An Optimal Ordering Decision-making Model with Random Demand Under Carbon Constraint

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Abstract

Under the increasing pressure to reduce carbon emission, the enterprises need to actively take account into carbon emission, and take low carbon action among daily business activities. This not only relate to the realization of carbon emission goal, but probably efficient solution to our country carbon emission targets. Applying to optimal theory under carbon-constrained, this paper comprehensively include economical cost and environment cost to construct random optimal decision-making model. Then using matlab numerical analysis, this paper reveals the decision-making mechanism of enterprise ordering making under carbon cap constrained, and provides management implications and future research direction. The result indicates that: An enterprise can significantly reduce carbon emission without significantly increasing cost through adjusting ordering quantity. Enterprise’s carbon emissions show a certain correlation with order quantity when carbon emission cap is within the scope of threshold value, but optimal ordering decision-making has no correlation with carbon emission cap when cap is beyond the scope of threshold value. Caron emission cap is set voluntarily by an enterprise’s decision maker or put forward mandatorily by an external regulatory agency, and its setting should be reasonable and scientific.

Keywords: Random demand; Carbon cap; Constraint; Ordering model

Introduction

When dealing with the global warming issues, almost all researchers and managers in different fields agree that we need to take action to effectively curb the carbon emissions. The Intergovernmental Panel on Climate Change (IPCC) estimates an increase of 1.8-4.1°C in Earth’s temperature by the end of this century because of increased CO₂, methane and nitrous oxide. U.S.-China Joint Announcement Climate change on 12 Nov. 2014, the United States intends to achieve an economy-wide target of reducing its emissions by 26%-28% below its 2005 in 2025 and to make best efforts to reduce its emission by 28%. China intends to achieve the peaking of CO₂ emission around 2030 and to make best efforts to peak early and intends to increase the share of non-fossil fuels in primary energy consumption to around 20% by 2030. The announcement targets of the United States and China urgently require enterprises to tackle corresponding measures to reduce carbon emission. At the same time, many evidences make increasing clear that low carbon enterprises can drive innovation, yield significant business opportunities, strengthen economic growth and bring broad benefits - from sustainable development to increase energy security, reduce cost, and strengthen competitive advantage. These latest regulations and research suggest the future cannot resemble the past, and enterprises may have change traditional operational decision-making with regard to procurement, production, and inventory management. So enterprises should integrate carbon emission reduction into their thinking, and modified their strategies and daily activities.

