

# A Simplified Approach for Stormwater Drainage Networks Sizing

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## Abstract

In this work, a modification and a generalization of the “Italian-Storage method” (ISM), a renowned method for sizing rainstorm drainage systems, are proposed and applied. The approach adopts a steady non-linear rainfall-runoff transformation probabilistic model within a “variational” or “maximizing” procedure. In this model, the transformation of rainfall in “excess (or effective) rainfall” is accomplished by means of the Rational Method, while the travel time is neglected. The flow within each link of the system, induced by a rectangular shaped discharge hydrograph for any rainfall duration, is assumed to be unsteady but locally uniform (Kinematic modeling). Combining the hydrological and hydraulic sub-models, and considering different couples of rainfall heights-durations, consistent with the local IDF curve, a “critical duration” is evaluated. Once the critical duration has been obtained, a maximum of maxima instantaneous flow discharges is evaluated for such duration, and subsequently, with such a discharge each link of the storm water drainage system can be designed.

**Keywords:** Italian-storage method; Sewer/rural drainage networks

## Introduction

In the last century, several methods have been proposed in the technical literature in order to allow the sizing and verification of storm water drainage networks, given a probabilistic framework. In particular, starting from several pioneering works [1-6], a few approaches, based on the adoption of optimization procedures, have been proposed for the optimal sizing of drainage networks (see, among others, [7-11]). Independently on the specific procedure adopted for the optimization process (e.g., linear or quadratic programming, heuristic algorithms, etc.), the considerable number of evaluations of network performances required in optimization problems imply the adoption of a tool for the fast evaluation of the network hydraulic features (e.g., discharges and flow depths values). As a consequence, very often simplified approaches are preferred to the most accurate, and meanwhile slow, ones [9].

Starting from these considerations, in this work a slight modification and generalization of a method for the sizing of rainstorm drainage networks, well-known in Europe as the “Italian-Storage method” (ISM), is proposed and then applied to some case studies. This method was firstly proposed [12] for sewer systems and then extended [13] for drainage networks. Subsequently the approach was developed in order to give the possibility to straightly and quickly carry out the design of both sewer/rural drainage networks [14,15]. In particular, one of these approaches is nowadays widely adopted in Italy by technicians in the design of drainage systems, especially given the noticeable simplicity in software implementation [14]. Moreover, it allows the estimation of the hydraulic features of the whole design network in a very limited time, allowing its adoption in optimization procedures where it is usually needed the iterative modification of the network physical characteristics (e.g., longitudinal slopes, shapes and sizes of pipes, roughness parameters, upstream/downstream elevations or crown/bed elevations, etc.).

Given that this method has, ‘in nuce’, the whole characteristics of whatever semi-distributed, probabilistically based, hydrologic model, it has been already adopted, within several papers of these authors, together with an ‘extremal’, or ‘variational’, procedure [16-18] for choosing the ‘critical value’ for the rainstorm duration, in order to allow the comprehension of the whole procedure.

## Fundamentals

Generally speaking, if lateral inflow/outflows are left aside, the *de Saint-Venant's* unsteady flow equations could be written as:

$$\frac{\partial \Omega}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad (1)$$

$$\frac{1}{g} \frac{\partial u}{\partial t} + \frac{V}{g} \frac{\partial u}{\partial x} + \frac{\partial h}{\partial x} + (i - j) = 0 \quad (2)$$

where  $i = \sin \beta$  and  $\beta$  = angle between the bed and the horizontal (Figure 1).

If the kinematic wave approximation could be considered, equation (2) becomes  $i = j$  (i.e., locally and instantaneously uniform flow), and then a resistance formula, such as the one proposed by Chezy and shown hereinafter, could be applied:

$$U = k_c \cdot \sqrt{R \cdot i} \Rightarrow Q = k_c \cdot \omega \cdot \sqrt{R \cdot i} \cong \mu \cdot \omega^\alpha \quad (3)$$

where the coefficient  $\mu$  and the exponent  $\alpha$  could be considered constant during the filling/emptying process (with the value of the constants to be defined properly), and calibrated by an iterative procedure.

Applying the equation (1) to a discrete reach of length  $L$  (which could be assumed, for instance, equal to the pipe length), the governing equations become:

$$\frac{d}{dt} \left( \frac{w}{L} \right) + \frac{[q_{out} - q_{in}]}{L} = 0 \quad \text{or} \quad q_{in} - q_{out} = \frac{dw(t)}{dt} \quad (4)$$

and

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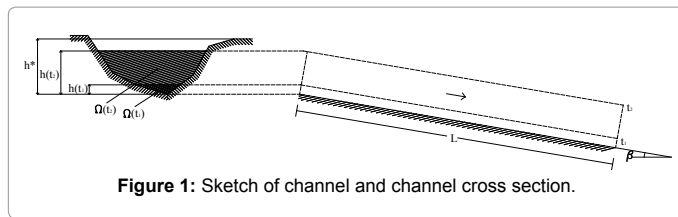


Figure 1: Sketch of channel and channel cross section.

$$q_{out} = \mu \cdot \omega^\alpha \quad \text{or} \quad w = k \cdot q_{out}^V \quad (5)$$

In the previous equations,  $w=L\omega$  is the volume of water stored in the reach of length  $L$  when the discharge flowing out from the channel at time  $t$  is  $q_{out}$  and the area of cross section is  $\omega$ ;  $v = 1/\alpha$ ,  $k = (W^*/Q^{*V})$ ,  $W^* = L\Omega^*$  is the maximum volume of water that can be stored in the reach when the discharge flowing out from the reach itself is the maximum flow discharge capacity  $Q^*$ , and  $\Omega^*$  is the corresponding channel cross section.

