastructure defects are catered for in this study.

2. BRIEF REVIEW OF RELATED LITERATURE

The locomotive scheduling problem is defined by Ahuja et al. (2005) as the assignment of a set of locomotives to each train in a train schedule that is pre-planned in order to provide them sufficient power to pull them from their origins to their destinations. In railroad scheduling problems locomotive scheduling models are among the crucial problems (Kraay et al., 1991; Kraay and Harker, 1995; Winston, 2004; Ahuja et al., 2005; Baceler and Garcia 2006; Tormos et al., 2008; Lusby et al., 2009; Corman et al., 2010; Mu and Dessouky; 2011).

Transportation of goods by railroads is a crucial part of the economy of most developing countries such as Zimbabwe. Cooper (1990) described the rail transportation industry as very rich in terms of problems that can be modelled and solved using mathematical optimisation techniques. Slow growth, however, in research related to railroad scheduling has been experienced and most of the early models failed to incorporate the characteristics of real-life applications.

According to Godwin et al (2006) a common occurrence in many developing countries is that locomotive assignment and freight train scheduling considers the movement of freight trains through a passenger rail network. One of the constraints to be considered in locomotive assignment problems is that passenger trains run according to a fixed schedule while freight trains must be accommodated on the same track without interfering with passenger train movements which are fixed. Aronsson et al. (2006) expressed that the flexibility of scheduling rail freight timetables is greater than that of passenger railways as long as customer requirements are catered for. From the end of the 20th century significant advances in rail freight and passenger transportation have been made (Brannlund et al., 1998; Newton et al., 1998; Baceler and Garcia, 2006; Carey and Carville, 2003; Zhou and Zhong, 2007; Ahuja et al., 2005; Dessouky et al., 2006; Fuchsberger, 2007; Borndörfer and Schlechte, 2008; Cacchiani et al., 2010; Corman et al., 2010; Mu and Dessouky, 2011).

A closely related locomotive scheduling problem is that of Ahuja et al. (2005) who studied the CSX Transportation locomotive assignment problem. CSX Transportation is a major railroad company in the United States of America. Ahuja et al. (2005) studied the problem of assigning locomotives to various trains using mixed integer programming (MIP) which they extended to problem decomposition, integer programming, and very large-scale neighbourhood search in order to get the optimal solution in acceptable running times. Their model involved assigning of a set of locomotives to each train in the weekly train schedule so as to avoid congestion arising from locomotive breakdown resulting in stranded trains blocking the movements of other trains. The assignment of a set of locomotives to a single train is highly restricted to not more than 4 locomotives per train in our National Railways of Zimbabwe (NRZ) assignment problem because only a limited number of locomotives are functioning. The main objective in Ahuja et al. (2005) study was to minimise total cost which comprised the sum of active locomotive costs, deadheading costs and locomotive usage costs. Their solution was intended to provide sufficient power to every train so as to meet their prescribed schedules.

In our NRZ locomotive scheduling problem we use a similar approach to that of Ahuja et al. (2005), where further references are made, but we give more priority to the reduction of idle wagons than reducing deadheading time since high contract penalties for late arrivals have to be paid. The other deviation of our study, as mention earlier, is the inclusion of the aspect of reduction of the number of deployed locomotives.

Other studies relating to locomotive scheduling problems using mixed integer programming are that of Aronsson et al. (2006) who studied locomotive scheduling for Green Cargo Railways, the largest rail operator in Sweden, and Fügenschuh et al. (2006) who studied the locomotive and wagon scheduling for Deutsche Bahn AG in Germany. In all these studies the large-size mixed integer programming (MIP) optimal or near optimal solution could not be found in reasonable running times using commercially available software, so the authors would avoid this problem by using problem decomposition, integer programming, and very large-scale neighbourhood search. This study also uses integer programming to solve the problem of locomotive scheduling at NRZ, but on a smaller scale.

The locomotive scheduling problem at NRZ is similar to the locomotive scheduling problems discussed in this paper where different locomotives are assigned to different wagons. The planning version of the locomotive scheduling problem at NRZ is also similar to the well known South African Railways (SAR) locomotive assignment problem, and the operational version of NRZ is similar to that of Green Cargo locomotive scheduling problem. However, the locomotive scheduling model (LSM) for NRZ although it uses similar approaches to those of the previous studies discussed in this paper, cost functions of having idle wagons and deadheading locomotives are introduced since long idle times of wagons are undesirable. Additionally, the locomotive scheduling model in this paper also include a decision variable that seeks to find the optimal number of locomotives that could potentially be saved if some wagon transfers are missed. Furthermore, the locomotive scheduling problem for NRZ is formulated as an integer multi-commodity flow problem with side constraints. This means that the NRZ model only assigns an integer number of locomotives to trains.

