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A General System of Regularized Non-convex Variational Inequalities

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Abstract

In this communication, we introduced a general system of regularized non-convex variational inequalities (GSRNVI) and established an equivalence between this system and fixed point problems. By using this equivalence we define a projection iterative algorithm for solving GSRNVI, we also proved existence and uniqueness of GSRNVI. The convergence analysis of the suggested iterative algorithm is studied.

Keywords: General system of regularized non-convex variational inequalities; Uniformly t-prox regular sets; Iterative schemes; Convergence analysis

Introduction

The originally variational inequality problem introduced by Stampacchia [1] in the early sixties has a great impact and in influence in the development of almost all branches of pure and applied sciences and has witnessed an explosive growth in theoretical advances, algorithmic development. As a result of interaction between different branches of mathematical and engineering sciences, we now have a variety of techniques to suggest and analyze various algorithms for solving variational inequalities and related optimizations [2-6]. Verma [7-10] studied some systems of variational inequality with single valued mappings and suggest some iterative algorithms to compute approximate solutions of these systems in Hilbert spaces. Agarwal et al. [11] studied sensitivity analysis for a system of generalized nonlinear mixed quasi variational inclusions with single valued mappings. Several authors studied different kinds of systems of variational inequalities and suggested iterative algorithms to find the approximate solutions of the systems [12-15]. We remark that the results regarding the existence of solutions and iterative schemes for solving the system of variational inequalities and related problems are being considered in the setting of convex sets and the technique defined on the characteristics of the projection operator over convex a set which does not hold in general when the sets are non-convex. It is well known that the uniform prox regular sets are convex and include the convex set as special cases. Wen [16] considered a system of non-convex variational inequalities with different nonlinear operator and asserted that this system is equivalent to the fixed point problem and suggested an iterative algorithm for the system of non-convex variational inequalities. The convergence analysis of the proposed iterative algorithm under some certain assumption is also studied. In [17] point out the equivalence formulation used by Wen [16] is not correct. Inspired and motivated by the works of [18-26], we introduced and studied a general systems of regularized non-convex variational inequalities. By using the equivalence, we defined a projection iteration algorithm for solving GSRNVI. Further, we proved the existence and uniqueness of solutions of general system of regularized non-convex variational inequalities. The convergence analysis of the proposed iterative algorithm is also studied.

Basic Foundation

Let H be a real Hilbert space endowed with norm $\|\cdot\|$ and an inner product $<.,.>$ respectively. Let Ω be nonempty closed subsets of H. We represent $d_\Omega(.)$ or $d(.;\Omega)$ the distance function from a point to a set Ω that is

$$d_\Omega(u) = \inf_{v\in\Omega} \|u-v\|$$

Definition 2.1: Let u 2 H be a point not lying in Ω. A point $v \in \Omega$ is called a closed point or a projection of u onto Ω if $d_\Omega(u)=\|u-v\|$: The set of all such closed points is denoted by $P_\Omega(u)$, that is

$$P_\Omega(u) = \{V \in \Omega : d_\Omega(u) = \|u-v\|\}$$

Definition 2.2 The proximal normal cone of Ω at a point $u \in \Omega$ is given by

$$Q_\Omega^P(u) = \{\zeta \in H : u \in Q_\Omega(u+\alpha\zeta)\}$$

where $\alpha>0$ is a constant.

Lemma 2.3 [26] Let Ω be a nonempty closed subset of H. Then $\zeta \in Q_\Omega^P(u)$ if and only if there exists a constant $\alpha = \alpha(\zeta, u) > 0$ such that

$$<\zeta, v-u \le \alpha\|u-v\|^2 \ \forall v \in \Omega$$

Lemma 2.4 [27] Let Ω be a nonempty closed and convex subset of H. Then $\zeta \in Q_\Omega^P(u)$ if and only if

$$<\zeta, v-u \le 0 \ \forall v \in \Omega$$

Definition 2.5 [4] Let $f : H \to R$ be a locally Lipschitz near a point x. The Clark's directional derivative of f at x in the direction v, denoted by $f^o(x; v)$ is define by

$$f^o(x;v) = \limsup_{y\to x, \tau\downarrow 0} \frac{f(y+\tau v) - f(y)}{\tau}$$

where y is a vector in H and τ is a positive scalar.

The tangent cone to Ω at a point $x \in \Omega$, denoted by $T_\Omega(x)$ is defined by

$$T_\Omega(x) = \{v \in H : d_\Omega^o(x;v) = 0\}$$

The normal cone to Ω at $x \in \Omega$; denoted by $Q_\Omega(x)$ is defined by

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$$Q_\Omega(\mathrm{x})=\{\zeta\in H:<\zeta,v>\le 0\ \forall v\in T_\Omega(\mathrm{x})\}$$

The Clarke normal cone denoted by $Q^C_\Omega(\mathrm{x})$ is defined by

$$Q^C_\Omega(\mathrm{x})=\overline{\mathrm{co}}[Q^P_\Omega(\mathrm{x})]$$

where $\overline{\mathrm{co}}(S)$ denotes the closure of the convex hull of S and $Q^P_\Omega(\mathrm{x})\subseteq Q^C_\Omega(\mathrm{x})$; where $Q^C_\Omega(\mathrm{x})$ is a closed convex cone and $Q^P_\Omega(\mathrm{x})$ is convex but may not be closed.

Definition 2.6: [4] For a given t € (0, + ∞]; a subset t of H is called the normalized uniformly prox-regular (or uniformly t-prox-regular) if every nonzero proximal normal to Ω_t can be realized by an t-ball.

That is for all $\bar{x}\in\Omega_t$ and $0\neq\zeta\in\Omega^P_{\Omega t}(\bar{\mathrm{x}})$

$$<\frac{\zeta}{\|\zeta\|},\ x-\bar{x}>\le\frac{1}{2t}\|x-\bar{x}\|^2\ \ x\in\Omega_t$$

Therefore, for all $\bar{x}\in\Omega_t$ and $0\neq\zeta\in\Omega^P_{\Omega t}(\bar{\mathrm{x}})$ with $\|\zeta\|=1$ we have

$$<\zeta,x-\bar{x}>\le\frac{1}{2t}\|x-\bar{x}\|^2,\ x\in\Omega_t$$

Lemma 2.7 [23] A closed set $\Omega\subseteq H$ is convex if and only if it is uniformly t-prox-regular for every t>0 Proposition 2.8 [25] Let t>0 and Ω_t be a nonempty closed and uniformly t-prox -regular subset of H. Let U(t)={u € H : 0 ≤ $d_{\Omega t}$(u)<t} Then the following statements are hold:

(a) for all x € U(t); $P_{\Omega t}$ (x) ≠0;

(b) for all t0 2 (0; t); Pt is Lipschitz continuous mapping with constant $\frac{t}{t-t'}$ on $U(t') = \{u\in H : 0\le d_{\Omega_t}(u) < t'\}$.

