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Music, the Body in Time, and Self-Similarity Concepts

Daniel J. Schneck

Virginia Polytechnic Institute and State University, Blacksburg, Virginia, USA
Address correspondence to Daniel J. Schneck, adanielj@vt.edu

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Abstract It is shown in this paper that the structural configuration of music exactly parallels the structural configuration of the human body. Music and biological systems share *time* as a common denominator, since both derive from, and exist in the time dimension. Music and biological systems also share as a common denominator certain features of *self-similarity* that can be quantified by power-law formulations. Discussed herein is how further investigations into these two common features—*self-similarity* as an architectural property of both music and the human body, and *space-time* as common dimensions of perception—can provide some insights into specific mechanisms by which the six elements of music—rhythm, melody, harmony, timbre, dynamics, and form—succeed in eliciting profound physiological responses. Once these mechanisms are understood, they can be exploited to develop diagnostic protocols which, in turn, can provide a scientific basis for using music as a clinical intervention in a variety of diagnosed populations.

Keywords music; anatomy/physiology; space-time; self-similarity; power-law formulations; music therapy; cardiovascular system

1 Introduction

There is no question that music *derives* from and exists in the domain of time (more accurately, temporal *frequencies*). Indeed, *rhythm*—the "beat" (*pulse*), the "tempo" (*pace*) and "rhythmic patterns" (*pulse trains*)—is the most basic element of music (Schneck and Berger [12]). Sound *frequency* establishes musical *pitch*; frequency *ratios* define musical *intervals* (and associated harmonics and overtones); frequency *sequencing* produces *melodic lines*; super-imposed frequency *series* give sound its *quality* (timbre), and so on. Thus, whereas there is definitely a *spatial* architectural aspect to the structure of music, its existence in the dimension of *time* plays a major role in terms of physiologic responses to musical inputs. That is to say, there is also no question that biological systems, too, exist and *function* in the domain of time (Schneck [9]).

Indeed, *biorhythms* (circadian, circannual, etc.) are perhaps the most basic element of physiology; and nerve action-potential *frequencies*, morse-code-like frequency *patterns*, and super-imposed frequency *series* that derive from nerve *networks* determine physiologic responses to light, heat, sound pressure, and so on. Thus, there is no doubt that the two—music and biological systems—share *time* and temporal *frequency* as common denominators, which already establishes a symbiotic connection between them. However, what needs to be worked out in greater detail are specific *mechanisms* by which

(a) the time-dependent elements of music, interact with
(b) the time-dependent functional processes in biological systems, to
(c) elicit profound effects that can be exploited for many purposes, including clinical intervention in a variety of diagnosed populations (Schneck and Berger [12]).

A great deal of good work is being done by scientists endeavoring to identify these mechanisms, rigorously quantify them, and apply them for specific purposes (Schneck and Berger [12], and, see also, among many others, Zatorre and Peretz [17] and Hodges [6]). However, as of this writing, success in this effort still depends on the need to know much more than is currently known about the structure and function of biological systems such as the human body. As described in the next section, some of the connections between music and physiologic function are obvious; others, as we will see, are more subtle, but no less significant. With respect to the latter, it will be shown in this paper that some basic insights can be gained by relating physiologic *time* considerations to anatomic *spatial* considerations, and vice versa, that is, extrapolating to what is happening in *time* from what we learn about what is going on in *space*. It is further suggested that a plausible explanation for these space-time and music/physiology connections is embedded in the general concept of self-similarity.

2 "Obvious" connections between music and physiologic function

The hearing architecture of humans is generally capable of perceiving sound energy in the frequency range from 20 to 20,000 cycles per second (Hertz, Hz). Within this range lie the human singing voice (80–1050 Hz), speech sounds (300–3000 Hz) and a wide variety of musical instruments, encompassing the ten-octave range from C_0 (16 Hz) to C_{10} (16,384 Hz), but more commonly, in the tempered range of the 88-key piano, that is, from A_0 (27.5 Hz) to C_8 (4166 Hz based on $A_4 = 440$ Hz).

What 'jumps out' at one is the one-to-one correspondence of these frequency ranges with, specifically, neural action potentials, synaptic transmission rates, and physiological information-processing rates that typically fall in the range 0.5–2500 cycles per second. Now, it is true that the overall range of cyclic bodily functions is actually much wider than the range of human hearing. For example, cardiovascular, respiratory, and other "vital signs," along with gait patterns and activities of daily living, typically extend the lower frequencies well below 20 Hz; and many biochemical reactions proceed at rates well above 20,000 Hz. However, other biochemical reaction rates, together with most cellular transport processes and neuro-musculo-skeletal periodicities do correspond to musical frequencies (Schneck [9, 10]).