3. METHODOLOGY

The model is formulated as a mixed integer programming model. At Mpopoma Station, there is always a little flexibility in the departure and arrival of the trains, which has to be negotiated with the customers. The model presented aims at a support of strategic simulations for the future, for example, simulating the network load in freight transport in the year 2015, where it is hoped the Zimbabwean economy would have picked up so the National Railways of Zimbabwe will be performing at maximum possible capacity. The corresponding model is called Cyclic Vehicle Scheduling Problem with Time Windows (CVSPTW). This model has to take care of the synchronisation of the train departures, the wagon schedules and transfer of wagons between trains.

3.1. Model Notation

The following notations and terminology are used in this paper as denoted in Ahuja et al. (2005).

(i) Train data: L denotes a set of trains and the index l represents a specific train; T_l is tonnage required of train l; B_l denotes horsepower per tonnage for train l; H_l denotes horsepower requirement of train l; E_l is the penalty for using a single locomotive consist for train l.

(ii) Locomotive data: K denotes the set of all locomotives and the index k represents a specific train; $h^{(k)}$ represents horsepower provided by a locomotive of type k; $\propto^{(k)}$ denotes number of axles in a locomotive of type k; $G^{(k)}$ denotes ownership cost for a locomotive of type k; $F^{(k)}$ is the fixed cost for light travelling; $\beta^{(k)}$ denotes fleet size of locomotives of type k; that is the number of locomotives available for assignment.

(iii) Active and deadheaded locomotives (train-locomotive type combinations data): $c_l^{(k)}$ denotes the cost incurred in assigning an active locomotive of type k to train l; $d_l^{(k)}$ denotes the cost incurred in a deadheaded locomotive of type k to train l; $t_l^{(k)}$ denotes the tonnage pulling capacity provided by an active locomotive of type k to train l.

The active cost $c_l^{(k)}$ comprises the economic asset cost of the locomotive for the duration of the train, the fuel and maintenance costs. The deadheaded cost $d_l^{(k)}$ captures the same economic asset cost, a reduced maintenance cost and zero fuel cost.

For each train l three disjoint sets of locomotives are specified as follows: $Most\ Preferred$ $[l]$ denoting the preferred classes of locomotives; $Less\ Preferred$ $[l]$ denoting the acceptable (but not preferred) classes of locomotives; $Prohibited$ denoting the prohibited classes of locomotives. Only locomotives from the classes listed as $Most\ Preferred$ $[l]$ and $Less\ Preferred$ $[l]$ are assigned to a train. There is also a penalty associated with using $Less\ Preferred$ $[l]$ locomotives (Ahuja et al., 2005).

(iv) Decision variables for the mixed integer programming model (Ahuja et al., 2005): $x_l^{(k)}$ denotes the integer variable representing the number of active locomotives of type $k \in K$; $y_l^{(k)}$ denotes the integer variable indicating the number of non-active locomotives deadheading, light-travelling or idling of type $k \in K$; z_l is the binary variable which takes value 1 if at least one locomotive is connected, and 0 otherwise; w_l denotes the binary variable which takes value 1 if there is a flow of a single locomotive on train routes (arcs), and 0 otherwise; $s^{(k)}$ denotes the integer variable indicating the number of unused locomotives of type $k \in K$.

3.2 Data Collection

The data given in the following Tables 1, 2, 3 and 4 were supplied by the Strategic Planning Branch of National Railways of Zimbabwe. For train data the specific trains (l) are HSI, DSI and COV while for locomotive data the particular locomotives (k) are DE6 and DE9.

Table 1: Required minimum tonnage and horsepower for trains

l	Required Tonnage (tonnes): T_l	Horsepower per 20 tonnes (Watts): B_l	Horsepower (Watts): $H_l = B_l T_l$	Penalty (US$): E_l
HIS	180	756	136080	250
DSI	165	756	124740	200
COV	175	756	132300	230

Table 2: Locomotive properties data

k	Horsepower: $h^{(k)}$	Number of axles: $\alpha^{(k)}$	Ownership costs: $G^{(k)}$	Fleet size: $\beta^{(k)}$	Fixed cost for light travelling: $F^{(k)}$
DE6	39690	16	550	40	400
DE9	30240	14	300	38	400