If $Q^P_\Omega(\cdot)$ is a closed set valued mapping, hence

$$Q^C_{\Omega_t}(\mathrm{x})=\mathrm{Q}^P_{\Omega_t}(\mathrm{x})$$

and

$$Q_{\Omega_t}(\mathrm{x})=\mathrm{Q}^C_{\Omega_t}(\mathrm{x})=\mathrm{Q}^P_{\Omega_t}(\mathrm{x})$$

The union of two disjoint intervals [a,b] and [c,d] is uniformly t-prox-regular with $t=\frac{c-b}{2}$ The infinite union of disjoint intervals is also uniformly t-prox-regular and t depends on the distance between the intervals.

Basic Remarks and Formulations

Let t be an uniformly Ω_t prox-regular (nonconvex) set and g_i: $\Omega_t\to\Omega_t$ be a given mapping for i = 1; 2; 3: For given mappings T_1, T_2, T_3: $\Omega_t\to\Omega_t$ we consider the following problems of finding (x^*, y^*, z^*) € $\Omega_t\times\Omega_t\times\Omega_t$ such that

$$\begin{aligned}&< r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*),\ g_1(x)-g_1(x^*)>\ge 0,\ g_1(x)\in\Omega_t, r_1>0\\ &\langle r_2T_2(z^*,y^*)+g_2(y^*)-g_2(z^*),\ g_2(x)-g_2(\mathrm{y}^*)\rangle\ge 0,\ g_2(x)\in\Omega_t, r_2>0\\ &\langle r_3T_3(\mathrm{x}^*,z^*)+g_3(z^*)-g_3(x^*),\ g_3(x)-g_3(\mathrm{x}^*)\rangle\ge 0,\ g_3(x)\in\Omega_t, r_3>0\end{aligned}\tag{3.1}$$

The problem (3.1) is called a general system of regularized non convex variational inequalities. We note that if T_1=T_2=T_3=T : $\Omega_t\to\Omega_t$ is an univariant nonlinear operator, g_i = I; i = 1; 2; 3 (the identity operator) and x*=y*=z*=u, then the system (3.1) reduces to the following classical variational inequalities defined on the nonconvex set Ω_t find u € Ω_t such that

$$\langle Tu,v-u\rangle\ge 0,\ \forall v\in\Omega_t\tag{3.2}$$

and (3.2) is equivalent to find u € Ω_t such that

$$0\in Tu+Q^P_{\Omega_t}(\mathrm{u})\tag{3.3}$$

Where $Q^P_{\Omega_t}(\mathrm{u})$ denotes the normal cone of Ω_t at u over the non convex set.

Lemma 3.1 (x^*, y^*, z^*) € $\Omega_t\times\Omega_t\times\Omega_t$ is a solution set of problem (3.1) if and only if

$$\begin{aligned}g_1(x^*)&=P_{\Omega_t}[g_1(y^*)-r_1T_1(y^*,x^*)],\\ g_2(y^*)&=P_{\Omega_t}[g_2(z^*)-r_2T_2(z^*,\mathrm{y}^*)],\\ g_3(z^*)&=P_{\Omega_t}[g_3(x^*)-r_3T_3(x^*,z^*)],\end{aligned}\tag{3.4}$$

where $P_{\Omega t}$ is the projection of H on to the uniformly t-prox-regular set Ω_t. In the proof of Lemma 3.1, there occur three fatal errors. First in view of Proposition 2.8, for any t € (0, 1) the projection of points in the tube $U(t') = \{u\in H : 0 < d_{\Omega_t}(u) < t'\}$ onto the set Ω_t exists and unique, that is for any x € U(t'), the set $P_{\Omega t}$ (x) is nonempty and singleton. From the Lemma 3.1 and Proposition 2.6 the points $g_1(y^*)$ - $r_1T_1(y^*; x^*)$; $g_2(z^*)$ - $r_2T_2(z^*, y^*)$ and $g_3(x^*)$ - $r_3T_3(x^*, z^*)$ should be in U(t') for some t' € (0, t) it is not necessary true, hence (3.4) are not necessarily well defined. If $r_1<\frac{t'}{1+\|T_1(\mathrm{y}^*,\mathrm{x}^*)\|}$, $r_2<\frac{t'}{1+\|T_2(z^*,y^*)\|}$, and $r_3<\frac{t'}{1+\|T_3(x^*,z^*)\|}$, and t' € (0, t). Then we have

$$d_{\Omega_t}(\mathrm{g}_1(\mathrm{y}^*)-\mathrm{r}_1\mathrm{T}_1(\mathrm{y}^*,x^*))\le d_{\Omega_t}(\mathrm{g}_1(\mathrm{y}^*))+\mathrm{r}_1\|\mathrm{T}_1(\mathrm{y}^*,x^*)\|$$

$$\le\frac{t'\|T_1(\mathrm{y}^*,\mathrm{x}^*)\|}{1+\|T_1(\mathrm{y}^*,\mathrm{x}^*)\|}<t',\ \text{for }\mathrm{g}_1(\mathrm{y}^*)\in\Omega_t,$$

Hence, $(\mathrm{g}_1(\mathrm{y}^*)-\mathrm{r}_1\mathrm{T}_1(\mathrm{y}^*,x^*))\in U(t')$

Similarly we have $(\mathrm{g}_2(z^*)-\mathrm{r}_2\mathrm{T}_2(z^*,y^*))\in U(t')$ and $(\mathrm{g}_3(x^*)-\mathrm{r}_3\mathrm{T}_3(x^*,z^*))\in U(t')$

If $r_1<\frac{t'}{1+\|T_1(\mathrm{y}^*,\mathrm{x}^*)\|}$, $r_2<\frac{t'}{1+\|T_2(z^*,y^*)\|}$, and $r_3<\frac{t'}{1+\|T_3(x^*,z^*)\|}$,

for t' ε (0; t); then the equation (3.4) are well defined.