By differentiating the first of equations (7), one obtains:

$$d\omega = \frac{\Omega^*}{\alpha Q^{*V/\alpha}} \cdot q_{out}^{V/\alpha-1} dq_{out} \quad \text{or} \quad dw = \frac{W^*}{\alpha Q^{*V/\alpha}} q_{out}^{V/\alpha-1} dq_{out} \quad (6)$$

Substitution of Equation (8) into the Equation (6) gives:

$$(q_{in} - q_{out}) = \frac{W^*}{\alpha Q^{*V/\alpha}} \cdot q_{out}^{V/\alpha-1} \frac{dq_{out}}{dt} \quad (7)$$

and then:

$$dt = \frac{W^*}{\alpha Q^{*V/\alpha}} \cdot \frac{q_{out}^{V/\alpha-1}}{q_{in} - q_{out}} dq_{out} \quad (8)$$

or

$$t_2 - t_1 = \frac{W^*}{\alpha Q^{*V/\alpha}} \cdot \int_{(q_{out})_1}^{(q_{out})_2} \frac{q_{out}^{V/\alpha-1}}{(q_{in} - q_{out})} dq_{out} \quad (9)$$

If, at the beginning of the rainstorm ( $t_1=0$ ), the discharge flowing out from the reach is  $(q_{out})_1=0$  and, at time  $T_f$ , the maximum permissible discharge flowing out from the reach is  $(q_{out})_2=Q$ , the Equation (11) becomes:

$$T_f = \frac{W^*}{\alpha Q^{*V/\alpha}} \cdot \int_0^Q \frac{q_{out}^{V/\alpha-1}}{(q_{in} - q_{out})} dq_{out} \quad (10)$$

where  $T_f$ , namely filling time, is the time needed to reach the maximum permissible discharge  $Q$ .

If  $q_{in}$  = constant in time and the "attenuation ratios"  $z=q_{out}/q_{in}$  and  $Z=Q/q_{in}$  are considered, the time  $T_f$  could be evaluated as:

$$T_f = \frac{W^*}{q_{in}} \cdot \frac{1}{\alpha Z^{*V/\alpha}} \int_0^Z \frac{z^{V/\alpha-1}}{1-z} dz = \frac{W^*}{q_{in}} \cdot \Phi_\alpha(Z^*) \quad (11)$$

where

$$\Phi_\alpha(Z^*, Z) = \frac{1}{\alpha Z^{*V/\alpha}} \int_0^Z \frac{z^{V/\alpha-1}}{1-z} dz = \frac{1}{\alpha Z^{*V/\alpha}} \int_0^Z z^{V/\alpha-1} (1+z+z^2+\dots) dz \quad (12)$$

Equation (13) describes the hydraulic response; to an input discharge  $q_{in}$  constant in time, of a reach where the volume  $W^* = L \cdot \Omega^*$  can be stored.

In order to evaluate  $q_{in}$ , a hydrologic approach could be used. In the approach proposed by Supino and Chow [14,15] the evaluation of discharge  $q_{in}$  is attained considering:

1) Storms characterized by a return period  $T$ , constant rainfall intensity  $i_{d,T}$  in the time interval  $t[0,d]$  and constant rainfall intensity  $0$  for  $t>d$ ;

2) As the input values at most upstream cross section of channels considered in the calculations, the runoff discharges drained from the whole area  $A$  subtended by the most downstream cross section of the channel reach;

3) The infiltration processes, by using the runoff coefficient  $c=A_{imp}/A$ , being  $A_{imp}$  and  $A$ , respectively, the impervious and total drainage area of the basin;

4) Initially neglecting lag phenomenon due to the formation and subsequent arrival of the surface runoff to the upstream section of the reach considered in the analysis, for which is  $q_{in} = c i_d A$ ;

An Intensity-Duration-Frequency relationship having the simple structure

$$i_{d,T} = (a \cdot d^{n-1}) \cdot k_T \Leftrightarrow i_{d,T} = (a \cdot k_T) \cdot d^{n-1} = a_T \cdot d^{n-1} \quad (13)$$

where  $a$  and  $n$  have to be evaluated by using regional and/or local rainfall data (partial series of the annual maximum rainfall depth in the duration  $d$ ).

If the storm duration  $d$  was higher than the channel filling time  $T_f$  ( $d>T_f$ ), the Surface Runoff  $SR=q_{in}d$  would be higher than the storage availability  $W^*$ , and, then, the maximum value of the flow depth  $h'_{max}$ , corresponding to the maximum available flow area  $\Omega^*$ , would be exceeded (causing, for instance, a pressurized flow instead of a free surface flow).

On the other hand, if  $d<T_f$  also  $SR<W^*$ . As a direct consequence the maximum cross-sectional flow area,  $\Omega$ , and the maximum flow depth,  $h_{max}$ , both attained during the filling process, should follow the following relations:  $\Omega < \Omega^*$ ,  $h_{max} < h'_{max}$  and  $(Q_{out})_{max} < (Q^*)_{max}$ . In this case, the sizes and/or slope considered should be higher than those strictly needed, and, as a consequence, the construction costs would grow.