Table 3: Data for active and deadheaded locomotives

k	l	Costs for active locomotives: $c_l^{(k)}$	Costs for deadheaded locomotives: $d_l^{(k)}$	Tonnage pulling capacity: $t_l^{(k)}$
DE6	HSI	350	500	350
DE6	DSI	350	500	350
DE6	COV	350	500	350
DE9	HSI	350	500	350
DE9	DSI	350	500	350
DE9	COV	350	500	350

Table 4: Preferences of different trains for different locomotives

l	$Most\ Preferred\ [l]$	$Less\ Preferred\ [l]$	$Prohibited\ [l]$
HIS	DE6	-	DE9
DSI	DE9	DE6	-
COV	DE6	DE9	-

For the HSI wagon, the DE9 locomotive is prohibited to haul it due to the fact that the HSI wagon usually hauls heavier tonnages than the DE9 locomotive is capable to haul. In most cases, if a DE9 locomotive is allowed to haul the HSI wagon it fails before it reaches its destination especially with long distances. The DE9 locomotive is preferred with the DSI wagon and less preferred with the COV wagon.

3.3. Mixed Integer Programming (MIP) Model

The constraints used in this paper for the locomotive scheduling model (LSM) are same as those given and explained in Ahuja et al. (2005) except that for the consist size constraints at most 4 locomotives (including the active and deadheaded) can be assigned to a train according the NRZ business policy. The objective function also comprise the terms given in Ahuja et al. (2005), that is; cost of ownership, maintenance, and fuelling of locomotives; cost of active and deadheaded locomotives; cost of light travelling locomotives; penalty for consist-busting; penalty for inconsistency in locomotive assignments and train-to-train connections; and penalty for using single locomotive consists.

The general objective function for the mixed integer programming (MIP) model is given as (Ahuja et al., 2005):

$$Min\ Z = \sum_{l\in L}\sum_{k\in K} c_l^{(k)} x_l^{(k)} + \sum_{l\in K}\sum_{k\in K} d_l^{(k)} y_l^{(k)} + \sum_{k\in K}\sum_{l\in L} F^{(k)} z_l + \sum_{l\in L} Bz_l + \sum_{l\in L} E_l w_l - \sum_{k\in K} G^{(k)} s^{(k)},$$

where the first term denotes the cost of actively pulling the wagons on train routes; the second term captures the cost of deadheading locomotives on train and light travelling routes, cost of idling locomotives and cost of consist-busting; the third term denotes the fixed cost of light travelling locomotives; the fourth term $\sum_{l\in L} Bz_l$ represents the fixed cost of consist-busting; the fifth term denotes the penalty associated with single locomotive consists; and the sixth term denotes the savings accrued from not using all the locomotives i.e. saving some other locomotives.

Using the data given in Tables 1, 2, 3, and 4 we get a mixed integer programming (MIP) model with 20 decision variables and 32 constraints. The simplified objective function for the MIP model (referred to in this paper as Model 1) is:

$$\begin{aligned} Min\ Z(Total\ costs) &= 350\left[x_{HSI}^{(DE6)} + x_{DSI}^{(DE6)} + x_{COV}^{(DE6)} + x_{HSI}^{(DE9)} + x_{DSI}^{(DE9)} + x_{COV}^{(DE9)}\right] \\ &+ 500\left[y_{HSI}^{(DE6)} + y_{DSI}^{(DE6)} + y_{COV}^{(DE6)} + y_{HSI}^{(DE9)} + y_{DSI}^{(DE9)} + y_{COV}^{(DE9)}\right] + 756[z_{HSI} + z_{DSI} + z_{COV}] \\ &+ 250w_{HSI} + 200w_{DSI} + 230w_{COV} - \left[550s^{(DE6)} + 300s^{(DE9)}\right] \end{aligned}$$

Subject to the constraints

Constraint set **1**

$$350\left[x_{HSI}^{(DE6)} + x_{HSI}^{(DE9)}\right] \geq 180$$
$$350\left[x_{DSI}^{(DE6)} + x_{DSI}^{(DE9)}\right] \geq 165$$
$$350\left[x_{COV}^{(DE6)} + x_{COV}^{(DE9)}\right] \geq 175$$

This constraint set ensures that the locomotives assigned to a train provide the required minimum tonnage.

Constraint set **2**

$$39690x_{HSI}^{(DE6)} + 30240x_{HSI}^{(DE9)} \geq 136080$$
$$39690x_{DSI}^{(DE6)} + 30240x_{DSI}^{(DE9)} \geq 124740$$
$$39690x_{COV}^{(DE6)} + 30240x_{COV}^{(DE9)} \geq 132300$$

This constraint set ensures that the assigned locomotives provide the required minimum horsepower.