Secondly the following general system of regularized non convex variational inclusions is equivalence to the system (3.1):

$$\begin{aligned}&0\in r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*)+r_1Q^P{}_{\Omega t}(g_1(x^*))\\ &0\in r_2T_2(\mathrm{z}^*,\mathrm{y}^*)+g_2(\mathrm{y}^*)-g_2(\mathrm{z}^*)+r_2Q^P{}_{\Omega t}(g_2(\mathrm{y}^*))\\ &0\in r_3T_3(\mathrm{x}^*,\mathrm{z}^*)+g_3(\mathrm{z}^*)-g_3(\mathrm{x}^*)+r_3Q^P{}_{\Omega t}(g_1(\mathrm{z}^*)),\end{aligned}\tag{3.5}$$

Since $Q^P{}_{\Omega t}(g_1(x^*))$, $Q^P{}_{\Omega t}(g_2(\mathrm{y}^*))$ and $Q^P{}_{\Omega t}(g_3(\mathrm{z}^*))$ are cone, the system (3.1) is equivalent tothe following system:

$$\begin{aligned}&0\in r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*)+Q^P{}_{\Omega t}(g_1(x^*))\\ &0\in r_2T_2(\mathrm{z}^*,\mathrm{y}^*)+g_2(\mathrm{y}^*)-g_2(\mathrm{z}^*)+Q^P{}_{\Omega t}(g_2(\mathrm{y}^*))\\ &0\in r_3T_3(\mathrm{x}^*,\mathrm{z}^*)+g_3(\mathrm{z}^*)-g_3(\mathrm{x}^*)+Q^P{}_{\Omega t}(g_1(\mathrm{z}^*)).\end{aligned}\tag{3.6}$$

The system (3.1) is equivalent to the system (3.5) which is not true in general.

Example 3.2 Let H=R and t=[0, b] [c, d] be the union of two disjoint intervals [0, b] and [c, d] where 0 <b<c<d: Then t is an uniformly t-prox-regular set with t = cℵb 2. Define $T_i:\Omega_t\times\Omega_t\to\Omega_t$ and $g_i:\Omega_t\to\Omega_t$ by

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$T_i(x,y,z)=\theta_i e^{s_i xyz}$,

$g_i(x)=kx^m \quad x,y,z\in\Omega_t$

where for $i=1,2,3,\ \ si,m\in\mathbb{R},\theta_i<0 \ and \ \ b^{1-m}\le k<\frac{c}{b^m}$ are arbitrary but fixed.

Assume $x^*=y^*=z^*=b$ and $r_i>0$, i = 1, 2, 3

$$\alpha>\max\left\{\frac{-r_1\theta_1e^{s_1b^2}}{c-kb^m},\frac{-r_2\theta_2e^{s_2b^2}}{c-kb^m},\frac{-r_3\theta_3e^{s_3b^2}}{c-kb^m}\right\}$$

be the fixed arbitrary. Hence for all w ε Ω_t

$$\left\langle r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*),w-g_1(x^*)\right\rangle+\alpha\|w-g_1(x^*)\|^2$$
$$=r_1\theta_1e^{s_1b^2}(w-kb^m)+\alpha(w-kb^m)^2$$
$$=(w-kb^m)+(\alpha(w-kb^m)+r_1\theta_1e^{s_1b^2}).$$

If w € [0, b], then

$$kb^m\le w-kb^m\le b-kb^m=b(1-kb^{m-1})$$

and

$$-k\alpha b^m+r_1\theta_1e^{s_1b^2}\le\alpha(w-kb^m)+r_1\theta_1e^{s_1b^2}\le r_1\theta_1e^{s_1b^2}+\alpha b(1-kb^{m-1})$$

For w € [c, d] we have

$$c-kb^m\le w-kb^m\le d-kb^m$$

and

$$\alpha(c-kb^m)+r_1\theta_1e^{s_1b^2}\le\alpha(w-kb^m)+r_1\theta_1e^{s_1b^2}\le\alpha(d-kb^m)+r_1\theta_1e^{s_1b^2}$$

$$(w-kb^m)\ (\alpha(w-kb^m)+r_1\theta_1e^{s_1b^2})\ge0\ \ \forall_w\in\Omega_t \qquad (3.8)$$

From (3.7)-(3.8), we have

$$\left\langle r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*),w-g_1(x^*)\right\rangle+\alpha\|w-g_1(x^*)\|^2\ge0\ \forall_w\in\Omega_t$$

Since $r_1\theta_1e^{s1b^2}(w-kb^m)<0$ for all w € [c, d] i.e.,

$$\left\langle r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*),w-g_1(x^*)\right\rangle<0\ \forall_w\in\Omega_t$$

Hence

$$\left\langle r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*),w-g_1(x^*)\right\rangle\ge0,\ w\in\Omega_t$$

cannot hold. Similarly we have

$$\left\langle r_2T_2(z^*,y^*)+g_2(y^*)-g_2(z^*),w-g_2(y^*)\right\rangle+\alpha\|w-g_2(y^*)\|^2\ge0\ \forall_w\in\Omega_t$$

while the inequality

$$\left\langle r_2T_2(z^*,y^*)+g_2(y^*)-g_2(z^*),w-g_2(y^*)\right\rangle\ge0,\ \forall_w\in\Omega_t$$

cannot hold. Again in similar way we have

$$\left\langle r_3T_3(x^*,z^*)+g_3(z^*)-g_3(x^*),w-g_3(z^*)\right\rangle+\alpha\|w-g_3(z^*)\|^2\ge0\ \forall_w\in\Omega_t$$

the inequality

$$\left\langle r_3T_3(x^*,z^*)+g_3(z^*)-g_3(x^*),w-g_3(z^*)\right\rangle\ge0$$

cannot hold. Therefore we can see that every solution of (3.2) is a solution of (3.3) but converse need not be true in general. On the basis of example we define as the general system of regularized non-convex variational inequality. For given nonlinear mappings $T_i:H\times H\to H$ and $g_i:H\times H\to H$ i=1, 2, 3 we consider the general system of regularized non-convex variational inequality for finding $(x^*,y^*,z^*)\in H\times H\times H$ such that $(g_i(x^*),g_i(y^*),g_i(z^*))\in\Omega_t\times\Omega_t\times\Omega_t$ and

$$\left\langle r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*),\ g_1(x)-g_1(x^*)\right\rangle$$
$$+\frac{\|r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*)\|}{2t}\|g_1(x)-g_1(x^*)\|^2\ge0,\ for\ x\in\Omega_t,$$
$$\left\langle r_2T_2(z^*,y^*)+g_2(y^*)-g_2(z^*),\ g_2(x)-g_2(y^*)\right\rangle$$
$$+\frac{\|r_2T_2(z^*,y^*)+g_2(y^*)-g_2(y^*)\|}{2t}\|g_2(x)-g_2(y^*)\|^2\ge0,\ for\ x\in\Omega_t,$$
$$\left\langle r_3T_3(x^*,z^*)+g_3(z^*)-g_3(x^*),\ g_3(x)-g_3(x^*)\right\rangle$$
$$+\frac{\|r_3T_3(x^*,z^*)+g_3(z^*)-g_3(x^*)\|}{2t}\|g_3(x)-g_3(x^*)\|^2\ge0,\ for\ x\in\Omega_t, \qquad (3.9)$$

Proposition 3.3: Let Ω_t be an uniformly t-prox regular set. The system (3.9) is equivalent to the system (3.6).