Moreover, whereas many physiologic time scales are *endogenous* to the body (i.e. independent of external cues), and are *autonomic* (i.e. "self-governed," not driven by ambient rhythms), they are, nevertheless amenable to being *influenced* (driven) by external stimulation that can derive from the elements of music (Schneck and Berger [12]). For example, because of the approximately 0.0005 s (range 0.4–1.0 ms) that it takes for a nerve to "reload" enough to "fire" again once it has completely discharged (called the *absolute refractory period*), nerves are generally constrained to fire, at most, 1000–2500 times per second—corresponding to musical pitches from around B_5 (987.767 Hz in a tempered scale based on $A_4 = 440$ Hz), to $D_7^{\#}$ (2489.016 Hz). However, in "real life," maximum neural transmission rates beyond 700–800 action potential "spikes" per second ($\approx F_5(698.457\,\text{Hz}) - G_5(783.991\,\text{Hz})$) are rarely encountered, and *most* neurons fire at rates closer to 300 temporally-sequenced impulses per second, which corresponds to an A-440 pitch of around D_4 (293.665 Hz). Furthermore, when we take into account attenuation and consolidation through nerve networks, and sorting and filtering by the Reticular Activating System (Schneck and Berger [12]), it is more likely that responses derived from any given input modality result from "higher centers" having actually attended to and processed only 50, to as few as 5 *integrated* pieces of information per second, per modality, with corresponding time scales on the order of 20–200 ms. The significance of these numbers is that they take us down to the threshold of human hearing, and are consistent with the facts that:

- Fast-twitch skeletal muscle fibers achieve maximum contracted lengths in 15–50 ms (time-to-peak) following a single stimulation (Schneck [10]);
- slow-twitch muscle fibers require 60–120 ms to reach peak contraction (Schneck [10]);
- it takes about 100 ms to establish a good image on the retina;
- 50–100 ms are required for the hearing apparatus to "lock onto" and identify an incident sound frequency;
- a typical blink is completed in about 200 ms;
- at rest, the human heart actively completes a single "beat" in about 250 ms, passively "resting" for some 500 ms in-between beats;
- it takes about 5 s to inhale (2 s) and exhale (3 s) once, at rest;
- arterioles contract and relax in 2–8 s cycles.

Of further significance here is the concept of *phase regulation*—an optimization scheme whereby the activity of one set of anatomical structures is *coordinated* and *synchronized* with that of another (leading to a phenomenon known as *co-acting*), in order to minimize the energetic demands of physiologic function.

In phase-regulation, we find that the operating *frequency bandwidths* of individual organs or tissues are not *independent* of one another. Rather, they are *co-dependent*—meaning that the operating frequencies of said organs and tissues are related to one another by, in most cases, simple whole number ratios. This is not unlike those frequency *ratios* that characterize musical *intervals*, such that, like music, there is most likely a type of *harmonic order* to physiologic function (Alexjander and Deamer [1]). As an illustration of phase regulation, consider that up to 30% of the heart's energy is conserved by allowing

- the ejection of blood from this organ to be *timed* so that it is in-phase with the inherent rhythmic oscillations of the elastic arterial system and
- the beginning of the respiratory cycle to be *timed* to coincide with the end of one of the ejection phases of the cardiac cycle.

Such energy-conservation is enhanced by maintaining the *ratio* of the cardiac period to the arterial pulsation period at approximately 2:1 (a perfect octave in the musical realm!).

Consider further that the musculature of the stomach "cranks out" a new peristaltic wave every 20 s, but the wave *lasts* about a minute, so that three waves at a time travel down this organ, and the ratio of peristaltic-duration to muscular-period is about 3:1. Moreover, for the cardiovascular and respiratory systems, the whole-number-ratio for several system parameters is optimized at 4:1 (a "double-octave"). For instance, although during the day, the ratio of pulse-rate-to-respiration-rate can vary from 2.5 to 7.0, it equilibrates to a value of 4.0 between midnight and 3 a.m., when physiologic function is optimized according to a so-called *nightly normalization of the rhythmic functional order* principle that prevails during restful sleep. In fact, harmonic co-acting (synchronization) of *all* physiologic processes is especially prevalent during sleep—as is a decrease in physiological *complexity*, commensurate with the absence of conscious activity. This allows harmonic ratios to be the simplest ones possible, thus reducing and optimizing the energetic demands of the organism. Result: a state of *physiologic consonance* that also optimizes healing. On the other hand, *stress*, during one's wakeful hours upsets this balance/equilibrium, leading to *physiologic dissonance*, with all of its adverse consequences on health and well-being.

Bottom line: there are some obvious connections between the elements of music—rhythm, melody, harmony, timbre, dynamics, form—and physiologic function, as embedded in *biorhythms* that include, for example,

- the cyclic beating of the heart;
- periodicity of the firing rates in neurons;
- rhythmic breathing patterns;
- oscillating peristaltic movements in the gastrointestinal system;
- diurnal variations in body temperature;
- timed release of urine (bladder function) and fecal wastes (rectal function);
- timed release of endocrine hormones to correspond with their role in circadian metabolic processes;
- sleep/wake cycles;
- periodic 90-min sleep stage cycles;
- female menstrual cycles;
- patterns of gait;
- the 4-to-6-hr cravings for food (hunger cycles).

But the connections actually go much deeper than that, as we glean from an examination of *spatial* anatomic engineering design principles.