However, most of the existing literature focus on such issues as product recycling or reuse, or equipment of alternative energy through R&D, re-designing products and packaging, and using new energy - namely using cleaner energy with low pollution from the source, and carbon capture technology, but they neglect that it is possible, by making only adjustments in the ordering decision, to significantly reduce carbon emission without significantly increasing cost. According to Benjaafar et al. [1] research, little document on top international academic periodicals (e.g. Management Science, Operation Research) published by Informs have focused on issues about operation management incorporating carbon emission through extended retrieval of these periodicals. With the deepening of studies, several scholars have recognized this problem and have attempted to construct quantitative model, which typically focus on either minimizing cost or maximizing profit, to include carbon emission cap. Benjaafar et al. [1] use EOQ model to study the extent to which carbon reduction requirements can be addressed by operational adjustments, as an alternative (or a supplement) to costly investments in carbon reducing technologies. Based on Benjaafar’s enlightening study, Chen et al. [2] provide analytical support for the notion that it may be possible, via modifying order quantities by using the EOQ model under a variety of environmental regulations including strict carbon caps, carbon tax, cap-and-offset, and cap-and-price, to significantly reduce emissions without significantly increasing cost. Abis et al. [3] discuss multi-sourcing lot-sizing problems under different perspectives including periodic carbon emission constraint, cumulative carbon emission constraint, and global carbon emission constraint and rolling carbon emission constraint. He et al. [4] establishes an optimization decision-making model in production and store constraints of carbon emission, acquires optimla production decision, carbon emission right trading decision and purification decision, clarifies the affection of product market demand fluctuation, carbon right price and product price changes on enterprise’s optimized decision, and discusses the conversion conditions whether enterprises carry out purification of...
carbon emission or trading carbon emission right. Bozorgi et al. [5] propose a new inventory to find the optimal order quantity based on cost and emission functions minimization, and the mathematical proof of the optimality of the solutions are presented. Arikun and Jannernegg [6] consider single period inventory model with product carbon footprint constraint, and an upper bound for the carbon constraint is specified as a benchmark derived either from the company’s environmental target or form an industry standard. Lan et al. [7] use the improved EOQ model considering three-level production-inventory system under two circumstances of decentralized and centralized supply chain, and find that the influence of carbon price is more significant when the manufacture adopts centralized supply chain through heuristic algorithm. Chollette and Venkat [8] use web-based tool to calculate the energy and carbon emissions associated with each transportation link and storage echelon, and find that supply chain configurations can result in vastly different energy and emission’ profiles. Xie and Zhao [9] adopt neoclassical economics and game theory to analyze the influence and profits under differ cooperation of upstream and downstream enterprises to reduce carbon emission, and find that the different cooperation of carbon emission and product price can result in vastly different both cost and emission functions. The above literature assumes that customer tastes and need are deterministic can be expressed in a precise function, but in reality customer requirements could vary in the temporal space between product conceptualization and market introduction. Thus, these studies leave a gap to be bridged and the gap reveals a critically important research problem to be resolved, that is, how to consider optimal ordering quantity with random demand under carbon emission constraint and dynamic customer needs? This paper, therefore, adopts random theory to comprehensively analyze optimal ordering quantity. The quantitative analysis of our study can help managers to ensure that they are not overpaying for storage, administrative, financing and insurance cost by holding excessive levels of stock.

This paper constructs enterprises’ optimal ordering decision-making model with random demand under carbon emission constraint, to find whether significantly reducing carbon emission without significantly increasing cost through adjusting ordering quantity.

Model Description and Problem Assumptions

The following subsections summarize the main model characteristic and problem assumptions.

Model description

Because of carbon emission awareness and regulations have increased the pressure on enterprises to take carbon emission considerations into account in their operation management. This paper applies life cycle assessment (LCA) to analyze carbon emission impacts on the major part of enterprise value activities. The model is organized in four main activities including ordering, production, and inventory activities to efficiently and effectively develop low carbon, and the model function aims to maximize its benefit (profit, utility, etc.) from these activities and carbon emission resulting from these activities under random customer demand. Through this model, our object is to draw attention to the strong connection between operational decisions and the carbon emission constraint, and the extent to which concerns about about carbon emission can be addressed by adjusting operational decision. Using this model, we explore how carbon emission constraint could be integrated into operational decision-making, and acquire useful insights by carrying out numerical experiments.

This model includes ordering cost, production cost, inventory cost and carbon emission cost resulting from these activities. They show how much to ordering is very important in a single period when considering carbon emission constraint. The enterprise makes ordering decision that minimizes not only ordering cost, production cost, inventory holding cost, and inventory shortage cost, but also carbon emission cost associated with these activities. Ordering cost may include procurement cost, transportation cost, negotiation cost, process setup cost etc. Inventory holding cost are cost incurred if ordering quantity exceeds demand in one period, while inventory shortage cost are cost if customer demand cannot be fulfilled in one period. In the presence of carbon emission consideration, we calculate carbon emission cost deriving from ordering, production and inventory holding.

Problem assumptions

To construct a randomly optimal ordering decision-making model under carbon emission constraint, this paper makes the following assumptions: (a) Cost function includes ordering cost, production cost, inventory holding cost and inventory shortage cost, and carbon emission cost arising from these business operation activities. (b) Emissions are associated with each unit ordering or production and with the storage of each unit held in inventory in each period. (c) It is not considered that excess inventory can be sold in the next period or unsatisfied demand can be satisfied in the next period. (d) Replenishment from the supplier and ordering lead time are not considered. (e) Production capability limitation is not considered. (f) Customer demand is random. (g) Carbon cap can be mandated by a government department, and enterprise must adhere to this limits on carbon emission. (h) Carbon price is fixed.