Constraint set **3**

$$16x_{HSI}^{(DE6)} + 14x_{HSI}^{(DE9)} \geq 24$$
$$16x_{DSI}^{(DE6)} + 14x_{DSI}^{(DE9)} \geq 24$$
$$16x_{COV}^{(DE6)} + 14x_{COV}^{(DE9)} \geq 24$$

This constraint set models the constraint that the number of active axles assigned to a train does not exceed 24.

Constraint set **4**

$$x_{HSI}^{(DE6)} + x_{HSI}^{(DE9)} + y_{HSI}^{(DE6)} + y_{HSI}^{(DE9)} \leq 12$$
$$x_{DSI}^{(DE6)} + x_{DSI}^{(DE9)} + y_{DSI}^{(DE6)} + y_{DSI}^{(DE9)} \leq 12$$
$$x_{COV}^{(DE6)} + x_{COV}^{(DE9)} + y_{COV}^{(DE6)} + y_{COV}^{(DE9)} \leq 12$$

This constraint set ensures that every train is assigned no more than 12 locomotives.

Constraint set **5**

$$y_{HSI}^{(DE6)} + y_{DSI}^{(DE6)} + y_{COV}^{(DE6)} + y_{HSI}^{(DE9)} + y_{DSI}^{(DE9)} + y_{COV}^{(DE9)} \leq 4[z_{HSI} + z_{DSI} + z_{COV}]$$

This constraint set (5) ensures that no more than 4 locomotives are assigned to a single train according to the business policy of National Railways of Zimbabwe (NRZ). The fixed charge variable on the right hand side of the inequality z_l becomes 1, that is, the whole bracket translates to 1, whenever a positive flow takes place on a connection route or a light route.

Constraint set **6**

$$z_{HSI} + z_{DSI} + z_{COV} = 1$$

This constraint states that for each inbound train, all the inbound locomotives use only one connection route, that is, either all locomotives go to the associated ground node (in which case consist-busting takes place) or all locomotives go to another outbound train (in which case there is train-to-train connection and no consist-busting) (Ahuja et al. 2005).

Constraint set **7**

$$x_{HSI}^{(DE6)} + x_{HSI}^{(DE9)} + y_{HSI}^{(DE6)} + y_{HSI}^{(DE9)} + w_{HSI} \geq 2$$

$$x_{DSI}^{(DE6)} + x_{DSI}^{(DE9)} + y_{DSI}^{(DE6)} + y_{DSI}^{(DE9)} + w_{DSI} \geq 2$$
$$x_{COV}^{(DE6)} + x_{COV}^{(DE9)} + y_{COV}^{(DE6)} + y_{COV}^{(DE9)} + w_{COV} \geq 2$$

This constraint set (7) makes the variable w_l equal to 1 whenever a single locomotive consist is assigned to train l.

Constraint set **8**

$$x_{HSI}^{(DE6)} + y_{HSI}^{(DE6)} + s^{(DE6)} = 40$$
$$x_{HSI}^{(DE9)} + y_{HSI}^{(DE9)} + s^{(DE9)} = 38$$
$$x_{DSI}^{(DE6)} + y_{DSI}^{(DE6)} + s^{(DE6)} = 40$$
$$x_{DSI}^{(DE9)} + y_{DSI}^{(DE9)} + s^{(DE9)} = 38$$
$$x_{COV}^{(DE6)} + y_{COV}^{(DE6)} + s^{(DE6)} = 40$$
$$x_{COV}^{(DE9)} + y_{COV}^{(DE9)} + s^{(DE9)} = 38$$

This constraint set counts the number of locomotives used in a week. The difference between the number of locomotives available and the number of locomotives used gives the number of locomotives saved $\left(s^{(k)}\right)$.

Constraint set **9**

$$x_{HSI}^{(DE9)} = 0$$

This constraint set ensures that the prohibited locomotives are never used on train routes.

Constraint set **10**

$$x_{HSI}^{(DE6)}, x_{DSI}^{(DE6)}, x_{COV}^{(DE6)}, x_{HSI}^{(DE9)}, x_{DSI}^{(DE9)}, x_{COV}^{(DE9)} \geq 0 \; and \; integer$$
$$y_{HSI}^{(DE6)}, y_{DSI}^{(DE6)}, y_{COV}^{(DE6)}, y_{HSI}^{(DE9)}, y_{DSI}^{(DE9)}, y_{COV}^{(DE9)} \geq 0 \; and \; integer$$

This constraint set ensures that all the numbers of active and non-active (light traveling or deadheading) locomotives are non-negative integers.