3 Spatial considerations that can be deduced by examining the human arterial system

Back in the mid-1970's, I set out to develop an engineering model of the morphometry of the human arterial system (Schneck [11]). So, off I went to map, for several anatomical vascular beds, the following:

(i) branching *configuration*, B (e.g. $B = 2$ defines a bifurcating network, $B = 3$, a trifurcating one, etc.);

(ii) total number, N, of branching *generations* per vascular tree, where a "generation," m ($m = 0, 1, 2, \ldots, N$), is defined to be any one of a number of successive downstream branchings—from the parent vessel ($m = 0$) spawning a specific vascular bed, to the downstream distribution of capillaries ($m = N$) that derive from that particular parent vessel;

(iii) total number, n, of *daughters* in any generation m;

(iv) mean internal-diameter, D_m, and characteristic length, L_m, for a "typical" blood vessel in generation m.

I postulated (see later) that D_m and L_m could be expressed in terms of parent dimensions, D_0 and L_0 by the *power-law expressions*: $D_m = D_0 r^m$ and $L_m = L_0 R^m$, where "r" represents a characteristic *scaling factor for internal diameter*, and "R" a characteristic *scaling factor for mean vessel length*; n, B, and m were also made to fit the *power-law equation*: $n = B^m$.

After I accumulated an enormous amount of published data for the following circulatory pathways:

- pulmonary—the lung, specifically, its blood-*processing* pathways,
- renal: the kidney,
- coronary: the heart, specifically, its *feeding* network,
- bronchial—the lung again, but this time, its *nourishing* pathways,
- and cerebral: the brain

an interesting pattern began to emerge which, in fact, carries over one-to-one into the time domain, and into music, as we will see later. That is to say, with remarkable consistency, r fell within the narrow range 0.600 (for the coronary circulation) and 0.616 (for the renal circulation). Given the margin of error in both accumulating the anatomical data and calculating these values for "r", there was a glaring similarity between the values of the scaling factors, r, and the value of a scaling factor that routinely appears in studies of *self-similar systems*, that is, the factor $r = 0.618$.

Actually, 0.618 is rounded-down from the more exact irrational number $[2 \div (1 + \sqrt{5})] = 0.618033989\ldots = (1/x)$, where $x = 1.618033989$. The latter is the *positive* solution to the equation, $x^2 - x - 1 = 0$. This equation derives from the observation of the Greek mathematician Euclid (c. 365–265 B.C.) that, given any two quantities, x and y, where $x > y$, the ratio of $(x + y)$-to-x is equal to the ratio of x-to-y, that is, $[(x + y)/x] = (x/y)$, which, for y normalized to unit value, yields, exactly, the above quadratic equation. Any two quantities for which these ratios hold are said to be in *Golden Ratio*, which is the irrational number, $x = 1.618033989\ldots$, whose reciprocal, as shown above, is equal to $(x - 1)$, that is, $(1/x) = (x - 1) = 0.618\ldots$ (note that this relationship, too, yields the above quadratic equation).

As shown further by a leading European mathematician of the late Middle Ages (c. 1202), Leonardo of Pisa (also known as Filius Bonacci), the Golden Ratio also happens to be the limiting value of sequential ratios in the numerical series, $0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55\ldots$. In this series, the numbers after 0, 1 obey a recursion formula, that is, each successive number is the sum of the previous two: $1 = 0+1$; $2 = 1+1$; $3 = 1+2; \ldots, a_j = a_{j-1} + a_{j-2}$ for $j > 2$. Moreover, as the series progresses, the *ratio* of each number to its immediate predecessor begins to oscillate around, and eventually approach x as a limiting value, for example, $5 \div 3 = 1.6667$, $13 \div 8 = 1.625$, $34 \div 21 = 1.619$, $55 \div 34 = 1.6176$, and so on.

More formally, *Fibonacci Numbers*, as they are now called, are defined by the series: $F(n) = (1/\sqrt{5})[\phi^n - (1-\phi)^n]$, $n = 0, 1, 2, \ldots, \infty$; and $\phi = 1/2(1+\sqrt{5})$ is precisely the *Golden Ratio*.

4 The relation of Fibonacci ratios to musical intervals

Moving from anatomical space considerations into time and musical considerations, observe, first, that the Fibonacci ratio, $F(3)/F(2) = 2:1$ defines the frequency-ratio of the tempered musical octave, $F(4)/F(3) = 3:2$ defines the musical "perfect fifth" interval, $F(5)/F(4) = 5:3$ defines the musical "major 6th" and $F(6)/F(5) = 8:5$ defines the musical "minor 6th"—all *consonant* frequency-ratios (musical *intervals*) that are pleasing to the human hearing apparatus. This one-to-one correspondence between low-number Fibonacci-ratios and consonant-musical-intervals lends further credence to the intimate connection between physiologic function and the basic elements of music—one that may be rooted (at least in part) in the underlying *self-symmetry* that prevails in the human body across various scales of perception (Schneck and Berger [12]).

The Golden Ratio appears frequently as a *geometric* scaling-parameter that characterizes systems displaying an inherent self-similarity. Indeed, many investigators actually associate the Hausdorff (or Hausdorff-Besikovich; see later) dimension *strictly* with a fractal, *space-filling* dimension; but the relationship of this parameter to *time* and *musical* intervals, and through them, to human physiology, has not yet been entirely explored. It definitely deserves further consideration.