This underlying precondition in these assumptions is that carbon emissions can be measured and quantified, and emission data can be extracted with ease. It is lucky that many enterprises have started to make arduous efforts to record carbon footprints of their activities, because they can show that they strictly comply with requirements of regulation policy and better display carbon footprints of their products to customers. And these enterprises have managed to obtain support from institutions- some international NGOs or third-party organizations have defined carbon emission types of products and processes, put forward feasible carbon footprint measurement and evaluation methods, and conducted certification of relevant carbon footprints of processes, products and services. Relevant symbols and definition are as shown in Table 1.

Optimal Ordering Decision-making Model under Carbon Emission Constraint

This section considers optimal ordering decision-making models on deterministic demand and random demand respectively under carbon emission constraint, and uses dual method to solve the models by converting random demand into deterministic demand. First of all, comprehensive cost model includes ordering cost, production cost, inventory cost and shortage cost under carbon emission constraint, and then constructs deterministic decision based on theoretical models and assumptions above. Finally the market’s random demand is considered- if a product cannot satisfy market demand, the enterprise should bear shortage loss; if product supply exceeds demand, the enterprise shall bear inventory cost and associated carbon emission cost arising from overstocked products.

Optimal ordering decision-making model on deterministic demand under carbon emission constraint

In the case of deterministic demand, when customer demand in the
market is \( d \), an enterprise needs to determine the order quantity \( x \) in order to satisfy customer demand. Per unit ordering cost is indicated with \( c_p \) and total ordering cost is indicated with \( TC_p \). Unit production cost is indicated with \( c_m \) and total production cost is indicated with \( TC_m = c_m x \). If customer demand exceeds ordering quantity \( (d \geq x) \), a certain shortage cost will be incurred, and total shortage cost is indicated with \( TC_s \). The function can be expressed as:

\[
TC_p = \begin{cases} 
(c_p(d - x)), & d > x \\
0, & d \leq x 
\end{cases}
\]

If \( d \leq x \), excess ordering quantity will incur inventory cost \( TC_s \). The function can be expressed as:

\[
TC_s = \begin{cases} 
(c_s(x - d)), & x > d \\
0, & x \leq d 
\end{cases}
\]

An enterprise’s total cost is indicated with \( TC = TC_p + TC_m + TC_s \). Considering environmental cost is incurred due to carbon emission during ordering, production and inventory storage, and total cost function is indicated with \( G(x,d) \), the function can be expressed as:

\[
G(x,d) = (c_m + c_s)x + (c_m + c_p)d + c_s[d - x]^+ + (c_k + c_m + c_g)d[x - d]^-
\]

\( (1) \)

Decision-making optimization objective in real scenario is indicated with \( Min G(x,d) \). Decision-making model under carbon cap constraint is expressed as:

\[
Min \ G(x,d)
\]

\( s.t. \ 0 \leq x \leq \frac{Q}{g_1 + g_2 + g_3}
\]

(2)

In formula (1), objective function \( G(x,d) \) is equivalent to:

\[
Max \ \{c_m + c_g + c_m g_1 + c_m g_2 - c_p \} x - c_p d, c_m + c_p g_1 + c_m g_2 + c_m g_3 \} \} x - (c_m + c_g) d\}
\]

So model (2) under carbon emission restriction can be further expressed as:

\[
Min \{c_m + c_g + c_m g_1 + c_m g_2 - c_p \} x - c_p d, c_m + c_p g_1 + c_m g_2 + c_m g_3 \} \} x - (c_m + c_g) d\}
\]

\( s.t. \ 0 \leq x \leq \frac{Q}{g_1 + g_2 + g_3}
\]

(3)

Optimal ordering decision-making model on random demand under carbon emission constraint