Constraint set **11**

$$z_{HSI} \in \{0,1\}$$
$$z_{DSI} \in \{0,1\}$$
$$z_{COV} \in \{0,1\}$$

Constraint set **12**

$$w_{HSI} \in \{0,1\}$$
$$w_{DSI} \in \{0,1\}$$
$$w_{COV} \in \{0,1\}$$

3.4. Sensitivity Analysis

In order to obtain high quality feasible solutions and to keep the total running time of the algorithm small, the fixed charge variables from mixed integer programming (MIP) Model 1 were eliminated using heuristics. In Model 1 there are two kinds of fixed charge variables; one corresponding to light travelling and the other corresponding to deadheading. In Model 2 we consider the fixed charge variables corresponding to light travelling, and in Model 3 we consider the fixed charge variables corresponding to deadheading.

In order to determine the effect of light travelling (Model 2) we eliminate the fixed charge variables corresponding to light travelling in the procedure. The mixed integer programming (MIP) Model 2 is the same as Model 1 except that Model 2 excludes all fixed charge variables corresponding to light travelling. Model 2 has been reduced to 17 decision variables and 29 constraints. The objective function for Model 2 will then be given as:

$$\begin{aligned} &Min \; Z(Total \; costs) \\ &\quad = 350\left[x_{HSI}^{(DE6)} + x_{DSI}^{(DE6)} + x_{COV}^{(DE6)} + x_{HSI}^{(DE9)} + x_{DSI}^{(DE9)} + x_{COV}^{(DE9)}\right] \\ &\quad + 500\left[y_{HSI}^{(DE6)} + y_{DSI}^{(DE6)} + y_{COV}^{(DE6)} + y_{HSI}^{(DE9)} + y_{DSI}^{(DE9)} + y_{COV}^{(DE9)}\right] + 250w_{HSI} + 200w_{DSI} \\ &\quad + 230w_{COV} - \left[550s^{(DE6)} + 300s^{(DE9)}\right] \end{aligned}$$

All the constraint sets remain the same as in Model 1 with the exception of constraint sets 6 and 11 which are be eliminated.

To determine the effect of deadheading (Model 3) the fixed variables corresponding to deadheading are eliminated in the procedure. The MIP Model 3 is also same as Model 1 except that Model 3 excludes all fixed charge variables corresponding to deadheading. This model has 14 decision variables and 21 constraints. The objective function for Model 3 will then be given as:

$$\begin{aligned}Min\ Z(Total\ costs) &= 350\left[x_{HSI}^{(DE6)} + x_{DSI}^{(DE6)} + x_{COV}^{(DE6)} + x_{HSI}^{(DE9)} + x_{DSI}^{(DE9)} + x_{COV}^{(DE9)}\right] + 756[z_{HSI} + z_{DSI} + z_{COV}] \\ &+ 250w_{HSI} + 200w_{DSI} + 230w_{COV} - \left[550s^{(DE6)} + 300s^{(DE9)}\right]\end{aligned}$$

All the other constraint sets in Model 1 are retained in Model 3 except for constraint set 4, 5, 7 and 8 which are eliminated. The following section presents the results of all the three models as well as the results of the existing model for National Railways of Zimbabwe.

4. PRESENTATION AND ANALYSIS OF RESULTS

The data provided by National Railways of Zimbabwe (NRZ) specified that there are 528 trains, each of which operates several days in a week and 2 locomotive types available for its day to day business. The optimal solutions to the models given in section 3 are presented in this section. LINGO 10, an optimization package, was used to arrive at the optimal solutions of the mixed integer programming models. The number of trains running with different frequencies in a week for NRZ was analysed for the purpose of making improvements to the model. The results are presented in Table 5. Results of sensitivity analysis were also presented and comparison of models were done in order to find improvements to the model (i.e. model 1).

Table 5: Analysis of trains and their frequencies.

Train Frequency (days): P	Number of trains: N	P x N	Cumulative sum of P x N	Cumulative Percentage of P x N
7	201	1407	1407	55%
6	54	324	1731	67%
5	60	300	2031	79%
4	48	192	2223	86%
3	65	195	2418	94%
2	60	120	2538	98%
1	40	40	2578	100%

The first column in Table 5 gives the train frequency in a week, that is, how often the train runs in a week. The second column of Table 5 gives the number of trains in the train schedule, for example, there are 201 trains that run all 7 days in a week. Table 5 shows that 79% of the trains corresponds to the trains that run at least 5 days a week, that is, 5, 6, or 7 days. Trains that run at least 5 days in a week are important in our modelling for consistency.