Given the previous framework, it is evident that, in order to avoid both under- and over- sizing of cross-sections/slopes/conveyance capability, it would be preferable to consider the following conditions:

$$d=T_f, \quad \Omega=\Omega^*, \quad h_{max}=h'_{max}, \quad Q_{out}=Q^*_{out} \quad \text{and} \quad Z = \frac{Q_{out}}{q_{in}} = \frac{Q^*_{out}}{q_{in}} = Z^*$$

As a consequence, it would result:

$$\Phi_\alpha(Z^*, Z) = \Phi_\alpha(Z) = 1 + \frac{Z}{1+\alpha} + \frac{Z^2}{1+2\alpha} + \dots + \frac{Z^j}{1+j\alpha} + \dots \quad (14)$$

By substituting equation (16) in equation (13), together with the condition  $d = T_f$ , one would obtain:

$$Q_{out} = \frac{(c a A)^{V/\alpha}}{W^{V/\alpha-1}} \cdot Z \cdot [\Phi_\alpha(Z)]^{n-1} \quad (15)$$

or

$$Q_{out} = \left[ \frac{\mu^{\frac{1}{\alpha} \left( \frac{1}{n} - 1 \right)} (c a A)^{\frac{1}{n}}}{L^{\frac{n-1}{n}}} \cdot Z \cdot [\Phi_\alpha(Z)]^{\frac{n-1}{n}} \right]^{\frac{\alpha n}{1-n(1-\alpha)}} \quad (16)$$

If the ‘udometric coefficient’ (i.e., the contribution of the basin unitary area to the formation of peak discharge),  $u = Q_{out}/A = Q/A$ , and the specific storage (i.e., volume of water stored in the reach of length  $L$  per basin unitary area),  $w = W/A$ , are introduced, equation (17) becomes:

$$u = \frac{(ca)^{1/n}}{w^{1/n-1}} \cdot Z \cdot [\Phi_\alpha(Z)]^{n-1} \quad (17)$$

For a fixed value of  $Z$ , is it possible to evaluate:

- by equation (17), the values of  $Q_{out}$  ;
- by equation (18), the corresponding value of  $u$ ;
- by the following equation

$$d = \left( \frac{u}{Zca} \right)^{\frac{1}{n-1}} \quad (18)$$

the corresponding rainstorm duration.

Then, by changing the  $Z$  value (i.e.: by varying the storm duration  $d$  and, subsequently, inflow discharge  $q_{in}$ ), the evaluation of the maximum discharge value,  $Q_{max} = \max_Z [Q_{out}(Z)]$ , becomes possible. This is the main reason why in the technical literature the procedure above described is known as ‘variational’ (or ‘extremal’) approach. In particular, the duration  $d_{crit}$  for which  $Q_{out} = Q_{max}$  is given by:

$$d_{crit} = \left[ \frac{Q_{max}}{Z_{crit} ca A} \right]^{\frac{1}{n-1}} \quad (19)$$

and is usually defined as ‘critical storm duration’.

As a consequence, the maximum  $u_{max} = \max_Z [u(Z)] = \max_d [u(d)] = u(d_{crit})$  could be evaluated by:

- a) differentiating equation (18) with respect to  $Z$ ;
- b) putting the above obtained derivative equal to zero, in order to evaluate  $Z_{crit}$  (and, then, the critical rainstorm duration  $d_{crit}$ );

c) Substituting this value of  $Z$  within equation (18).

Following these steps, it is possible to show that, at ‘critical’ conditions, it needs:

$$(1 - Z_{crit}) \cdot \Phi_\alpha(Z_{crit}) = \frac{1 - n}{1 + n \cdot (\alpha - 1)} \quad (20)$$

Using the Equation (21) for each couple of  $n$  and  $\alpha$  values, it is possible to evaluate, by a trial-and-error approach, corresponding value of  $Z_{crit}$  and, then, corresponding value of the product  $Z_{crit} \cdot [\Phi_\alpha(Z_{crit})]^{n-1}$ , both present in equation (18).

A few values of  $Z_{crit}$  and  $Z_{crit} \cdot [\Phi_\alpha(Z_{crit})]^{n-1}$  are summarized, for fixed  $\alpha$  and  $n$  values, in Table 1.

It is worth noticing that the values of the product  $Z_{crit} \cdot [\Phi_\alpha(Z_{crit})]^{n-1}$  can be approximated by using different interpolation formulas. Puppini and Supino [13,15] proposed the following expression:

$$Z_{crit} \cdot [\Phi_\alpha(Z_{crit})]^{n-1} \cong (\lambda_1 \alpha + \lambda_2) \cdot n \quad (21)$$

where  $\lambda_1=0.259$  and  $\lambda_2=0.518$  (with maximum errors about 5%) for [13] formulation and  $\lambda_1=0.221$  and  $\lambda_2=0.574$  for [15] one, with a maximum error lower than 3%.

Other and better approximation can also be obtained if the following relation is considered [15]:

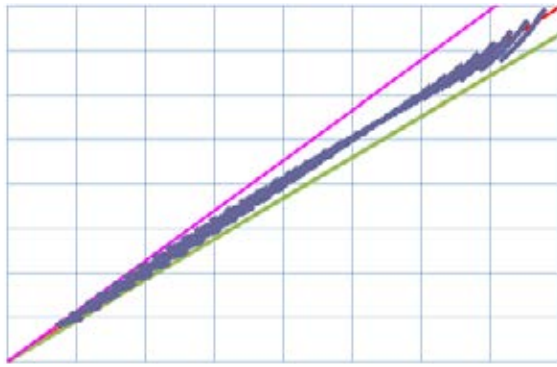
$$Z_{crit} \cdot [\Phi_\alpha(Z_{crit})]^{n-1} \cong (43,3 \cdot \alpha + 40,7) \cdot n - (47,2 \cdot \alpha + 68,5) \cdot n^2 \quad (22)$$

where the variables involved are expressed considering the following units of measurement:  $u$  [l/(s·ha)];  $a$  [m/day] and  $w$  [m].