In real commercial activities, customer demand \( d \) is usually highly uncertain and somewhat random. The method of two-phase scenario planning is used to describe random demand in this section because data on continuous demand are usually not easy to be obtained and discrete probability distribution of market demand can be usually obtained. Customer’s random demand in the market is \( D \). Assuming that there are \( d_k \) \( (k = 1,2,3, \ldots) \) kinds of scenarios, corresponding probability is \( \{p_1, p_2, p_3, \ldots \} \) respectively and \( \sum p_k = 1 \). Objective function is \( G(x,D) \). Decision-making model takes expected cost minimization as the objective function. Total cost includes two parts: one part is enterprise’s ordering cost, production cost and corresponding carbon emission cost determined by ordering quantity, and another part is enterprise’s inventory holding cost or shortage cost and carbon emission cost arising from extra inventory; the former is related with order quantity \( x \), and the latter is related with actual demand \( d_k \). Optimal objective function can be expressed as:

\[
E[G(x,D)] = G(x,D) \sum_{k=1}^{K} p_k
\]

(4)

As it is relatively complicated to solve the model (4), dual method needs to be adopted. The condition after satisfying the demand is considered here; that is to say, demand is \( d \), and corresponding probability of occurrence is 1. In this case, random demand can be converted into deterministic demand. Through dual transformation of the model (3) in the case of deterministic demand, the following equivalent model can be obtained:

\[
Min \ y_k
\]

\( s.t. \ y_k \geq (c_m + c_g + c_m g_1 + c_m g_2 - c_p) x - c_p d \)

\( y_k \geq (c_m + c_g + c_m g_1 + c_m g_2 + c_m g_3) x - (c_m + c_g) d \)

\( x \in X \)

(5)

Thus, after determining random demand \( D \), if \( x \) in model (5) is determined, model (5) is equivalent to the optimal value of model (3) on deterministic demand. So when there are \( K \) possibilities of random demand scenarios, model (4) is equivalent to a two-phase random linear programming model.

\[
Min \ y_k
\]

\( s.t. \ y_k \geq (c_m + c_g + c_m g_1 + c_m g_2 - c_p) x - c_p d \)

\( y_k \geq (c_m + c_g + c_m g_1 + c_m g_2 + c_m g_3) x - (c_m + c_g) d \)

\( 0 \leq x \leq \frac{Q}{g_1 + g_3}, \ y_k \geq 0 \) \( (k = 1,2,3, \ldots) \)

(6)

Variable \( x \) in model (6) is phase-I decision variable, and variable \( y_k (k = 1,2,3, \ldots) \) is phase-II decision variable. In objective function, as phase-I decision variable \( x \) is put in constraints; \( x \) coefficient in objective function is 0.

Empirical Application and Main Conclusions

This section applies the above model to numerical simulation.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Descriptions</th>
<th>Notation</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>Scenario set</td>
<td>( c_p )</td>
<td>Unit product penalty cost arising from shortage</td>
</tr>
<tr>
<td>( d )</td>
<td>Customer’s deterministic demand</td>
<td>( c_g )</td>
<td>Unit product inventory cost</td>
</tr>
<tr>
<td>( D )</td>
<td>Customer’s random demand</td>
<td>( c_m )</td>
<td>Unit emission cost</td>
</tr>
<tr>
<td>( Q )</td>
<td>Carbon emission cap</td>
<td>( g_1 )</td>
<td>Unit carbon emissions during production</td>
</tr>
<tr>
<td>( d_k )</td>
<td>Customer’s deterministic demand in k scenarios, ( k = 1,2,3, \ldots )</td>
<td>( g_2 )</td>
<td>Unit carbon emissions during ordering</td>
</tr>
<tr>
<td>( p_k )</td>
<td>Probability of occurrence in scenario k, ( k = 1,2,3, \ldots )</td>
<td>( g_3 )</td>
<td>Unit carbon emissions during inventory holding</td>
</tr>
<tr>
<td>( c_u )</td>
<td>Unit manufacturing cost</td>
<td>( x )</td>
<td>Ordering decision variable</td>
</tr>
<tr>
<td>( c_s )</td>
<td>Unit ordering cost</td>
<td>( y_k )</td>
<td>Decision variable in scenario k</td>
</tr>
</tbody>
</table>
We could study the behavioral characteristic of the simulation from different angles based on the practical problems, such as increase and decrease of carbon emission cap. In this paper, we compare and analyze the enterprise overall cost and ordering quantity under different presumed circumstances, aiming to provide reference and consultancy for the decision-making of operational adjustment under carbon emission constraint.