4.1. Computational Results of Model 1

Model 1 was coded into LINGO 10 and the summary of the results of the weekly scheduling problem is given in Table 6, 7 and 8. Table 6 shows that the active locomotive is the DE6 which is connected to any of the 3 types of wagons. No non-active locomotives are obtained by this model (model 1) for both the DE6 and DE9 locomotives as shown in Table 6. Table 7 shows that the possibility of having at least one locomotive connected to wagons is only possible for the COV wagon, and the possibility of flowing of a single locomotive is also only possible for the COV wagon. Table 8 shows the number of locomotives saved using Model 1. Relative to the number of locomotives available for assignment at National Railways of Zimbabwe, the number of saved locomotives using Model 1 is quite reasonable. The objective function value solution for Model 1, which is the total cost in the weekly scheduling, is $35 156. The problem size of Model 1 took 2 minutes of solution time using LINGO 10 to obtain the optimal integer solution.

Table 6: Summary results of the number and type of locomotives used in the weekly schedule

Type of locomotives	Type of wagon	Number of active locomotives	Number of non-active locomotives
DE6	HSI	2	0
DE6	DSI	2	0
DE6	COV	2	0
DE9	HSI	0	0
DE9	DSI	0	0
DE9	COV	0	0

Table 7: Summary of the number and type of wagons used in the weekly schedule

Type of wagon	At least on locomotive connected	Flow of a single locomotive
HSI	0	0
DSI	0	0
COV	1	1

Table 8: Summary of the number and type of unused locomotives used in weekly schedule

Type of locomotives	Number of unused locomotives
DE6	38
DE9	38

4.2. Computational Results of Model 2 (Light Travelling)

In this subsection we summarise the results of the effect of light travelling, Model 2. The results for the number and type of locomotives used in the weekly schedule were exactly the same as those of Model 1 in Table 6. Thus results in Model 2 also indicate that the active locomotive is DE6 which is connected to any of the 3 types of wagons. Also no non-active locomotives were obtained by this Model. Results for Model 2 showed no possibility of flowing of a single locomotive for all wagons as opposed to Model 1. In Model 2 the number of unused locomotives of type DE6 and DE9 were 25 and 30 respectively, which is quite sensible relative to the number of locomotives available for assignment at NRZ. The objective function value solution for Model 2, which is the total cost in the weekly scheduling after eliminating the fixed charge variables of light travelling, is $34 400. The problem size of Model 2 took 1 minute to obtain the optimal integer solution using LINGO 10.

4.3. Computational Results for Model 3 (Deadheading Effect).

A summary of the results of the number and type of locomotives used in the weekly schedule of Model 3 is given in Table 9. Results in Table 9 show a random distribution of the number of active locomotives which are connected to any of the 3 types of wagons. The DE9 locomotive has no assignments of the HSI wagon allocated to it since it is prohibited to haul the HSI wagon. As was the case with Model 2, Model 3 also has no possibility of flowing of a single locomotive on any connections to wagons. The number of saved (unused) locomotives in Model 3 of type DE6 and DE9 were 30 and 32 respectively which is quite sensible relative to the number of available locomotives for assignment. The objective function value solution for Model 3 is $28 900. This is the total cost in the weekly scheduling after eliminating the fixed charge variables for deadheading. Model 3 problem size took only 45 seconds using LINGO 10 to obtain the optimal integer solution.

Table 9: Summary results of the number and type of locomotives used in the weekly schedule for Model 3

Type of locomotives	Type of wagon	Number of active locomotives	Number of non-active locomotives
DE6	HSI	2	0
DE6	DSI	1	0
DE6	COV	1	0
DE9	HSI	0	0
DE9	DSI	2	0
DE9	COV	2	0

4.4. Comparison between the three models and the existing system

The three models proposed (Model 1, Model 2 and Model 3) were compared with the existing system in order to find the effect of the inclusion or omission of fixed charge variables in the models. Figure 1 shows the comparison of active DE6 locomotives connected to 3 different types of wagons for the 3 models and the existing system at NRZ. From Figure 1 it can easily be seen that Model 1 and Model 2 have equal numbers of active locomotives for DE6 connected to 3 different types of wagons per time period. Model 3 on the other hand, has lesser active locomotives compared to all the other models. The existing number of active locomotives is notably greater than that of all the proposed models. Thus the proposed models achieved a dramatic decrease in the number of locomotives in comparison to the existing system.