In order to evaluate the maximum peak discharges, flow depths and volumes stored in the reach of length  $L$ , the following first order approximation relation can be adopted, starting from equation (21), (with errors ranging within the interval [-7.80%, +13.44%], (Figure 2):

n	Z <sub>crit</sub>					Z <sub>crit</sub> · [Φ <sub>α</sub> (Z <sub>crit</sub> )] <sup>n-1</sup>				
	α = 1.00	α = 1.25	α = 1.50	α = 1.75	α = 2.00	α = 1.00	α = 1.25	α = 1.50	α = 1.75	α = 2.00
0.100	0.187	0.206	0.224	0.241	0.258	0.075	0.083	0.091	0.099	0.107
0.150	0.271	0.296	0.319	0.340	0.360	0.113	0.125	0.137	0.148	0.159
0.200	0.350	0.378	0.404	0.427	0.449	0.153	0.168	0.182	0.196	0.209
0.250	0.423	0.453	0.480	0.504	0.526	0.193	0.211	0.228	0.244	0.259
0.300	0.491	0.522	0.549	0.573	0.594	0.233	0.254	0.273	0.291	0.307
0.350	0.554	0.584	0.610	0.633	0.653	0.275	0.298	0.318	0.338	0.355
0.400	0.612	0.641	0.666	0.687	0.705	0.318	0.342	0.364	0.384	0.403
0.450	0.666	0.693	0.715	0.734	0.751	0.362	0.387	0.410	0.431	0.449
0.500	0.715	0.740	0.760	0.777	0.791	0.407	0.433	0.456	0.477	0.496
0.550	0.761	0.782	0.800	0.814	0.827	0.454	0.480	0.503	0.524	0.542
0.600	0.802	0.820	0.835	0.848	0.858	0.502	0.528	0.551	0.570	0.588
0.650	0.839	0.855	0.867	0.877	0.886	0.552	0.577	0.599	0.618	0.635
0.700	0.873	0.886	0.895	0.904	0.910	0.604	0.628	0.648	0.666	0.681
0.750	0.903	0.913	0.920	0.927	0.932	0.658	0.680	0.699	0.715	0.729
0.800	0.930	0.937	0.942	0.947	0.950	0.715	0.735	0.751	0.765	0.777

Table 1: A few values of  $Z_{crit}$  and  $Z_{crit} \cdot [\Phi_\alpha(Z_{crit})]^{n-1}$  for fixed  $\alpha$  and  $n$  values.



**Figure 2:** Comparison between the values of  $Z_{crit} \cdot [\Phi_{\alpha}(Z)_{crit}]^{\frac{n-1}{n}}$  obtained by numerical code and the values calculated by equation (24).

$$u = [(\lambda_1 \alpha + \lambda_2)] \cdot n \cdot \frac{(ca)^{\frac{1}{n}}}{W^{\frac{1}{n-1}}} \quad (23)$$

with  $n \in [0.10, 0.80]$ ,  $\alpha \in [1, 2]$ ,  $\lambda_1 = 0.140$ ;  $\lambda_2 = 0.709$ .

A second order approximation is reported hereinafter, (with errors ranging within the interval  $[-2.66\%, +1.26\%]$ , (Figure 3):

$$u = [(\lambda_3 \alpha + \lambda_4) + (\lambda_5 \alpha + \lambda_6) \cdot n] \cdot n \cdot \frac{(ca)^{\frac{1}{n}}}{W^{\frac{1}{n-1}}} \quad (24)$$

with:  $\lambda_3 = 0.347$ ;  $\lambda_4 = 0.364$ ;  $\lambda_5 = -0.340$ ;  $\lambda_6 = 0.566$ .

A third order of approximation has the following structure, (errors ranging within the interval  $[-0.44\%, +1.07\%]$ , (Figure 4):

$$u = [(\lambda_7 \alpha + \lambda_8) + (\lambda_9 \alpha + \lambda_{10}) \cdot n + (\lambda_{11} \alpha + \lambda_{12}) \cdot n^2] \cdot n \cdot \frac{(ca)^{\frac{1}{n}}}{W^{\frac{1}{n-1}}} \quad (25)$$

with:  $\lambda_7 = 0.358$ ;  $\lambda_8 = 0.389$ ;  $\lambda_9 = -0.384$ ;  $\lambda_{10} = 0.464$ ;  $\lambda_{11} = 0.041$ ;  $\lambda_{12} = 0.094$ .

Finally, a fourth order approximation leads to the following form (errors ranging within the interval  $[-0.38\%, +0.82\%]$ , see (Figure 5):

$$u = [(\lambda_{13} \alpha + \lambda_{14}) + (\lambda_{15} \alpha + \lambda_{16}) \cdot n + (\lambda_{17} \alpha + \lambda_{18}) \cdot n^2 + (\lambda_{19} \alpha + \lambda_{20}) \cdot n^3] \cdot n \cdot \frac{(ca)^{\frac{1}{n}}}{W^{\frac{1}{n-1}}} \quad (26)$$

with:  $\lambda_{13} = 0.368$ ;  $\lambda_{14} = 0.364$ ;  $\lambda_{15} = -0.456$ ;  $\lambda_{16} = 0.646$ ;  $\lambda_{17} = 0.193$ ;  $\lambda_{18} = -0.291$ ;  $\lambda_{19} = 0.099$ ;  $\lambda_{20} = 0.250$ .