**Model parameter setting and numerical simulation**

Assuming that an enterprise’s ordering cost, production cost, inventory holding cost and inventory shortage cost and associated carbon emissions cost arising from these activities. As shown in Tables 2 and 3, there are 10 possibilities of customer demand, and $K$ and corresponding probability of occurrence $p_k$ in demand scenario set. Assuming that carbon cap is 200 units, optimal order quantity after using matlab is 33.33 units and corresponding cost is 518.67 units.

**Main conclusions**

If other parameters are unchanged and carbon emission cap is changed, some management conclusions can be obtained based on numerical simulation of matlab (Figures 1 and 2). Changes of carbon emission cap are as shown in Table 4 below.

**Conclusion**

1. An enterprise can significantly reduce carbon emission without significantly increasing cost through adjusting ordering quantity.

First, Figure 1 shows that enterprise cost curve is flat first and then steep as carbon emissions are from high to low. When carbon emission cap is [450-700], enterprise’s total cost remains unchanged and is stable at about 485.21, which means that an enterprise can reduce total carbon emissions without greatly increasing total cost. But if carbon emissions are further reduced, the enterprise’s total cost may be increased because carbon emissions reduction is mainly realized by adjusting order quantity and decrease of order quantity per time means that frequency of ordering satisfying the same demand will increase which will cause increase of ordering expense and possible shortage cost and thereby cause increase of comprehensive cost. Thus, further adjusting order quantity for carbon emission reduction will cause increase of cost. In this case, measures for reducing carbon emission that may be taken by an enterprise are making low-carbon technology innovations, such as finding special materials or equipment of alternative energy through R&D and using new energy-namely using cleaner energy with low pollution from the source.

Second, Figure 1 also shows that change rate of enterprise’s total cost increase is lower than change rate of carbon emission reduction. Based on data in the case, when the enterprise’s carbon emissions decrease from 700 units to 50 units, the enterprise’s cost increases from 485.22 to 525.33 only, so carbon emissions decrease by 13 times while the enterprise’s cost increases by 7.6% only. It should be noted that carbon emission reduction is realized by simply adjusting operation decision; that is to say, operating cost can be reduced greatly by changing order quantity in each period without greatly increasing total cost, which gives enlightenment to enterprises which want to reduce carbon emission but are afraid of increasing cost- that is to say, a certain carbon emission reduction can be realized by adjusting operation decision. Moreover, cost saved by different enterprises may be different.

**Table 2: Demand Scenario Set and Probability of Occurrence.**

<table>
<thead>
<tr>
<th>$K$</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
<th>110</th>
<th>120</th>
<th>130</th>
<th>140</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_k$</td>
<td>0.02</td>
<td>0.04</td>
<td>0.05</td>
<td>0.03</td>
<td>0.17</td>
<td>0.03</td>
<td>0.16</td>
<td>0.12</td>
<td>0.15</td>
<td>0.18</td>
</tr>
</tbody>
</table>

**Table 3: Model Parameter Setting.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$Q$</th>
<th>$c_m$</th>
<th>$c_r$</th>
<th>$c_h$</th>
<th>$c_p$</th>
<th>$c_s$</th>
<th>$g^T$</th>
<th>$g^T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set Value</td>
<td>200</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>0.3</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>
under the condition of reducing the same carbon emissions, which brings the possibility of trading. Enterprises which bear high carbon emissions reduction cost but realize little carbon emission reduction can conduct market transaction with enterprises which bear low carbon emission reduction cost but realize much carbon emission reduction to realize minimum carbon emission reduction cost in the entire system, achieving the win-win goal.

2. Enterprise’s carbon emissions show a certain correlation with order quantity when carbon emission cap is within the scope of threshold value, but optimal ordering decision-making has no correlation with carbon emission cap when cap is beyond the scope of threshold value.