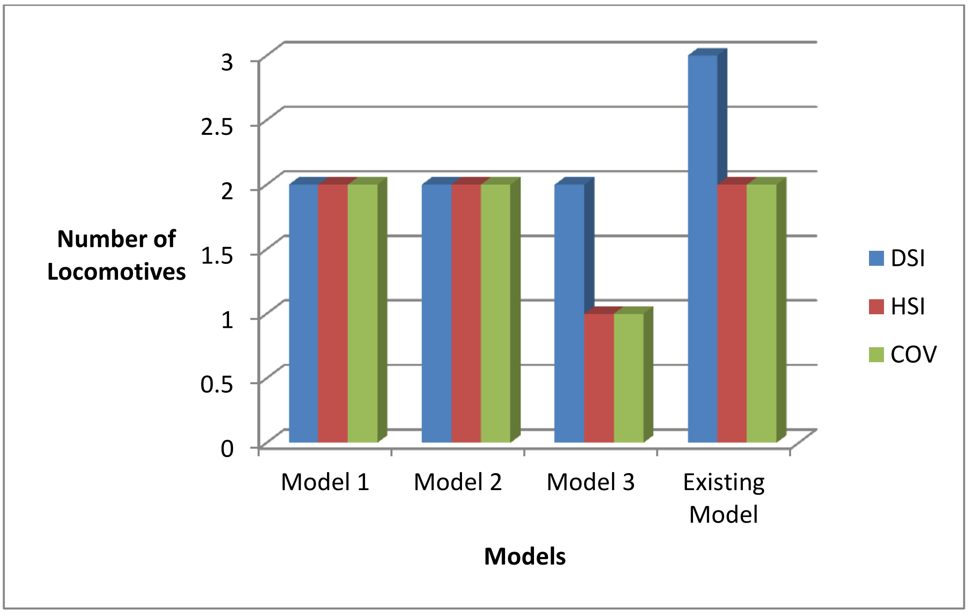

Figure 1: Number of active DE6 locomotives per model for the 3 types of wagons

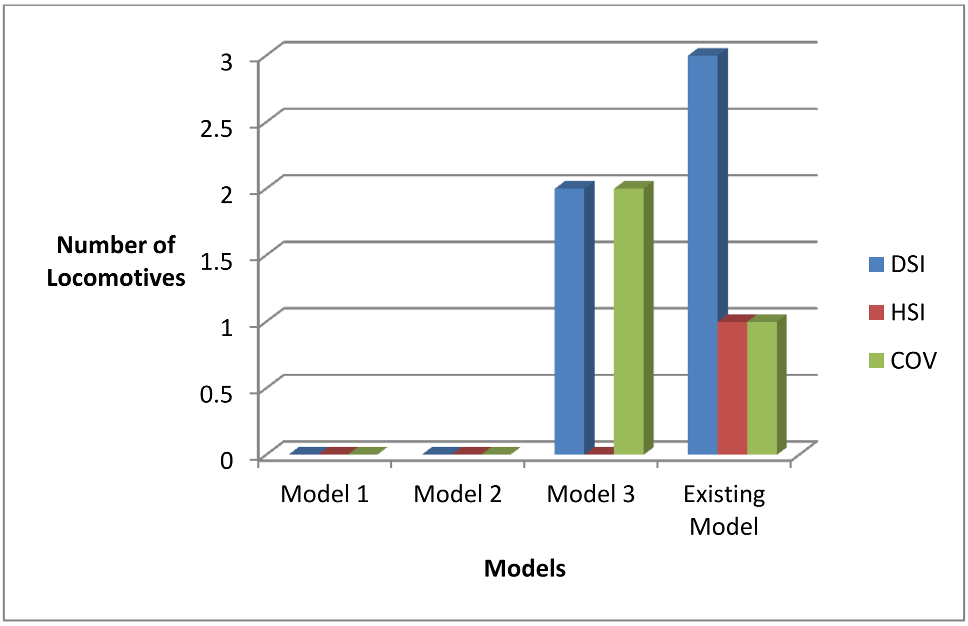

Figure 2: Number of active DE9 locomotives per model for the 3 types of wagons

Results from Figure 2 show that Model 1 and Model 2 have no active DE9 locomotives connected to 3 different types of wagons per time period. On the hand, Model 3 has more active DE9 locomotives compared to Model 1 and Model 2. The existing model, once again has generally the most number of active DE9 locomotives relative to the proposed 3 models. Figure 3 shows the results of the comparison of the number of unused (or saved) locomotives for the 3 models and the existing model. It can easily be seen from Figure 3 that Model 1 has the greatest number of unused locomotives, followed by Model 2. Model 3 does not consider unused locomotives (deadheading) thus it is not included in the graph. The existing model however has the least number of unused (saved) locomotives, meaning that it is wasteful.