5 Geometric self-similarity in nature

Consider the following question: what attribute do the: retina of the eye, kidney urinary-collecting-tubes, lining of the gastrointestinal tract, brain, bile-ducts, placenta, lungs, various neural networks ... and ... trees, hearts, coral formations, coastlines, turbulent flows, cumulus clouds ... all share in common?

Answer: When examined with stronger and stronger magnifying lenses, the smaller-scale structures appear remarkably "similar" to the larger-scale forms, in that, all *relative* proportions are uniformly preserved at any scale of observation. That is, basic patterns *within* a given level-of-perception are maintained across all magnification factors, resulting in a spatial/temporal/functional ordering that is called *proportional self-similarity*. Strictly speaking, by "self-similarity" we mean that, because of underlying physical constraints—such as minimum-energy considerations and optimization schemes (Schneck [10]) that prevail—*independent of the size of the unit of observation*—scaling relationships in biological (and other) systems require them to "look roughly the same" at any level of observation. That is to say, there is an infinite nesting of structure on all scales-of-perception, such that all of them must have consistent *proportions* when compared to one another across all such scales-of-perception. When the structure involved has an *irregular* geometry, it is called a *fractal*; and an important defining property of fractals is their self-similarity.

Based on my results for the arterial system (and related findings of many other investigators; see, e.g., West and Goldberger [16], Eberhart [3], Bassingthwaighte [2] and Marques de Sá [7]), there is a clear suggestion that the vascular system is no exception, that is, it is a *self-similar* system. In that respect, "R" too—though showing (compared to "r") somewhat wider variability among vascular beds (see later)—still had values between $2/3$ (for the pulmonary circulation) and $3/4$ (for the coronary circulation), suggesting at least some degree of constrained self-similarity. The former scaling ratio, $(2 \div 3)$ is equal to $[F(3) \div F(4)]$, and the latter, $(3 \div 4) = [F(4) \div F^2(3)]$. Indeed, anatomical/physiological-self-similarity has been shown to prevail in such highly-organized human systems as the following (West and Goldberger [16], Eberhart [3], Bassingthwaighte [2] and Marques de Sá [7]):

- the tree of airway passages that progressively branch from trachea (wind pipe) to the alveoli of the lungs; in this branching network, average bronchial-diameter follows an *inverse* power-law;
- the structure and organization of connective tissue;
- many nerve networks;
- the configuration of glandular duct-work;
- the cochlea, which retains its original proportions while *spiraling* in logarithmic, "snail-shell" fashion to form the cavity of the inner ear that houses auditory nerve-endings. Again, we point out the relationship of the *geometric* self-similar configuration of the hearing apparatus to musical-intervals in *time*; an observation worthy of further study.

6 Self-similarity and power-law formulations

As a *spatial*, anatomical engineering design principle, it appears reasonable to declare that, *in ratios of longer-to-shorter dimensions (or, perhaps, bigger-to-shorter time scales), or in ratios involving growing numbers, the human body is a self-similar system, showing generalized Golden-Ratio/Fibonacci-scaling proportionalities.* Further confirmation of this lies in the fact that *power-law* *relationships—which typically characterize self-similar systems—are highly successful in describing the structure-and-function of the human body* (Schneck and Simanowith [13]). Thus, it is not surprising that such power-law formulations do provide very effective paradigms to model and quantify both nonlinear *anatomic* complexity (including *scaling* laws), and nonlinear *physiologic* complexity (including *constitutive* laws that define the behavior of physiologic tissue), which explains why such a paradigm was assumed in this study.

In self-similar systems, any given parameter, α, defining a property/characteristic of that system, can be related to another one, β, by a scaling-law of the form: $\alpha = \beta^k$, where $k = (\log \alpha) \div (\log \beta)$ is the "dimension" of the scaling-law, known as the previously-mentioned Hausdorff dimension. As a simple example, suppose α_0 represents the geometric property (length, area, or volume measure) of an object residing in Euclidean space having dimension k. Then, if we *reduce* the linear size of this object by the scaling factor, β—that is, multiply each linear measure by the factor $[1/\beta]$ in each spatial direction—then its geometric property (measure, α) would *increase* to "N" times the original, that is, $\alpha = N\alpha_0$, according to the power-law relation, $\alpha = \beta^k$, so that, $N = \beta^k \div a_0$.

Allowing k to take on both integer and *fractional* values gives it sufficient mathematical degrees-of-freedom to account for the fact that—in addition to the *ordering constraints* that are embedded in the "principle of similitude"—realistic models of anatomic design, as mentioned earlier for the parameter "R," must also include the complementary "principal of variability" that typically accompanies such order in biological systems. One can call the latter a *structured randomness* (or "chaos") that is accounted for by applying *weighting factors* to the consistent proportionalities observed on different scales-of-perception. That is to say, examined in greater-and-greater detail, self-similar patterns reveal an increasing degree of *complexity*—devoid of a single, constant, "transmutational" scaling-factor—that is, the patterns are *structurally heterogenous* (i.e. "noisy") across scales-of-perception. Let's see how this works, again, by examining the human cardiovascular system, and then extrapolating to the time domain.