Proof: Let $(x^*,(y^*),(z^*)\in H\times H\times H\ (g_i(x^*),g_i(y^*),g_i(z^*)\in\Omega_t\times\Omega_t\times\Omega_t$ i = 1; 2; 3 be a solution set of the system (3.9). If $r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*)=0$ then $0\in r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*)+Q^P{}_{\Omega t}(g_1(x^*))$ **If** $r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*)\ne0$ then

$$\left\langle -r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*)),g_1(x)-g_1(x^*)\right\rangle$$
$$\le\frac{\|r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*)\|}{2}\|g_1(x)-g_1(x^*)\|^2,\forall x\in\Omega_t$$ **From Lemma 2.3,**

$$-(r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*))\in Q^P{}_{\Omega t}(g_1(x^*)),$$

Hence $\quad 0\in r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*)+Q^P{}_{\Omega t}(g_1(x^*))$

Similarly we have

$$0\in r_2T_2(z^*,y^*)+g_2(y^*)-g_2(z^*)+Q^P{}_{\Omega t}(g_2(y^*))$$

and

$$0\in r_3T_3(x^*,z^*)+g_3(z^*)-g_3(x^*)+Q^P{}_{\Omega t}(g_3(z^*))$$

Conversely, (x^*, y^*, z^*) € H×H×H with $(g_i(x^*); g_i(y^*); g_i(z^*))$ €$\Omega_t\times\Omega_t\times\Omega_t$; i=1; 2; 3be a solution set of the system (3.6) then from Definition 2.6, (x^*, y^*, z^*) € H×H×Hwith $(g_i(x^*); g_i(y^*); g_I(z^*))$ € $\Omega_t\times\Omega_t\times\Omega_t$; i=1, 2, 3 be a solution set of the system (3.9).

Lemma 3.4: For i=1, 2, 3 let $T_i; g_i; r_i$ be the same as in the system (3.9), then (x^*, y^*, z^*) € H × H × H with $(g_i(x^*); g_i(y^*); g_i(z^*))$ € $\Omega_t\times\Omega_t\times\Omega_t$ be a solution set of the system (3.9) if and only if (x^*,y^*,z^*) satisfies the system (3.4) with $r_1<\frac{t'}{1+\|T_1(y^*,x^*)}$, $r_2<\frac{t'}{1+\|T_2(z^*,y^*)}$ $and\ r_3<\frac{t'}{1+\|T_3(x^*,z^*)\|}$ for t' € (0,t)

Proof: Let (x^*, y^*, z^*) € H × H × H with $g_i(y^*); g_i(z^*))$ € $\Omega_t\times\Omega_t\times\Omega_t$; i = 1, 2, 3 be a solution set of the system (3.9). Since $g_i(y^*); g_i(z^*))$ € $\Omega_t\times\Omega_t\times\Omega_t$ $r_1<\frac{t'}{1+\|T_1(y^*,x^*)}$, $r_2<\frac{t'}{1+\|T_2(z^*,y^*)}$ $and\ r_3<\frac{t'}{1+\|T_3(x^*,z^*)\|}$ it follow that the equations (3.4) are well defined. By using $P_{\Omega t}=(I+Q^P_{\Omega t})^{-1}$ and Proposition 3.3 we have

$$0\in r_1T_1(y^*,x^*)+g_1(x^*)-g_1(y^*)+Q^P{}_{\Omega t}(g_1(x^*))$$
$$\Leftrightarrow g_1(y^*)-r_1T_1(y^*,x^*)\in g_1(x^*)+Q^P{}_{\Omega t}(g_1(x^*))$$

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$$\Leftrightarrow g_1(y^*) - r_1T_1(y^*, x^*) \in (I + Q^P_{\Omega t})(g_1(x^*))$$

$$\Leftrightarrow g_1(x^*) = P_{\Omega t}\,[g_1(y^*) - r_1T_1(y^*, x^*)],$$

where I is an identity mapping. Similarly we have

$$0 \in r_2T_2(z^*, y^*) + g_2(y^*) - g_2(z^*) + Q^P_{\Omega t}(g_2(y^*))$$

$$\Leftrightarrow g_2(z^*) - r_2T_2(z^*, y^*) \in g_2(y^*) + Q^P_{\Omega t}(g_2(y^*))$$

$$\Leftrightarrow g_2(z^*) - r_2T_2(z^*, y^*) \in (I + Q^P_{\Omega t})(g_2(y^*))$$

$$\Leftrightarrow g_2(y^*) = P_{\Omega t}\,[g_2(z^*) - r_2T_2(z^*, y^*)],$$

and

$$0 \in r_3T_3(x^*, z^*) + g_3(z^*) - g_3(x^*) + Q^P_{\Omega t}(g_3(z^*))$$

$$\Leftrightarrow g_3(x^*) - r_3T_3(x^*, z^*) \in g_3(z^*) + Q^P_{\Omega t}(g_3(z^*))$$

$$\Leftrightarrow g_3(x^*) - r_3T_3(x^*, z^*) \in (I + Q^P_{\Omega t})(g_3(z^*))$$

$$\Leftrightarrow g_3(z^*) = P_{\Omega t}\,[g_3(x^*) - r_3T_3(x^*, z^*)]$$

This completes the proof.