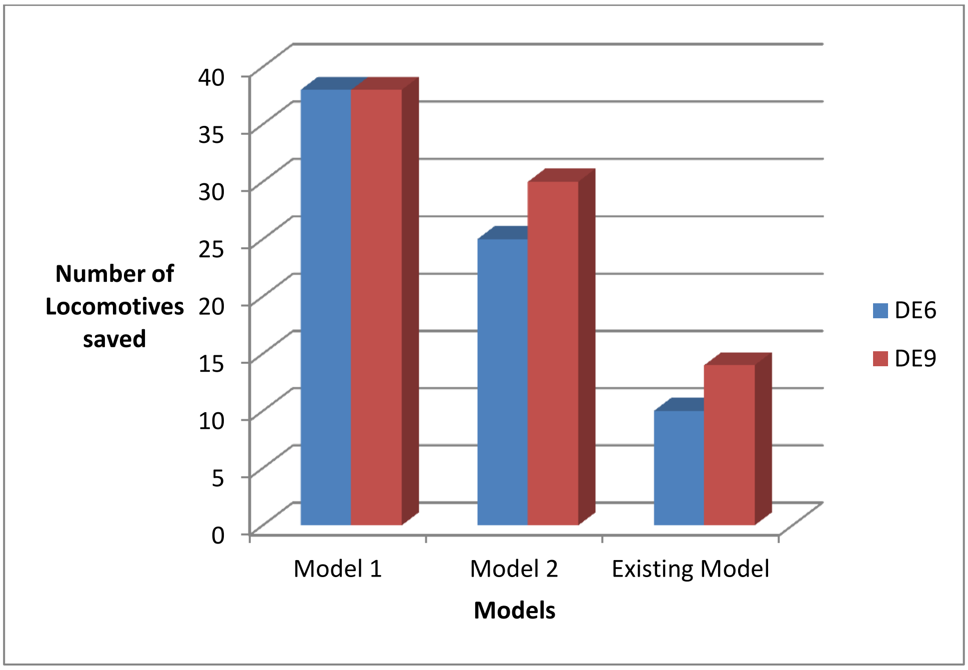

Figure 3: Number of unused (saved) locomotives per model

5. DISCUSSION AND CONCLUSION

The results obtained in this study reveal that the 3 models significantly increase the locomotive productivity. The dramatic decrease in the number of locomotives for the proposed models relative to the existing system can be attributed to the fact that currently there is no particular schedule used at NRZ for assigning locomotives to wagons, that is, if enough wagons are assembled any locomotives available are assigned irrespective of the possibilities of light travelling, deadheading or the associated costs. The other reason for the significant decrease in the number of active DE9 locomotives for the proposed models relative to the existing model is that the proposed models consider the fact that the DE9 locomotive is weaker in terms of haulage capacity and speed as compared to DE6.

In order to satisfy the first objective which is to reduce operating costs, the model that does not include deadheading and the costs associated with it will be the most optimal, followed by the model that excludes light travelling. To satisfy the second objective which is to use as few locomotives as possible, which is equivalent to increasing the number of unused locomotives, the most optimal model will be the weekly schedule model (Model 1) that includes both light travelling and deadheading, followed by the model that excludes deadheading. To satisfy the final objective which is to reduce the possibility of deadheading and light travelling only Model 1 is feasible as it is the only model that includes both the variables of light travelling and deadheading.

In light of the results presented in section 4 and discussions in this section the optimal model that can be recommended to National Railways of Zimbabwe is Model 1, that is to say most wagons should be connected to the DE6 locomotive as it has a greater hauling capability, greater speed and generally stronger than the DE9 locomotive. The DE9 locomotive is prone to frequent failures. If connection is made to the DE9 locomotive, some DE6 locomotives should be connected as well so that should the DE9's fail, there is immediate backup. In that way, quite a number of locomotives will be saved due to the fact that as few locomotives as possible are used.

The model helped quantify the network benefits of re-centring the fleet composition. In the future, NRZ should consider expanding on this work to include locomotive fuelling and servicing constraints into the model.

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