Recall from our earlier discussion that self-similarity derives from the need that physical systems have to adhere to certain underlying constraints—*regardless* of the size of the unit of observation. Thus, when blood is pumped through the anatomical architecture of the cardiovascular system, at any level of observation the pumping mechanism is constrained to adhere to an *optimization principle for energy expenditure*. This need to *economize on the "cost" of pumping* establishes specific, *self-similar vascular branching patterns* that prevail across all N generations (Fung [4]). But in the smallest blood vessels (*microvasculature*) an *additional* requirement must be met, which is to *optimize as well molecular diffusion and transvascular transport mechanisms* across the vessel wall. This introduces the need to have—*specific to this particular level of the cardiovascular system*—an additional constraint: the largest possible total surface-area-of-contact between the lateral walls of the microvasculature and the surrounding interstitial fluid, balanced against the amount of *time* that blood actually spends in the microvasculature. In other words, to optimize the transport process we desire to have the largest available lateral surface area, and the slowest possible perfusion rate through the microvasculature.

Thus, we find that the arterial system meets this additional need by creating a self-similar branching network that progressively generates more-and-more, smaller-and-smaller vessels as we move downstream from the heart (Schneck [11]). The *cumulative* cross-sectional area, A, of the *entire* arterial system (i.e. all n vessels) gradually increases at each generation, m, of vessels; thus creating a *diverging flow* that slows down the mean velocity, v, of flow, Q, according to the conservation of mass relationship for incompressible fluids: $Q = Av$. That is to say, since Q is constant (on the order of 5 liters per minute), flow velocity, v, gradually slows down as A increases—ultimately to some 800 times its original value—thereby satisfying the time-constraint for optimization of transport in the microvasculature. Eventually, more than 16 *billion* capillaries—each having a thin (0.1–$1\,\mu$m) wall, and measuring 3.5–$10\,\mu$m in internal-diameter, by 0.5–1.1 mm in average-length, where the mean velocity, v, in any one of the smallest vessels of the microvasculature is on the order of 0.4–0.5 millimeters per second—provide a microvasculature transmembrane surface area totaling up to $560\,\mathrm{m}^2$, thus satisfying as well the maximum-area constraint for optimum mass transport in this microvasculature. That area is more than twice the size of a doubles tennis court!

Similarly, the branching network of human lungs terminates in 300-million tiny air sacs called *alveoli* (from the Latin, *alveolus*, for, "a small hollow") that have a total air-sac-to-microvasculature contact area totaling 40-to-50-times the surface area of the adult-body's skin, which averages $1.75\,\mathrm{m}^2$ (range, 1.5–$2.0\,\mathrm{m}^2$). Indeed, the interior of the lungs is the most extensive body surface area in contact with the environment.

Without belaboring the point, we can extrapolate these considerations to more general engineering principles of biological scaling by noting that, depending on one's level of observation, *additional* criteria might have to be met to satisfy *differing functional objectives* on *different scales-of-perception*. Therefore, in self-similar systems—where basic *relative proportions* are preserved across all magnification levels—the *individual* scaling principles from level-to-level are nevertheless *weighted* to allow for a certain degree of *variability*, depending on which of several constraints take(s) precedence at any given scale-of-observation. In other words, although the general *topography* "looks the same" across all levels of magnification, when viewed "macroscopically," at one level, it might appear to be scaled to *half* the original dimensions of the pattern, whereas, when examined at another, more "microscopic" level, the pattern might reveal scaling to only *one-third* of its macroscopic dimensions; and even "tighter" focusing might show scaling to only *one-tenth* of the original pattern, while still maintaining the same *relative* proportions that prevail across *all* levels of perception. Therefore, moving from level to level, we observe that there is not a constant overall scaling factor that prevails across each step—which would, indeed, be the case for "traditional," *classical* similitude—but, rather, the fractal imperfection in real-life systems, together with additional constraining criteria, leads us to encounter a scaling factor that is variably *weighted* relative to the original pattern, even though, within each level and across all of them, the same *proportions* prevail.

So it is that while we observe the vascular tree to appear to incorporate a type of "classical" similitude—scaled according to the Golden-Ratio/Fibonacci model—this is not the whole story, because the vascular system also exhibits a certain scaling *variability* across branching generations. Then a more complete mathematical description (i.e. one that takes into account such variability) must go beyond simple similitude to the concept of *renormalization*, which introduces the additional feature of *fractal dimensions*.

7 Fractal dimensions in space and time

Suppose we have a collection of objects—a "mathematical set"—consisting of, say, the number of blood vessels, n, in any branching generation, m, of the arterial system, where $n = f(m)$. Such a set might also be composed of groups of notes in a piece of music, each group containing a number of specific notes—say, all C's, or all $G^{\#}$'s, and so on. Suppose further, that this mathematical set has dimensions (e.g. average vessel internal-diameter, D_m, length, L_m, and wall-thickness, δ_m, or note time values ... whole notes, half-notes, quarter notes, etc.) that get smaller and smaller with each succeeding generation, that is, the set "thins out" with increasing m. Such a set is called a *Cantor Set*, because its scaling properties across generations were first studied by the German mathematician Georg Cantor (1845–1918). Given any specific property, α, of the set, suppose further that α is related to another property, λ, by a power-law relationship of the type $\alpha = \lambda^k$. The value of "k" is the "dimension" of the domain within which the power-law expression holds. For example, in classical Euclidean *space*, λ is some characteristic length, L, and k can assume *integer* values—which is to say $k = 1$ establishes α as a one-dimensional line of length L, $k = 2$ establishes α as a two-dimensional plane having area A, proportional in some sense to L^2 and $k = 3$ establishes α as a three-dimensional volume proportional in some sense to L^3.