Summarizing:

a) the errors obtained by each approximation have a decreasing trend as the order of approximation and formal complexity increase (Figure 6);

b) the *udometric coefficient*  $u$  can be expressed as it follows:

$$u = f(\alpha, n) \cdot n \cdot \frac{(ca)^{\frac{1}{n}}}{W^{\frac{1}{n-1}}} \quad (27)$$

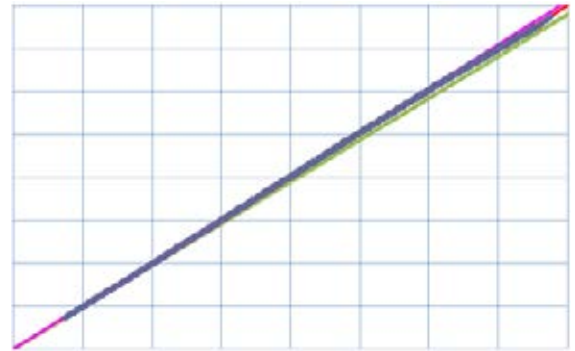
where  $f(\alpha, n)$  is a simple polynomial, function of  $\alpha$  and  $n$ .

Furthermore, also  $Z_{crit}$  can be expressed by means of different interpolation formulas. Indeed, the  $Z_{crit}$  values obtained by equation (21) can be approximated by the following:

$$Z_{crit} \cong (k_1 \cdot n^3 + k_2 \cdot n^2 + k_3 \cdot n + k_4) \cdot \alpha + (k_5 \cdot n^2 + k_6 \cdot n + k_7) \quad (28)$$

where  $k_1 = 0.839$ ;  $k_2 = -1.434$ ;  $k_3 = 0.609$ ;  $k_4 = 0.026$ ;  $k_5 = -0.600$ ;  $k_6 = 1.700$ ;  $k_7 = -0.058$ ; (with errors ranging within the interval  $[-0.04\%, +0.02\%]$  (Figure 7).

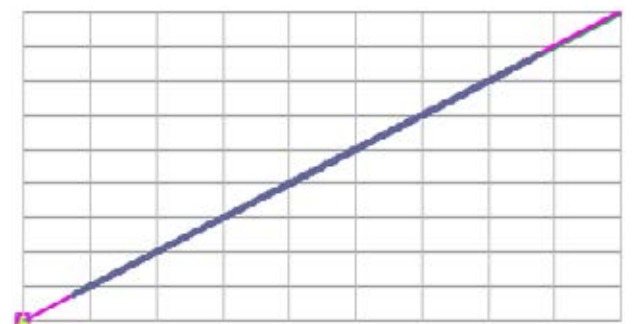
Equation (28) can be easily applied in order to evaluate the main flow characteristics (i.e: maxima of discharge, velocity and flow depth) of the drainage network source channels.



**Figure 3:** Comparison between values of  $Z_{crit} \cdot [\Phi_{\alpha}(Z)_{crit}]^{\frac{n-1}{n}}$  obtained by numerical code and the values calculated by equation (25).



**Figure 4:** Comparison between values of  $Z_{crit} \cdot [\Phi_{\alpha}(Z)_{crit}]^{\frac{n-1}{n}}$  obtained by numerical code and the values calculated by equation (26).



**Figure 5:** Comparison between the values of  $Z_{crit} \cdot [\Phi_{\alpha}(Z)_{crit}]^{\frac{n-1}{n}}$  obtained by numerical code and the values calculated by equation (27).

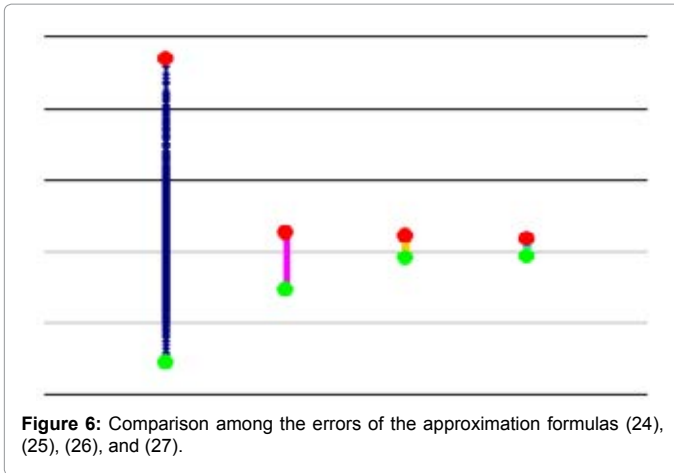


Figure 6: Comparison among the errors of the approximation formulas (24), (25), (26), and (27).

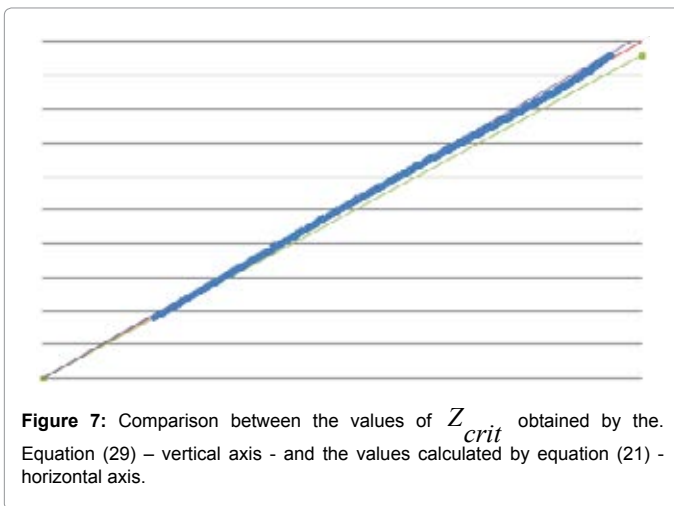


Figure 7: Comparison between the values of  $Z_{crit}$  obtained by the Equation (29) – vertical axis - and the values calculated by equation (21) - horizontal axis.