Existence and Convergence Analysis

Definition 4.1 A mapping T→H× H×H is said to be

(i) monotone in the first variable if for all x, y € H

$$\langle T(x,u) - T(y,v), x - y\rangle \geq 0 \ \forall u, v \in H,$$

(ii) μ -strongly monotone in the first variable if there exists a constant μ>o such that

$$\langle T(x,u) - T(y,v), x - y\rangle \geq \mu \| x - y \|^2 \ \forall u, v \in H,$$

(iii) μ -cocoercive if there exists a constant μ>o such that

$$\langle T(x,u) - T(y,v), x - y\rangle \geq \mu \| T(x,u) - T(y,v) \|^2 \ \forall x, y \in H,$$

(iv) relaxed μ-cocoercive if there exists a constant μ>o such that

$$\langle T(x,u) - T(y,v), x - y\rangle \geq -\mu \| T(x,u) - T(y,v) \|^2 \ \forall x, y \in H,$$

(v) relaxed (μ,v)-cocoercive if there exists a constant μ,v>o such that

$$\langle T(x,u) - T(y,v), x - y\rangle \geq -\mu \| T(x,u) - T(y,v) \|^2 + v \| x - y \|^2 \ \forall x, y, u, v \in H,$$

(vi) μ-Lipschitz continuous in the first variable if there exists a constant μ>o such that for all x, y € H

$$\| T(x,u) - T(y,v) \leq \mu \| x - y \| \ \forall x, y \in H$$

(i) μ-strongly monotone if there exists a constant μ>0 such that

$$\langle g(x) - g(y), x - y\rangle \geq K \| x - y \|^2 \ \forall u, v \in H$$

(ii) ξ-Lipschitz continuous if there exists a constant ξ>o such that for all x, y € H

$$\langle g(x) - g(y), x - y\rangle \geq \xi \| x - y \| \ \forall x, y \in H$$

Now we prove that existence and unique solution set of general system of regularized non-convex variational inequalities.

Theorem 4.3: Let the mappings T_i, g_i and r_i; i=1, 2, 3 be the same as in the system (3.1) such that $g_i(H)$ Ć Ω_t: Let gi be the μ_i-cocoercive with constant ξ_i>0 and Lipschitz continuous mapping with constant μ_i>0: Let T_i be the relaxed (η_i, v_i)-cocoercive with respect to the first variable with constants η_i, v_i>0 and λ_i-Lipschitz continuous mapping with constant λ_i>0: If the constant r_i for i=1; 2; 3: satisfy the following conditions:

$$r_1 < \frac{t'}{1 + \| T_1(y,x) \|},\ r_2 < \frac{t'}{1 + \| T_2(z,y) \|} \ \text{and} \ r_3 < \frac{t'}{1 + \| T_3(x,z) \|} \ \forall x, y \in H, \ for\ t' \in (0,t) \quad (4.1)$$

And

$$\left| r_1 - \frac{v_1 - \lambda^2{}_1\eta_1}{\lambda^2{}_1} \right| < \frac{\sqrt{(v_1 - \lambda^2{}_1\eta_1)^2\delta^2 - \lambda^2{}_1(\delta^2 - (1 - (1+\delta)p_1)^2)}}{\lambda^2{}_1\delta}$$

$$\left| r_2 - \frac{v_2 - \lambda^2{}_2\eta_2}{\lambda^2{}_2} \right| < \frac{\sqrt{(v_2 - \lambda^2{}_2\eta_2)^2\delta^2 - \lambda^2{}_2(\delta^2 - (1 - (1+\delta)p_2)^2)}}{\lambda^2{}_2\delta}$$

$$\left| r_3 - \frac{v_3 - \lambda^2{}_3\eta_3}{\lambda^2{}_3} \right| < \frac{\sqrt{(v_3 - \lambda^2{}_3\eta_3)^2\delta^2 - \lambda^2{}_3(\delta^2 - (1 - (1+\delta)p_3)^2)}}{\lambda^2{}_3\delta}$$

$$v_i > \lambda^2{}_i\eta_i \frac{\sqrt{\delta^2 - (1 - (1+\delta)p_i)^2)}}{\delta}\lambda_i,$$

$$v_i > \lambda^2{}_i\eta_i,$$

$$p_i = \sqrt{1 - 2\mu_i\xi^2{}_i + \xi^2{}_i},$$

$$2\mu_i\xi^2{}_i \leq 1 + \xi^2{}_i \quad \text{(4.2) for i= 1,2,3} \quad (4.2)$$

$\delta = \dfrac{t}{t-t'}$ then the system (3.9) admits a unique solutions.

Proof. Define $\phi, \varphi, \psi : H \times H \to H$ by

$$\phi(x,y) = x - g_1(x) + P_{\Omega t}[g_1(y) - r_1T_1(y,x)]$$

$$\varphi(y,z) = y - g_2(y) + P_{\Omega t}[g_2(z) - r_2T_2(z,y)]$$

$$\psi(z,x) = z - g_3(z) + P_{\Omega t}[g_3(x) - r_3T_3(x,z)], \quad \forall x, y, z \in H \quad (4.3)$$

Define $\|.\|$ $on\ H \times H \times H$ by

$$\|(x,y,z)\|_* = \|x\| + \|y\| + \|z\| \ \forall x, y, z \in H \quad (4.4)$$

Since $(H \times H \times H, \|.\|_*)$ is a Hilbert space, we define $\Im : H \times H \times H \to H \times H \times H$ by

$$\zeta(x,y,z) = (\phi(x,y), \varphi(y,z), \psi(x,z)), \ \forall x, y, z \in H \times H \times H \quad (4.5)$$

We claim that $\Im$ is a contraction mapping. Indeed let (x,y), (x^*,y^*) ε H× H× H, g_1 (y) ε Ω_t and $r_1 < \dfrac{t'}{1 + \| T_1(y,x)}$ for t'ε (0,t), hence

$$g_1(y) - r_1T_1(y,x) \in U(t')$$

and the t-prox-regularity of Ω_t implies that $P_{\Omega t}[g_1(y) - r_1T_1(y,x)]$ exists and unique.