Now think about a line ($k = 1$) that twists in such a complicated way that it starts to fill up a plane. As the twisting progresses, the line starts to "exist" in some *fractional* dimension, k, that exceeds unity (a simple straight line), and takes on values that get closer and closer to two (a simple plane), as the twisting configurations get progressively more complex. Classic examples of the latter are the fractal known as a *Sierpinski Triangle*, which has a k-value of 1.585, and is perfectly homogenous at all scales, showing no randomness, and the *Sierpinski Carpet*, which has a k-value equal to 1.89 (Gleick [5], Schroeder [14]). Similarly, think about a plane that contorts more and more into the third dimension—like taking a plane piece of paper and "crumpling it up" into a ball. Along the way, the plane progressively passes through fractional dimensions that exceed two (the original plane of the paper), and move gradually closer and closer to three (a simple volume) as the "crumpling" gets more and more complex and tighter (Gleick [5], Schroeder [14]). More generally, the "thinning dimension" of the "standard" Cantor Set works out to be $\ell n(2) \div \ell n(3) \approx 0.63093$—between a *point* and a *line* (compare this value to Golden Ratio considerations discussed earlier)—which brings to light the observation made in the early 1900s by another German mathematician, the previously-mentioned Felix Hausdorff (1868–1942).

Hausdorff determined that one could *classify* Cantor Sets by means of the mathematical concept of *fractional dimensionality*. That is to say, on the one hand, in "classical" similitude a general *homogeneity* prevails across all levels-of-perception, meaning that—as is the case for example, for an Archimedean spiral—a single, constant scaling factor suffices to quantify how the self-similar nature of the system is accounted for as we move from one observation level to another. On the other hand, if we are dealing with systems (like biological and musical ones) that have the *features* of self-similarity at each level of perception, but are heterogenous *across* levels-of-perception, we must resort to Hausdorff's "Fractal Similitude" concept in order to describe such systems.

Indeed, embedded in the concepts of *fractal* (as opposed to *classical*) *similitude* and *weighted renormalization* is one's ability to account for a multiplicity of scales across levels-of-perception, such as is the case, for example, in the human bronchial tree. By "self-similarity," then, we mean that the functional *form*, $\alpha = \lambda^k$, is frame-and-scale-of-perception-indifferent—that is, one "sees" the same thing, *relatively* speaking, at any scale of perception. However, the specific *details* at any particular magnification level—implicit in "m"—are *weighted*—implicit in the *fractional dimensionality* of k—compared to a corresponding "standard." Again, note from the equations above, that the fractional dimension can be calculated as $(\log \alpha) \div (\log \lambda)$.

For the vascular system in "branching space," k is evaluated by plotting $y = \log[D_m/D_0]$ versus $x = \log[n]$ (both functions of m), and then using a regression technique to calculate the mean slope of the resulting graph. The Hausdorff dimension we thus calculated was found to have a value between -0.37 and -0.40. The negative signs are consistent with the vascular system's "thinning-out with increasing m, in the branching space," that is, this is an *inverse power-law relationship*.

We see then that, morphometrically, natural fractal systems are not "smooth and homogenous" across all scales of perception. Rather, they represent what might be called a *constrained randomness* that is found everywhere in nature (yes, even in music, too)—a type of self-similarity that, on the one hand, is not purely random, yet, on the other hand, lacks a specific, *classical*, constant scaling factor that prevails across all levels of dimensional perception. The significance of the *Principle of Fractal/Fibonacci Scaling* in such self-similar systems is that it provides a reasonable *balance* between deterministic, *complex* anatomic order and seemingly random, statistical chaos. (Note that the operative word here is "complex", like the line twisting or the plane contorting as described earlier.) This *Principle* also allows one to quantify (and explain?) the body's remarkable ability to do more with less—to economize on the use of limited *space* while optimizing physiologic transport and other processes—which is to say, elaborate:

- k-dimensional branching networks—such as the bronchial airways and vascular channels;
- convoluted enfoldings—such as the brain's cerebrum and lobes of the liver;
- jagged, irregular surfaces—such as the Hilbert-curve-like circular folds, villi, and microvilli of the small intestine, which give it a surface transport area on the order of 200 m^2 and
- microscopic functional sub-units—such as kidney nephrons;

they all allow for limited volumes to be "carved-out" or otherwise "mapped" into regions having enormous transport areas. So, again, let us see how what we have learned primarily from *spatial* considerations can be applied with equal validity to temporal ones; and to illustrate this extrapolation, we will examine, of all things, noise!