Once the hydraulic (i.e.: longitudinal slope; cross section; roughness parameter), the hydrological (i.e.: the drainage area of the catchment; the surface runoff coefficient), the climatic (i.e.: the coefficient  $a$  and the exponent  $n$  of the IDF curve) and the design (i.e.: the return period  $T$ ) characteristics of the reach have been defined, the proposed design procedure follows the steps described hereinafter:

1) a first guess value of  $\alpha'$  of the exponent  $\alpha$  in equation (28) has to be assumed (e.g.: if  $\alpha'=\alpha=1$ , the routing is initially carried out by using the *linear reservoir* conceptual model);

2) a guess value  $u'$  of  $u$  is considered;

3) a first guess maximum peak discharge value  $Q'_{out}$  is evaluated (i.e.:  $Q'_{out} = u' \cdot A$ );

4) by means of the chosen steady and uniform roughness formula, the first guess values of the maximum peak flow depth  $h'_{max}$ , the maximum peak flow cross section area  $\Omega'_{out}$ , the maximum water volume stored in the reach  $W'= \Omega'_{out} \cdot L$ , and the maximum specific stored volume  $w'=W'/A$  are evaluated;

5) equation (28) is applied, and a new value of  $u$  (let say  $u''$ ) is evaluated;

$$6) \text{ then, if } \left| \frac{u'' - u'}{\left( \frac{u'' + u'}{2} \right)} \right| \leq \varepsilon_u \quad (29)$$

with  $\varepsilon_u \rightarrow 0$  (for instance,  $\varepsilon_u=0,0001$ ), the iteration process is stopped; otherwise

7) a second guess value of  $u$  ( $u=u''$ ) is considered, and the procedure is repeated from stage 1) to 6) until the relation (30) is satisfied;

8) the value of  $h_{max}$  evaluated during the first iteration process is then subdivided in  $N$  steps (for instance:  $N=100$ ), obtaining  $h_1 = 1 \times h_{max}/N$ ;  $h_2=2 \times h_{max}/N$ ; ...;  $h_N=h_{max}$ , and  $\Omega_1=\Omega_1(h_1)$ ;  $\Omega_2=\Omega_2(h_2)$ ; ...;  $\Omega_N=\Omega_N(h_{max})$ ;

9) by means of the roughness formula  $q = g(h)$ , the discharges  $q_1=g(h_1)$ ,  $q_2 = g(h_2)$ , ... ,  $Q_{max}=g(h_{max})$  are evaluated;

10) then, the values of couples  $(Q_r, \Omega_r)$  are interpolated by using the expression  $q = \mu \cdot \Omega^\alpha$ , thus obtaining a new guess value for  $\alpha$  ( $\alpha = \alpha''$ );

11) the iterative procedure is repeated again from step 1) to 11);

$$12) \text{ then, if } \left| \frac{\alpha'' - \alpha'}{\left( \frac{\alpha'' + \alpha'}{2} \right)} \right| \leq \varepsilon_\alpha \quad (30)$$

with  $\varepsilon_\alpha \rightarrow 0$  (for instance  $\varepsilon_\alpha=0,0001$ ), the second iterative stage is stopped; otherwise

13) a second guess value of  $\alpha$  ( $\alpha = \alpha''$ ) is considered, and the procedure is repeated from stage 1) to 12) until convergence is reached, equation (31).

Usually, after few iterations both the conditions (30) and (31) are satisfied. Thanks to modern computers, the whole procedure described usually takes less than (0.1 - 0.2) s<sup>(1)</sup>.

### Extension of the italian storage method to the whole channel network

In order to apply the approach illustrated above to the whole drainage network, the approach is modified introducing in the equation a new parameter  $w_r$ , obtained by splitting the volume  $w = W/A$  in two parts, namely  $W_0$  and  $W_r$ , for which  $w = w_0 + w_r$ :

- the first term,  $w_r = W_r/A_r$ , where  $W_r$  represents the maximum water volume actually stored within the reach considered in the calculation, and  $A_r$  is the area of the whole basin drained from the most downstream cross-section of the reach  $r$ ;

- the second term,  $w_0 = W_0/A_r$ , in which  $W_0$  represents the whole volume of water stored either in the reaches upstream the reach considered in the calculations or in other channels hydraulically linked to the reaches of the main network or at the surface of basins constituting the catchment whose ending drain is just the reach considered in the calculations. In particular:

<sup>1</sup> Please note that at the beginning of the computations, it is not possible to assign a correct guess value of  $u$ . As a matter of fact, because of the iteration processes obtain the solution oscillating around the final (true) values of  $u$  and  $\alpha$ , it is possible that: *i*) by using a first value  $u'$  too low, the second iteration could give a  $u''$  value too big, for which the second guess  $Q''_{out}$  value could be higher than the flow capability of the channel (and, then, the flow could overcome one or both the banks if the channel is open, or the flow could become 'pressurized flow' if the channel is closed); *ii*) on the opposite, by using a first value  $u'$  too high, the second iteration could give a  $u''$  value too low, for which the second guess  $Q''_{out}$  value could be lower than the flow capability of the channel (the channel itself becomes oversized). As a consequence, it is suggested to perform initial computation of a 'trial' value of  $u'$ . This value could be evaluated by reminding that the final (true) value of  $u$  is independent with respect to: the sizes, the roughness parameter/s, the longitudinal slope and the shape of the channel. Thus, a good' initial value  $u'$  could be obtained by considering, at beginning of the computations, a 'virtual' cross section, characterized by a very large size, and by carrying out the whole iteration process for this 'virtual channel'.



