

Figure 2: Comparison of the solutions via AGM and the fourth-order Runge-Kutta for  $\Theta(\zeta)$ – $\beta=0, \psi=0.5$

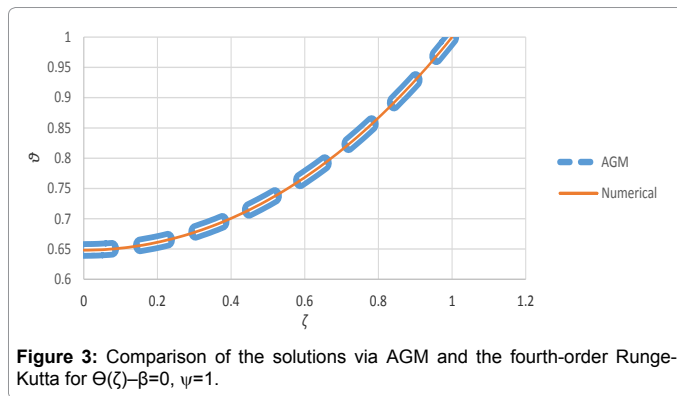


Figure 3: Comparison of the solutions via AGM and the fourth-order Runge-Kutta for  $\Theta(\zeta)$ – $\beta=0, \psi=1$ .

$\zeta$	$\beta=0, \psi=0.5$		
	AGM	Numerical	Error
0	0.648414986	0.648054499	0.000360487
0.1	0.651659539	0.651297468	0.000362071
0.2	0.661423401	0.661058833	0.000364568
0.3	0.677799078	0.677436294	0.000362783
0.4	0.700944784	0.700593758	0.000351026
0.5	0.731087896	0.730762995	0.000324901
0.6	0.768528415	0.768245949	0.000282466
0.7	0.813642421	0.813417762	0.000224659
0.8	0.866885533	0.866730522	0.000155011
0.9	0.928796369	0.928717806	7.86E-05
1	1	1	0

Table 1: The results of AGM and the fourth-order Runge-Kutta for  $\Theta(\zeta)$ .

### Validation of the AGM Approach Considering an Ordinary Differential Equation

In this section, we consider the following differential equation and solve it by HPM, VIM and AGM semi-analytical approaches to analyze the precision of AGM.

$$h(x); \frac{d^2 f}{dx^2} + \frac{df}{dx} + f^2 + f^4 = 0 \quad (23)$$

In this problem, the number of boundary conditions is one more than the order of the presented equation. The given equation should be derived so that the number of boundary conditions and the order of the differential equation will be equaled. Therefore, we will have:

$$g(x); \frac{d^2 f}{dx^2} + \frac{d^2 f}{dx^2} + 2f \frac{df}{dx} + 4f^3 \frac{df}{dx} = 0 \quad (24)$$

The corresponding boundary conditions are as below:

$$f(0) = 0, f(1) = 1, f'(1) = 0 \quad (25)$$

### Applying HPM approach to the given equation

Considering Eq. 23 and 24, we construct the homotopy function as below:

$$F(x, p) = (1-p) \left( \frac{d^3 f}{dx^3} \right) + p \left( \frac{d^3 f}{dx^3} + \frac{d^2 f}{dx^2} + 2f \frac{df}{dx} + 4f^3 \frac{df}{dx} \right) = 0 \quad (26)$$

Assuming  $f(x)$  as a summation of a power series of parameter  $p$ , we will have:

$$f(x) = \sum_{i=0}^4 p^i f_i(x) \quad (27)$$

Substituting Eq. 27 into Eq. 26 and rearranging the answer by powers of  $p$ , the multipliers of each power are obtained. Solving the obtained answers according to the given boundary condition which is constant for all of them, the consecutive terms of HPM solution are gained:

$$f_0(x) = -x^2 + 2x \quad (28)$$

$$f_1(x) = -\frac{1}{90}x^{10} + \frac{1}{9}x^9 + \frac{3}{7}x^8 + \frac{16}{21}x^7 - \frac{17}{30}x^6 + \frac{1}{5}x^5 - \frac{1}{3}x^4 + \frac{1}{3}x^3 + \frac{2}{315}x^2 - \frac{23}{315}x \quad (29)$$