As can be seen from Figure 2, when carbon emission cap is 50, 100, 200, 300, 400, 500, 600 and 700, order quantity is 8.33, 16.67, 33.33, 50, 66.67, 72.73, 72.73 and 72.73 respectively. Based on data and curve, carbon emissions show a certain correlation with order quantity and higher order quantity brings higher carbon emissions when cap is [50, 450]. If an enterprise wants to reduce carbon emissions without changing order quantity and production plan, it needs to invest in carbon emission reduction technology which usually weakens the enterprise’s willingness of low carbon innovation, so the government needs to take some measures to encourage or subsidize it. It should be pointed out that an enterprise cannot shift all low carbon requirements to supplier or customer by relying on its market power in value chain but should require stakeholders’ cooperation and coordination. It shows that same activity and same process can realize carbon emission reduction of supplier and consumer and realize provision of low-carbon products with price competitiveness for customers through such methods as system incentive, coordination and even vertical integration. However, when cap is [450, 700], order quantity reaches the maximum value and remains unchanged. The reason is that ordering decision-making is affected by both customer’s random demand and carbon emission cap. Even unlimited increase of carbon emissions could not bring increase of order quantity as customer demand is limited instead of being unlimited.

3. Carbon emission cap is set voluntarily by an enterprise’s decision maker or put forward mandatorily by an external regulatory agency, and its setting should be reasonable and scientific. If the cap constraint is too loose, it is not binding on the enterprise; if the cap constraint is too strict, it will increase the enterprise’s operating cost and affect business development.

As can be seen from Figures 1 and 2, when carbon emission cap is higher than 500 units, the enterprise’s operating cost remains unchanged. As can be seen from Figure 2, when carbon emission cap is higher than 450 units, the enterprise’s order quantity remains unchanged. These show that operation system is very sensitive and can timely make corresponding adjustment based on degree of looseness and strictness of carbon emission cap constraint. These also show that it is very important to reasonably set carbon emission cap: too high cap does not affect an enterprise’s cost and ordering decision-making while too low cap can realize emission reduction within a short period but can seriously affect an enterprise’s cost, dampen the enterprise’s ordering enthusiasm and affect business development. These give two enlightenments to enterprises’ decision makers or policy makers: (1) Need to learn about different effects of different carbon emission caps on enterprise’s operating cost; (2) Need to provide more flexible measures about when and how to satisfy the specified cap, to make the enterprise satisfy carbon emission requirements at relatively low cost.

**Summary**

In this paper, enterprises’ optimal ordering decision-making model with random demand under carbon cap constraint is constructed by adopting random optimization theory and considering economic cost and environmental cost. Under the given resource and capability conditions, reasonable arrangement and decision optimization are conducted to ensure that the sum of ordering cost, production cost, inventory holding cost and inventory shortage cost and carbon emission cost arising from corresponding activities is the lowest. Then some manage conclusions are obtained through variable assignments by using mat lab. For example, adjusting order quantity can help reduce carbon emissions without greatly increasing cost and realize transformation of an enterprise’s objective function from only focusing on economic benefit to optimizing both economic benefit and environmental benefit. As carbon emission cap is a scarce resource and a production factor nowadays, enterprises need conduct top-level design to seek the optimal ordering path under carbon cap constraint, in order to enhance their environmental performance, reduce cost of complying with environmental regulation in the future and thereby enhance their sustainable competitiveness. Carbon emission reduction can generate some other invisible benefits such as improving corporate image which can enhance customer loyalty and promote suppliers’ sales enthusiasm; low carbonization of operation system can make enterprises better use differentiation strategy to develop niche market sensitive to the environment. Moreover, enterprises’ low-carbon operation is a component of steps to realize ecological civilization and a measure taken in response to green development, cyclic development and low-carbon development put forward in the 18th National Congress. This paper does not consider such aspects as trading of extra carbon emissions between enterprises; imposing carbon tax and taking carbon offset actions. At the next step, optimal ordering decision-making on random demand and change of corresponding management decision under different carbon emission regulation policies may be studied.

**References**