$$W_0 = \sum_{up=1}^{N_{up}^r} [W]_{up}^r + \left( \sum_{up=1}^{N_{up}^r} [W']_{up}^r + \sum_{up=1}^{N_{up}^r} [W''_0]_{up}^r \right) + ([W'_0]_r + [W''_0]_r) \quad (31)$$

where:

$[W]_{up}^r$  is the volume of water stored within the link  $up$  of the network, located upstream of the reach  $r$  considered in the calculations and, then, which does not flow into the reach  $r$  itself;

$[W'_0]_{up}^r$  is the volume of water stored on the surface of the basin directly drained from the reach  $up$  of the network and, then, which does not flow into the reach  $up$  itself;

$[W''_0]_{up}^r$  is the volume of water stored within the whole reaches/tanks directly linked to the reach  $up$  of the network, located upstream of the reach  $r$  considered, but which were not explicitly considered in the calculations ("ghost reaches"); also this volume does not flow into the reach  $r$ ;

$[W'_0]_r$  and  $[W''_0]_r$  are the same of  $[W'_0]_{up}^r$  and  $[W''_0]_{up}^r$ , but they apply to the reach  $r$  under consideration;

$A_{up}^r$  is the area of the basin directly drained from the  $up^{th}$  reach existing upstream the reach  $r$ ;

$A_r$  is the area of the basin directly drained from the reach  $r$ ;

$A = \sum_{up=1}^{N_{up}^r} A_{up}^r + A_r$  is the whole area drained from the most downstream cross section of the reach  $r$  considered in the calculation;

$N_{up}^r$  is the number of reaches located upstream the link under consideration.

As a consequence, the relationships (24), (25), (26), (27) and (28) become, respectively:

$$u = [(\lambda_1\alpha + \lambda_2)] \cdot n \cdot \frac{(ca)^{\frac{1}{n}}}{(w_0 + w_r)^{\frac{1}{n-1}}} \quad (32)$$

$$u = [(\lambda_3\alpha + \lambda_4) + (\lambda_5\alpha + \lambda_6) \cdot n] \cdot n \cdot \frac{(ca)^{\frac{1}{n}}}{(w_0 + w_r)^{\frac{1}{n-1}}} \quad (33)$$

$$u = [(\lambda_7\alpha + \lambda_8) + (\lambda_9\alpha + \lambda_{10}) \cdot n + (\lambda_{11}\alpha + \lambda_{12}) \cdot n^2] \cdot n \cdot \frac{(ca)^{\frac{1}{n}}}{(w_0 + w_r)^{\frac{1}{n-1}}} \quad (34)$$

$$u = [(\lambda_{13}\alpha + \lambda_{14}) + (\lambda_{15}\alpha + \lambda_{16}) \cdot n + (\lambda_{17}\alpha + \lambda_{18}) \cdot n^2 + (\lambda_{19}\alpha + \lambda_{20}) \cdot n^3] \cdot n \cdot \frac{(ca)^{\frac{1}{n}}}{(w_0 + w_r)^{\frac{1}{n-1}}} \quad (35)$$

$$u = f(\alpha, n) \cdot n \cdot \frac{(ca)^{\frac{1}{n}}}{(w_0 + w_r)^{\frac{1}{n-1}}} \quad (36)$$

In order to better understand the meaning of  $W_0$ , it is possible to observe, preliminarily, that to arrive to the relationships (33), (34), (35), (36) and (37), it needs to introduce in the equation (6) the sum  $[W_0 + w_r(t)]$  instead of  $w(t)$ , preserving  $W_0 = \text{constant}$  in time during the filling/emptying processes afflicting the reach  $r$  under consideration.

Usually, the values of  $w_{0,r} = \frac{[W'_0]_{up}^r + [W''_0]_{up}^r}{[A]_{up}^r}$  are considered

parameters of the model (though variable sub-basin by sub-basin) whereas the values of  $[W]_{up}^r$  are taken to be equal to the values of  $W_r$  already evaluated for all the reaches hydraulically upstream with respect to that considered in the calculations.

The last hypothesis (usually defined 'synchronism hypothesis', because it implies that the peak time for all the reach of the networks has to be the same) has to be considered together with the second (usually defined 'independence hypothesis'), which allows, by using equation (37), the performance of a reach upstream another (and also, the performance of a reach upstream a confluence) without evaluating the eventual influence of the filling/emptying phenomena which are simultaneously developing in the downstream reaches.