Similarly $P_{\Omega t}[g_2(z) - r_2T_2(z,y)]$ and $P_{\Omega t}[g_3(x) - r_3T_3(x,z)]$ also exists and unique. Using

Proposition 2.8, we have

$$\| \phi(x,y) - \phi(x^*,y^*) \| = \| x - g_1(x) + P_{\Omega t}[g_1(y) - r_1T_1(y,x)] - (x^* - g_1(x^*) + P_{\Omega t}[g_1(y^*) - r_1T_1(y^*,x^*)]) \|$$

$$\leq \| x - x^* - (g_1(x) - g_1(x^*)) \| + \delta \| g_1(y) - g_1(y^*) - r_1(T_1(y,x) - T_1(y^*,x^*)) \|$$

$$\leq \| x - x^* - (g_1(x) - g_1(x^*)) \| + \delta(\| y - y^* - (g_1(y) - g_1(y^*)) \| + \| y - y^* - r_1(T_1(y,x) - T_1(y^*,x^*)) \|) \quad (4.6)$$

Where $\delta = \dfrac{t}{t-t'}$ By μ_1-cocoercive of g_1 and $£_1$-Lipschitz continuity of g_1 we have

$$\leq \| x - x^* - (g_1(x) - g_1(x^*)) \|^2 = \| x - x^* \|^2 - 2\langle g_1(x) - g_1(x^*), x - x^* \rangle + \| (g_1(x) - g_1(x^*) \|^2$$

$$\leq \| x - x^* \|^2 - 2\mu_1 \| g_1(x) - g_1(x^*) \|^2 + \| (g_1(x) - g_1(x^*) \|^2$$

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$$\leq \| x-x^* \|^2 - 2\mu_1 \xi^2{}_1 \| x-x^* \|^2 + \xi^2{}_1 \| x-x^* \|^2$$

$$\leq (1-2\mu_1 \xi^2{}_1 + \xi^2{}_1) \| x-x^* \|^2$$

$$\Rightarrow \| x-x^* - g_1(x) - g_1(x^*)) \| \leq p_1 \| x-x^* \| \qquad (4.7)$$

Where, $p_1 = \sqrt{1-2\mu_1 \xi^2{}_1 + \xi^2{}_1}$ Similarly we obtain

$$\| y-y^* - g_1(\mathrm{y}) - g_1(\mathrm{y}^*)) \| \leq p_1 \| y-y^* \| \qquad (4.8)$$

Since T_1 is relaxed (η_1, v_1) cocoercive and λ_1 Lipschitz continuous mapping with first variable, we have

$$\| y-y^* - r_1(T_1(y,x) - T_1(y^*,x^*)) \| = \| y-y^* \|_2 - 2r_1 \langle T_1(y,x) - T_1(y^*,x^*), y-y^* \rangle + r^2{}_1 \| T_1(y,x) - T_1(y^*,x^*) \|^2$$

$$\leq \| y-y^* \|^2 - 2r_1(-\eta_1) \| T_1(y,x) - T_1(y^*,x^*) \|^2 - r_1(T_1(y,x) - T_1(y^*,x^*)) \| = \| y-y^* \|_2$$

$$+ v_1 \| y-y^* \|^2 - r^2{}_1 \| T_1(y,x) - T_1(y^*,x^*) \|^2$$

$$\leq (1 + 2r_1\eta_1 \lambda^2{}_1 - 2r_1 v_1 + r^2{}_1 \lambda^2{}_1) \| y-y^* \|^2$$

$$\Rightarrow \| y-y^* - r_1(T_1(y,x) - T_1(y^*,\mathrm{x}^*)) \| \leq \sqrt{1 + 2r_1\eta_1\lambda^2{}_1 - 2r_1 v_1 + r^2{}_1 \lambda^2{}_1} \| y-y^* \| \qquad (4.9)$$

From (4.3)-(4.9) we have

$$\| \phi(x,y) - \phi(x^*,y^*) \| \leq P_1 \| x-x^* \| + \theta_1 \| y-y^* \| \qquad (4.10)$$

Where $P_1 = \sqrt{1-2\mu_1\xi^2{}_1 + \xi^2{}_1}$ **and** $\theta_1 = \delta(p_1 + \sqrt{1 + 2r_1\eta_1\lambda^2{}_1 - 2r_1 v_1 + r^2{}_1\lambda^2{}_1})$

Since g_2 is μ_2-cocoercive and $£_2$- Lipschitz continuous mapping and T_2 is relaxed (η_2, μ_2) cocoercive mapping and λ_2 Lipschitz continuous mapping with first variable, we obtain

$$\| \varphi(\mathrm{y},\mathrm{z}) - \varphi(\mathrm{y}^*,\mathrm{z}^*) \| \leq P_2 \| y-y^* \| + \theta_2 \| z-z^* \| \qquad (4.11)$$

Where $p_2 = \sqrt{1-2\mu_2\xi^2{}_2 + \xi^2{}_2}$ and $\theta_2 = \delta(p_2 + \sqrt{1 + 2r_2\eta_2\lambda^2{}_2 - 2r_2 v_2 + r^2{}_2\lambda^2{}_2})$

Again, since g_3 is μ_3 cocoercive and $£_3$ - Lipschitz continuous and T_3 is relaxed (η_3, v_3)-cocoercive and λ_3-Lipschitz continuous with first variable, we obtain

$$\| \psi(\mathrm{z},\mathrm{x}) - \psi(\mathrm{z}^*,\mathrm{x}^*) \| \leq P_3 \| z-z^* \| + \theta_3 \| x-x^* \| \qquad (4.12)$$

Where $p_3 = \sqrt{1-2\mu_3\xi^2{}_3 + \xi^2{}_3}$ and $\theta_3 = \delta(p_3 + \sqrt{1 + 2r_3\eta_3\lambda^2{}_3 - 2r_3 v_3 + r^2{}_3\lambda^2{}_3})$ It follows that from (4.4)-(4.12), we have

$$\| \Im(x,y,z) - \zeta(x^*,y^*,z^*) \|_* = \| \phi(x,y) - \phi(x^*,y^* \| + \|$$

$$\varphi(\mathrm{y},\mathrm{z}) - \varphi(\mathrm{y}^*,\mathrm{z}^* \| + \| \psi(\mathrm{z},\mathrm{x}) - \psi(\mathrm{z}^*,\mathrm{x}^*) \|$$

$$\leq (p_1+\theta_3) \| x-x^* \| + (p_2+\theta_1) \| y-y^* \| + (p_3+\theta_2) \| z-z^* \|$$

$$\leq \ell \| (x,y,z) - (x^*,y^*,z^*) \|_*, \qquad (4.13)$$

and

$$\ell = \max\{ p + \phi_i : p = p_i, i = 1,2,3 \}.$$

The condition (4.2) implies that 0 ≤l<1 and (4.13) guarantees that $\Im$ is a contraction mapping. By Banach fixed point Theorem, there exists a unique point $(\hat{x},\hat{y},\hat{z}) \in \mathrm{H}\times\mathrm{H}\times\mathrm{H}$ such that $\Im(\hat{x},\hat{y},\hat{x}) = (\hat{x},\hat{y},\hat{z})$

From (4.3) and (4.4) we have

$$g_1(\hat{x}) = P_{\Omega t}[g_1(\hat{\mathrm{y}}) - r_1T_1(\hat{\mathrm{y}},\hat{\mathrm{x}})]$$

$$g_2(\hat{\mathrm{y}}) = P_{\Omega t}[g_2(\hat{\mathrm{z}}) - r_2T_2(\hat{\mathrm{z}},\hat{\mathrm{y}})]$$

$$g_3(\hat{\mathrm{z}}) = P_{\Omega t}[g_3(\hat{\mathrm{x}}) - r_3T_3(\hat{\mathrm{x}},\hat{\mathrm{z}})]$$

for the constant r_i>0, i=1, 2, 3, In same way deduce that the above equations are well defined.