$$f_2(x) = -\frac{1}{6885}x^{18} + \frac{2}{765}x^{17} - \frac{43}{2100}x^{16} + \frac{428}{4725}x^{15} - \frac{14081}{57330}x^{14} + \frac{341}{819}x^{13} - \frac{169}{378}x^{12} + \frac{12731}{34650}x^{11} - \frac{32}{81}x^{10} + \frac{509}{1134}x^9 - \frac{809}{2940}x^8 + \frac{533}{13230}x^7 + \frac{1}{1890}x^6 + \frac{61}{1050}x^5 - \frac{223}{3780}x^4 - \frac{2}{945}x^3 + \frac{456325}{12864852}x^2 - \frac{7498577}{482431950}x \quad (30)$$

The answers of the next terms and the ultimate answer of Eq. 25 are presented in Appendix 1.

### Applying VIM approach to the given equation

To apply the construction of the variational iteration method, we have to find the first sentence to start the loop.

By considering the linear sentence with maximum power of derivative  $\frac{d^3}{dx^3} f_0(x)$  which is also the multiplier of power  $p^0$  in HPM method) and applying the corresponding boundary conditions,  $f_0(x)$  can be found.

$$\begin{cases} \frac{d^3}{dx^3} f(x) + \frac{d^2}{dx^2} f(x) + 2f(x) \left( \frac{d}{dx} f(x) \right) + 4f(x) \left( \frac{d}{dx} f(x) \right) = 0 \\ \rightarrow f_0(x) = -x^2 + 2x \\ f(0) = 0, f(1) = 1, f'(1) = 0 \end{cases} \quad (31)$$

Based on the structure of VIM method, we construct the following formulation:

$$f_{n+1}(x) = f_n(x) + \lambda \left( \int_0^x \left( \frac{d^3}{d\tau^3} f_n(\tau) + \frac{d^2}{d\tau^2} f_n(\tau) + 2f_n(\tau) \left( \frac{d}{d\tau} f_n(\tau) \right) + 4f_n(\tau) \left( \frac{d}{d\tau} f_n(\tau) \right) \right) d\tau \right) \quad (32)$$

Considering an adequate Lagrange multiplier and setting  $n$  from 0 to 3, four terms of  $f(x)$  are calculated, the last cycle of this answer is known as the ultimate solution of the given equation.

$$f_1(x) = \frac{1}{3}x^3 + 2x - \frac{1}{90}x^{10} + \frac{1}{9}x^9 - \frac{3}{7}x^8 + \frac{16}{21}x^7 - \frac{17}{30}x^6 + \frac{1}{5}x^5 = \frac{1}{3}x^4 - x^2 \quad (33)$$

The answers of the next terms and the ultimate answer of Eq. 25 are presented in Appendix 2.

### Applying AGM approach to the given equation

We first consider the given equation in the form below:

$$U: \frac{d^3}{dx^3} f(x) + \frac{d^2}{dx^2} f(x) + 2f(x) \left( \frac{d}{dx} f(x) \right) + 4f(x)^3 \left( \frac{d}{dx} f(x) \right) = 0 \quad (34)$$

As it was discussed in the previous section, the answer of the differential equation is considered as a finite series of polynomials with constant coefficients:

$$f(x) = \sum_{i=0}^5 a_i \cdot x^i = a_0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 \quad (35)$$

The aforementioned unknown coefficients are capable of being computed by applying the boundary conditions.

a) Applying the boundary conditions on Eq. 36:

$$f(0) = 0 \rightarrow a_0 = 0 \quad (36)$$

$$f(1) = 1 \rightarrow a_0 + a_1 + a_2 + a_3 + a_4 + a_5 = 1 \quad (37)$$

b) Applying the boundary conditions on the main differential Eq. 35 and its derivatives:

$$U(f(x)) \rightarrow U(f(BC)) = 0, U'(f(BC)) = 0, \dots \quad (38)$$

$$U(f(0), f(1)) = \begin{cases} U(0) = 0 \\ U(1) = 0 \end{cases} \rightarrow 4a_0^3 a_1 + 2a_0 a_1 + 2a_2 + 6a_3 = 0 \quad (39)$$