8 Noise, frequency, and musical considerations

We know it when we hear it: the undesired, *disturbing*, intrinsically unpleasant sound that is often harsh and loud—to the point of interfering with and/or masking other sounds being listened to—because one's brain has difficulty "picking out" the *desired* acoustic signal from the *background noise*. In fact, the very word, *noise*, that describes such an objectionable, subjective acoustic experience, derives from the Latin, *nausea*, which generally refers to "unpleasant conditions of various kinds." Most notably, the unpleasant conditions referred to involve the stomach's 'queasiness'—associated with *sea*sickness (*-sea*), and the vessel (*nau-*) that caused it, from the Greek, *naûs*, meaning, "ship." Generalizing the meaning of *nausea*, then, we have in *noise* the "unpleasant condition" experienced by the hearing apparatus forced to listen to a disagreeable acoustic signal. But what makes this signal displeasing to the ear?

Answer: noise consists of *random*, *unpredictable* groups of sound waves that impact the outer ear drum (*tympanic membrane*) with no single, fundamental frequency ("pitch") or "pitch components." Rather, the sound contains *irregular vibrations*, which include a very wide spectrum (*broadband*) of many, aperiodic, nonharmonic frequency components of comparable (if not equal) amplitudes, randomly distributed throughout the signal. When this random signal impacts the acoustic architecture of the body, the brain has difficulty "making heads-or-tails of it," and it does not like that one bit! Hence, we experience an "unpleasant sensation," a feeling of discomfort. (More generally, "noise" refers to *any* disturbing sound, regular *or* otherwise, but that reference is a highly-subjective, individual-specific perception that is virtually impossible to quantify objectively, especially as it relates to pain, fatigue, discomfort, and the compromising of mental and motor efficiency that results therefrom.)

Taking a leaf out of the "Book of Light," for vision, *white* noise, in the *acoustic* literature, is defined to be a signal containing (hypothetically) *all sonic frequencies*, f, between 20 and 20,000 cycles per second. Thus, *pure* white noise produces a *flat* frequency spectrum—which is a graph of wave-amplitude versus wave-frequency—in *linear* space. Each differential frequency component, δf, is assumed to have equal power, and any finite sub-band, Δf, of the broad bandwidth is considered to contain the same amount of total acoustic energy. For example, in the range between 20 Hz and 50 Hz ($\Delta f = 30$ Hz), the signal would have the same amount of total energy as it does in the 30-Hz range between 9,220 and 9,250.

Of course, *simultaneously* sounding 20,000 *different* tones of equal energy is a purely "hypothetical" construct, because to randomly generate so many pitches with intensity distributed uniformly over all frequencies would require an enormous amount of energy (indeed, in the extrapolated limit of total, pure "white noise," composed of *all* frequencies, sonic, electromagnetic, and otherwise, as the frequency goes to infinity, so does power). Thus, in practice, a "noisy" signal is considered to be "white" if it has a uniform-or-nearly-uniform *energy spectral density function*, $P(f)$ [energy, or power, $P(f)$, *per Hertz*], plotted versus f, over any well-defined frequency band. Using light as an example, *white light*, then, is the mixture of all seven *visible* frequency bands in the ranges:

- 400–484 Tera(10^{12}) Hertz (THz) = red,
- 484–508 THz = orange,
- 508–526 THz = yellow,
- 526–606 THz = green,
- 606–668 THz = blue and
- 668–789 THz = violet,

with each bandwidth having approximately equal representation, that is, equal energy *per cycle* in the frequency spectrum of the "noisy" signal.

Now suppose we have a situation where equal power exists not in frequency bands of constant *increments*, Δf, but in bands that are *proportionally* wide, having a constant *proportionality factor*, β. For example, the power might be the same in frequency bands separated by a β-factor of two—expressed as "equal power *per octave*, 8^{ve}," rather than "equal power *per cycle*" (recall our discussion of self-similarity). Then, instead of having the same energy in each band of width $\Delta f = 30\,\text{Hz} =$ constant, as was the case earlier, we now have the same amount of energy in the band from 20 Hz to 40 Hz (bandwidth *doubling*, increasing by $\Delta f = 20$-Hz), as exists in the band from 9,220-to-18,440 (bandwidth again *doubling*, but this time, $\Delta f = 9,220$-Hz, and $k = 2$) . . . and so on.

In the above case, each bandwidth of equal energy involves a constant *factor* (an *octave*, twice the frequency), rather than a constant *increment*, such as $\Delta f = 30$-Hz. This means that—compared to *white* noise, where each frequency *component* is assumed to have equal power, yielding a *flat* energy spectrum—we now have a situation where each frequency component actually *loses* energy as we move up the frequency spectrum in octaves. That is to say, at the low end (20–40 Hz) the *same amount of energy* is shared by only 20 frequency components, whereas at the high end (9,220–18,440 Hz), that *same amount of energy* is shared by 9,220 frequency components, so each frequency *component* at the high end has correspondingly less energy than does one at the low end. For many noisy signals, such intensity variations with increasing frequency, often expressed as "decibels, dB, per octave," can be modeled by a *Power-law* relationship, wherein $P(f)$, per infinitesimal bandwidth, δf, is expressed in terms of center frequency, f, by an *inverse-power-law equation* of the type $P(f) = [1/f]^k$ (again, suggestive of self-similarity in the *frequency*, as opposed to the *spatial* domain).