These hypotheses, though seeming (together with that related to not consider the lag times) the worst present in the model, could, in certain cases, balance one each other, because the backing up prorogued by

Branch	Length	Long. Slope	Subcatch. Area	c	Cross Section Bottom Width	Cross Section Lateral slope	Manning Coeff.	Cross Section Max. Height
n.	m	m/m	m <sup>2</sup>	m <sup>2</sup> /m <sup>2</sup>	m	m/m	m <sup>-1/3</sup> s	m
1	220	0.001	14.9	0.2	3	1	0.025	1
2	530	0.0015	33.8	0.2	3.2	1	0.025	1
3	450	0.001	20.4	0.3	3.5	1	0.025	1.2
4	300	0.002	19.7	0.3	3.2	1	0.025	1
5	250	0.002	8	0.3	3.1	1	0.025	1.1
6	350	0.0008	12.1	0.3	3	1	0.025	1.5
7	450	0.0012	11.5	0.22	3.8	1	0.025	2
8	310	0.0016	12.1	0.25	3.2	1	0.025	1.2
9	260	0.002	19.4	0.25	3.1	1	0.025	1.2
10	260	0.0014	12	0.22	3.8	1	0.025	1.5
11	510	0.0009	19.5	0.2	4	1	0.025	2.2
12	350	0.0022	24	0.25	2.7	1	0.025	1.2
13	380	0.0017	24.5	0.3	2.5	1	0.025	1
14	150	0.0015	13	0.22	2.3	1	0.025	1
15	130	0.0018	19.6	0.22	2.5	1	0.025	1.2
16	150	0.002	17.4	0.2	2.5	1	0.025	1
17	180	0.0021	13.5	0.2	2.6	1	0.025	1.1

Table 2: Geometrical characteristics of the network.

filling phenomena occurring in the downstream reaches could enhance the flow depths in the upstream reaches at peak discharge flows.

### Application to a case study

In this subsection the ISM is applied to a case study. The case study consists of a 17 branch rural drainage network with each branch having a trapezoidal cross section. The network geometrical characteristics are summarized in Table 2, while a schematic representation of the network is reported in Figure 8.

The simulation with the ISM has been carried out by considering  $W_{0,i} = 40$  m for each sub-catchment while the following intensity duration frequency (IDF) curve has been considered:

$$i_{d,T} = \begin{cases} 0.0003674417526 d^{-0.4}, & 0 < d < 3600s \\ 0.0042862673310 d^{-0.7}, & d \geq 3600s \end{cases} \quad (37)$$

For Comparisons purpose, the same network has been simulated with numerical solution of the Kinematic Wave Model (KWM) within a variational approach (see, for instance, [7]) using equation 38 as IDF curve and evaluating the rainstorm runoff with the runoff coefficient method (see point 4 at section 2)

Results of the simulation are reported in Table 3 in terms of both peak discharge  $Q_{max}$  and maximum flow depth  $H_{max}$  obtained with ISM and KWM.

By inspection of Table 3 it is evident that the ISM model can be successfully employed in the design of rainstorm drainage networks.

### Conclusion

In this work a slight modification and a generalization of a well-known method for sizing rainstorm drainage networks, very common in European contexts, the "Italian-Storage method" (ISM), are proposed and applied. The application to a case study has shown that the ISM is equivalent to the numerical solution of the Kinematic Wave Model, used within a variational approach in order to find the maximum discharges and flow depths flowing through the branch of a drainage

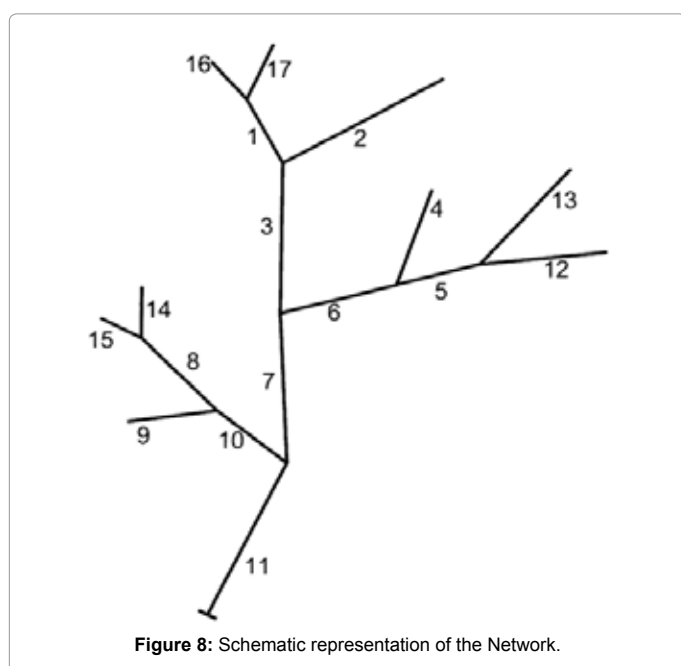


Figure 8: Schematic representation of the Network.

Branch	ISM		KWM	
	Qmax [m³/s]	Hmax [m]	Qmax [m³/s]	Hmax [m]
1	0.843	0.8	0.91	0.836
2	0.567	0.613	0.573	0.616
3	1.869	1.164	1.872	1.165
4	0.64	0.657	0.586	0.624
5	1.462	1.077	1.391	1.047
6	2.478	1.469	2.178	1.368
7	4.478	1.821	4.48	1.826
8	0.954	0.829	0.961	0.833
9	0.512	0.548	0.497	0.538
10	1.628	1.029	1.65	1.037
11	6.443	2.169	6.487	2.175
12	0.594	0.692	0.548	0.66
13	0.767	0.834	0.675	0.776
14	0.304	0.512	0.317	0.525
15	0.475	0.634	0.397	0.571
16	0.357	0.537	0.342	0.523
17	0.267	0.442	0.316	0.489

Table 3: Result of the simulation in terms of peak discharge  $Q_{max}$  and maximum flow depth  $H_{max}$ .

network. For this reason, the ISM can be satisfactory employed in the design of both urban and rural rainstorm drainage network.

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