Lemma 3.4 guarantees that $(\hat{x},\hat{y},\hat{z}) \in \mathrm{H}\times\mathrm{H}\times\mathrm{H}$ with $(\mathrm{g_i}(\hat{x}),\mathrm{g_i}(\hat{y}),\mathrm{g_i}(\hat{z})) \in \Omega_t\times\Omega_t\times\Omega_t, i=1,2,3$ i=1; 2; 3 is a solution set of general system of regularized non-convex variational inequalities. This completes the proof. By using Lemma 3.4, we suggest the following explicit projection iterative method for solving the general system of regularize non-convex variational inequalities. Algorithm 4.4 Assume that mappings T_i, g_i and constant r_i>0 for i=1; 2; 3 be the same as in the system (3.9) such that $g_i(\mathrm{H}) \subseteq \Omega_t$ For arbitrary initial points x_0, y_0, z_0ε H compute the sequences $\{x_n\}$,$\{y_n\}$ and $\{z_n\}$ in H in the following way:

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n[x_n - g_1(x_n) + P_{\Omega t}(g_1(y_n) - r_1T_1(y_n,x_n))]$$

$$y_{n+1} = (1-\alpha_n)y_n + \alpha_n[y_n - g_2(\mathrm{y}_n) + P_{\Omega t}(g_2(\mathrm{z}_n) - r_2T_2(\mathrm{z}_n,\mathrm{y}_n))]$$

$$z_{n+1} = (1-\alpha_n)z_n + \alpha_n[z_n - g_3(\mathrm{z}_n) + P_{\Omega t}(g_3(\mathrm{x}_n) - r_3T_3(\mathrm{x}_n,\mathrm{z}_n))], \qquad (4.14)$$

where {αn} is a sequence in [0,1]

Assume that g_i=I, i=1, 2, 3 is an identity mappings, then we have the following algorithm.

Algorithm 4.5: Let mapping Ti and constant r_i>0 for i=1; 2; 3 be the same as in the system (3.9). For arbitrary initial points x_0, y_0,z_0ε Ω_t compute the sequences $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ in $\Omega_t$in the following way:

$$x_{n+1} = P_{\Omega t}(\mathrm{y}_{n+1} - r_1T_1(\mathrm{y}_{n+1},\mathrm{x}_n))$$

$$y_{n+1} = P_{\Omega t}(\mathrm{z}_{n+1} - r_2T_2(\mathrm{Z}_{n+1},\mathrm{y}_n))$$

$$z_{n+1} = P_{\Omega t}(\mathrm{x}_{n+1} - r_3T_3(\mathrm{x}_{n+1},\mathrm{z}_n)),$$

where $\{\alpha^n\}$ is a sequence in [0, 1].

Again assume that g_i=I, i=1, 2, 3 is an identity mappings, then we have the following algorithm.

Algorithm 4.6 Let mapping Ti and constant r_i>0 for i=1, 2, 3 be the same as in the system (3.9). For arbitrary initial points x_0, y_0, z_0 ε Ω_t compute the sequences $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ in Ω_t in the following way:

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n P_{\Omega}(y_n - r_1T_1(y_n,x_n))$$

$$y_{n+1} = (1-\alpha_n)y_n + \alpha_n P_{\Omega}(\mathrm{z}_n - r_2T_2(\mathrm{z}_n,\mathrm{y}_n))$$

$$z_{n+1} = (1-\alpha_n)z_n + \alpha_n P_{\Omega}(\mathrm{x}_n - r_3T_3(\mathrm{x}_n,\mathrm{z}_n)),$$

where {αn} is a sequence in [0, 1]:

Now we prove the strong convergence of the sequences generated by Algorithm 4.4 to a unique solution set of the system.

Theorem 4.7 Let the mappings T_i, g_i and r_i> 0, i=1, 2, 3 be the same as in the Theorem 4.3. If the constant r_i, i=1; 2; 3 satisfy the conditions (4.1), (4.2) and $\sum_{n=0}^{\infty}\alpha_n = 0$ then the iterative sequences $\{(x_n, y_n, z_n)\}$ generated by Algorithm 4.4 converges strongly to a unique solutions (x^*, y^*, z^*) of the system (3.9).

Proof: Theorem 4.3 guarantees that the existence of a unique solution set (x^*, y^*, z^*) ε H × H × H

With $(g(x^*), g(y^*), g(z^*))$ ε Ωt × Ωt × Ωt for the system (3.9). Since

$$r_1 < \frac{t'}{1+\|T_1(y^*,x^*)\|}, \; r_2 < \frac{t'}{1+\|T_2(z^*,y^*)\|} \; and \; r_3 < \frac{t'}{1+\|T_3(\mathrm{x}^*,z^*)\|}, t' \in (0,t)$$

Therefore, from the Lemma 3.4, (x^*, y^*, z^*) satisfies the system (3.4). For each n ≥ 0 we have

$$x^* = (1-\alpha_n)x^* + \alpha_n\left[x^* - g_1(x^*) + P_{\Omega t}(g_1(y^*) - r_1 T_1(y^*, x^*))\right]$$

$$y^* = (1-\alpha_n)y^* + \alpha_n\left[y^* - g_2(y^*) + P_{\Omega t}(g_2(z^*) - r_2 T_2(z^*, y^*))\right]$$

$$z^* = (1-\alpha_n)z^* + \alpha_n\left[z^* - g_3(z^*) + P_{\Omega t}(g_3(x^*) - r_3 T_3(x^*, z^*))\right], \quad (4.15)$$

where {αn} is a sequence in [0, 1]: Since n ε N, $(g(y^*), g(yn))$ εΩt;

$$r_1 < \frac{t'}{1+\|T_1(y^*, x^*)\|}, \ r_2 < \frac{t'}{1+\|T_2(z^*, y^*)\|} \ and \ r_3 < \frac{t'}{1+\|T_3(x^*, z^*)\|}, t' \in (0, t)$$