If we take the logarithm of both sides of this equation we get

$$\log\big[P(f)\big] = k\log(1) - k\log(f); \quad \text{and since } \log(1) = 0,$$

this equation reduces to

$$\log\big[P(f)\big] = -k\log(f).$$

The log-log plot of the latter is a straight line of negative slope k. Note that $k = 0$ corresponds to the hypothetical case of pure *white noise*, where $P(f) = 1$ (i.e. a *flat* energy spectrum for all frequencies). If $k = 1$, and $[P(f)]$ is expressed as the non-dimensional ratio of *actual* sound intensity, I, to some *reference* value, I^* (perhaps the threshold of human hearing, around 10^{-16} Watts/cm^2), then $\text{Log}[P(f)]$ becomes $\log(I/I^*)$, which defines the *Bel*, B. Then, from above, between any two points, 1 and 2 on the log-log plot:

$$B_2 - B_1 = -\big[\log f_2 - \log f_1\big] = -\log\big(f_2/f_1\big).$$

Continuing with our octave-example, let f double, that is, let $f_2 = 2f_1$. Then $\log(f_2/f_1) = \log(2f_1/f_1) = \log 2 = 0.30$. $B_2 - B_1$, then, equals -0.30, so the power of this signal drops off at 0.3 Bels, or *three decibels, per octave*, equivalent to $[P(f_2)]/[P(f_1)] = 10^{-0.3} = 0.5$; so $[P(f_2)] = 0.5[P(f_1)]$, a 50% drop in energy per cycle for every octave traveled up the frequency spectrum. Again, by a loose analogy to light, "$1/f$ noise," as the case for $\beta = 1$ is called, is assigned the color *pink*, and known as *pink noise*. Pink ["$1/f$"] noise in music and speech has been studied extensively, from the pioneering work of Voss and Clark [15] in the early-to-mid 1970's, to the more recent analyses of Ro and Kwon [8]. Details are beyond the scope of this paper, so the reader is referred to the literature for more information, as we move on to note that, indeed, depending on the value of k, one might have:

- "white" noise: $k = 0$;
- "pink" noise: $k = 1$;
- "red" or Brownian ≡ "Brown" noise: $k = 2$, such that B falls off at 6 dB/8^{ve} [note: "pink" noise is so-called because $k = 1$ lies between "white," $k = 0$, and "red," $k = 2$ noise];
- "blue" or azure noise: $k = -1$, such that B *increases* at 3 dB/8^{ve};
- "violet" or purple noise: $k = -2$, such that B *increases* at 6 dB/8^{ve}.

There are also several "unofficial" colors, such as

- "grey" (random pink noise *weighted* over a given range of f);
- "orange" (quasi-stationary noise containing "pockets" of zero energy dispersed throughout a continuous spectrum);
- "green" (the "background noise" of the world, hence its name);

- “yellow”; (“sunlight” noise);
- “black” ($k > 2$).

It is worth emphasizing again that power-law relationships are typical of *self-similar systems*. It therefore follows that noise, in the time domain—and, by inference, music, as well—has many of the same properties of self-similarity as does geometric configuration, in the space domain. This means that as we move up the frequency spectrum from bandwidth to bandwidth, we encounter proportionally-scaled time-dependent behavior, related to energy considerations per bandwidth. For example, self-similarity in frequency (logarithmic) space helps to explain the auditory phenomenon of *pitch equivalence*. That is to say, as one travels up the frequency ladder, since the “$1/f$” *power is equal* among all octave bandwidths, all pitches separated by one or more perfect “octaves” (i.e. all acoustic frequencies that are related by powers of two) are *perceived* to be equivalent, regardless of the actual frequencies involved. That is why the upper note of an octave “sounds the same” as its lower note, regardless of the register in which the octave is sounded.

9 Concluding remarks

In this paper, some “food-for-thought” was presented that might help to elaborate the profound effects that music has on physiologic function. The two share “obvious” and, perhaps, “not-so-obvious” connections in the time domain . . . from an overlap in critical operating frequencies to self-similar function at various scales-of-perception. Self-similarity in physiologic function is best illustrated in the space domain, using the human vascular and respiratory systems as examples . . . and it is shown herein that spatial considerations can be easily extrapolated into the time domain. Indeed, the two share many of the same properties when viewed at different scales of perception.

Self-similarity in music is best illustrated in the time domain by examining the frequency spectra of noise, and easily extrapolated in the limit to the “organized sound” that forms the basis for the elements of music. Thus, self-similarity, as a property of nature, provides a “common ground” upon which both music and the human body stand and so, may very well be one of the “keys” to unlocking the mystery of *the music effect*. That is to say, within the context of the concepts of *self-similarity* and *power-law formulations*, we might find a common architecture between anatomic/physiologic function and musical structure—an architecture that might help to explain the symbiotic relationship between the two. In turn, such an explanation can be expected to shed some light on the physics of what is going on here, a type of physics that can be exploited in clinical applications. Certainly, it is worthy of further exploration.

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