It is easy to see that for each n ε N,

$$(g_1(y^*) - r_1T_1(y^*, x^*)), (g_1(y_n) - r_1T_1(y_n, x_n)) \in U(t')$$

From (4.14),(4.15) and Proposition 2.8, we have

$$20. \ \|(x_{n+1} - x^*)\| \le (1-\alpha_n)\|(x_n - x^*) + \alpha_n(\|x_n - x^* - (g_1(x_n) - g_1(x^*))\|$$

$$+\|P_{\Omega t}(g_1(y_n) - r_1T_1(y_n, x_n)) - P_{\Omega t}(g_1(y^*) - r_1T_1(y^*, x^*))\|$$

$$\le (1-\alpha_n)\|(x_n - x^*\| + \alpha_n \ \|x_n - x^* - (g_1(x_n) - g_1(x^*))\|$$

$$+\delta(\|g_1(y_n) - g_1(y^*) - r_1(T_1(y_n, x_n) - T_1(y^*, x^*))\|))$$

$$\le (1-\alpha_n)\|(x_n - x^*\| + \alpha_n \ \|x_n - x^* - (g_1(x_n) - g_1(x^*))\|$$

$$+\delta(\|y_n - y^* - (g_1(y_n) - g_1(y^*))\| + \|yn - y* - r_1(T_1(y_n, x_n) - T_1(y^*, x^*))\|)) \quad (4.16)$$

Since T_1 is relaxed (η_1, v_1) cocoercive mapping and λ_1-Lipschitz continuous mapping in the first variable and g_1 is μ_1-strong cocoercive mapping and Σ_1-Lipschitz continuous mapping, we have

$$\|x_n - x^* - (g_1(x_n) - g_1(x^*))\| \le P_1\|x_n - x^*\|, \quad (4.17)$$

$$\|y_n - y^* - (g_1(y_n) - g_1(y^*))\| \le P_1\|y_n - y^*\| \quad (4.18)$$

and

$$\|y_n - y^* - r_1(T_1(y_n, x_n) - T_1(y^*, x^*))\| \le \sqrt{1 - 2r_1(\upsilon_1 - \eta_1\lambda_1^2 + r_1^2\lambda_1^2}\ \|y_n - y^*\| \quad (4.19)$$

Combining (4.16)-(4.19) we get

$$\|x_{n+1} - x^*\| \le (1-\alpha_n)\|x_n - x^*\| + \alpha(p_1\|x_n - x^*\| + \theta\|y_n - y^*\|) \quad (4.20)$$

where p1 and $\theta_1$are same as in (4.10). By $(g_2(z^*), g_2(z_n)) \in \Omega$, $(n \in N)$

$$r_2 < \frac{t'}{1+\|T_1(y^*, x^*)\|}, \ r_2 < \frac{t'}{1+\|T_2(z_n, y_n)\|}$$

Therefore, for each n Σ N,

$$g_2(z^*) - r_2T_2(z^*, y^*), \ g_2(z_n) - r_2T_2(z_n, y_n) \in \mu(t').$$

Since T_2 is relaxed (η_2,v_2)-cocoercive mapping and λ_2-Lipschitz continuous mapping in the first variable and g_2 is _2-strong cocoercive mapping and _2-Lipschitz continuous mapping in a same way of (4.16) - (4.20), we get

$$\|y_{n+1} - y^*\| \le (1-\alpha_n)\|y_n - y^*\| + \alpha(p_2\|y_n - y^*\| + \theta_2\|z_n - z^*\| \quad (4.21)$$

where p2 and _2 are same as in (4.11). By $(g_3(x^*), g_3(x_n)) \in \Omega$, $(n \in N)$

$$r_3 < \frac{t'}{1+\|T_3(x^*, z^*)\|}, \ r_2 < \frac{t'}{1+\|T_3(x_n, z_n)\|}$$

Hence for each $n \in N$

$$g_3(x^*) - r_3T_3(x^*, z^*), \ g_3(x_n) - r_3T_3(x_n, z_n) \in \mu(t').$$

Since T_3 is relaxed (η_3, v_3)-cocoercive mapping and λ_3-Lipschitz continuous mapping in the first variable and g_3 is μ_3-strong cocoercive mapping and ξ_3-Lipschitz continuous mapping, as same way of (4.16) - (4.20), we get

$$\|z_{n+1} - z^*\| \le (1-\alpha_n)\|z_n - z^*\| + \alpha(p_3\|z_n - z^*\| + \theta_3\|x_n - x^*\|) \quad (4.22)$$

where p_3 and Θ_3 are same as in (4.12). Now

$$\|(x_{n+1}, y_{n+1}, z_{n+1}) - (x^*, y^*, z^*)\|_* \le (1-\alpha_n)\|(x_n, y_n, z_n) - (x^*, y^*, z^*)\|_*$$

$$+\alpha_n((p_1+\theta_3)\|x - x^*\| + (p_3+\theta_1)\|y - y^*\| + (p_3+\theta_2)\|z - z^*\|$$

$$\le (1-\alpha_n)\|(x_n, y_n, z_n) - (x^*, y^*, z^*)\|_* + \alpha_n((p_1+\theta_3)\|x - x^*\| + (p_3+\theta_1)\|y - y^*\| + (p_3+\theta_2)\|z - z^*\|$$

$$\le (1-\alpha_n)\|(x_n, y_n, z_n) - (x^*, y^*, z^*)\|_* + \alpha_n l\|(x_n, y_n, z_n) - (x^*, y^*, z^*)\|_*$$

$$\le (1-(1-l)\alpha_n)\|(x_n, y_n, z_n) - (x^*, y^*, z^*)\|_*$$

$$\le \prod_{i=0}^{n}(1-(1-l)\alpha_i)\|(x_0, y_0, z_0) - (x^*, y^*, z^*)\|_* \quad (4.23)$$

where ` is same as in (4.13). From condition (4.2) we get ` 2 (0; 1). Since $\sum_{n=0}^{\infty}\alpha_n = \infty$

we get

$$\lim_{x\to\infty}\prod_{i=0}^{n}(1-(1-l)\alpha_i) = 0 \quad (4.24)$$

Therefore from (4.23)-(4.24)

$$\|(x^n, y^n, z^n) - (x^*, y^*, z^*)\|_* \to 0 \ as \ n \to \infty$$

and the sequences {(x_n; y_n; z_n)} suggested by algorithm 4.4, converges strongly to a unique solution set (x^*, y^*, z^*) of the general system of regularized nonconvex variational inequalities. In a similar way, we can prove the convergence of iterative sequences generated by Algorithm 4.5 and Algorithm 4.6.

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