$$\begin{aligned} &80a_5 + 36a_4 + 12a_3 + 2a_2 + 2(a_5 + a_4 + a_3 + a_2 + a_1 + a_0) \\ &(5a_5 + 4a_4 + 3a_3 + 2a_2 + a_1) + 4(a_5 + a_4 + a_3 + a_2 + a_1 + a_0)^3 \\ &(5a_5 + 4a_4 + 3a_3 + 2a_2 + a_1) = 0 \end{aligned} \quad (40)$$

$$U'(f(0), f(1)) = \begin{cases} U'(0) = 0 \\ U'(1) = 0 \end{cases} \rightarrow 8a_0^2 a_2 + 12a_0 a_1^2 + 4a_2 a_1 + 2a_1^2 + 6a_3 + 24a_4 = 0 \quad (41)$$

$$\begin{aligned} &180a_5 + 48a_4 + 6a_3 + 2(5a_5 + 4a_4 + 3a_3 + 2a_2 + a_1)^2 + \\ &2(a_5 + a_4 + a_3 + a_2 + a_1 + a_0)(20a_5 + 12a_4 + 6a_3 + 2a_2) + \\ &12(a_5 + a_4 + a_3 + a_2 + a_1)^2 (5a_5 + 4a_4 + 3a_3 + 2a_2 + a_1)^2 + \\ &4(a_5 + a_4 + a_3 + a_2 + a_1 + a_0)^3 + (20a_5 + 12a_4 + 6a_3 + 2a_2) = 0 \end{aligned} \quad (42)$$

By solving a set of algebraic equations, six unknown coefficients are computed according to the existing six equations. By entering the values of the coefficients, the ultimate answer of AGM method is obtained.

Analogy between the methods is shown in Tables 3-6; Figures 4-15.

### Conclusion

In this paper, we have successfully developed semi-analytical methods HPM, VIM & AGM to compute the given equation. We have utilized the Maple Package for our calculations. AGM & HPM methods were the most precise methods in this matter, so that they can be applied to the numerous questions arising in the fields of science and engineering day in day out [20-26].

### Appendix 1

**Description of the HPM approach:** To elucidate on, consider the following equation:

$$A(u) - f(r) = 0; r \in \Omega \quad (43)$$

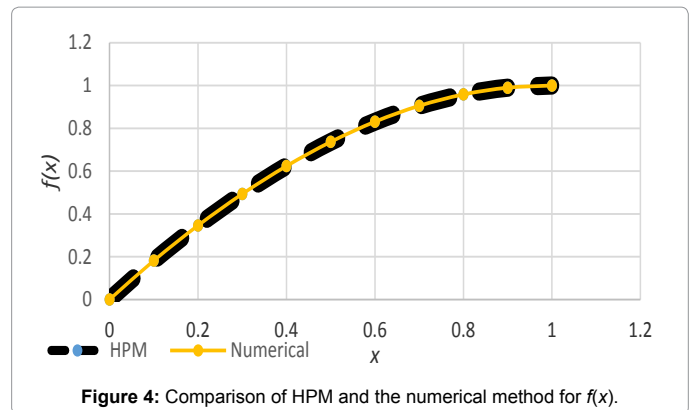


Figure 4: Comparison of HPM and the numerical method for  $f(x)$ .

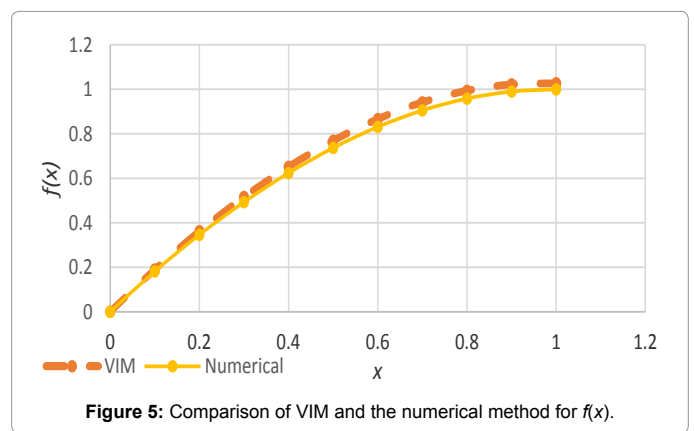


Figure 5: Comparison of VIM and the numerical method for  $f(x)$ .

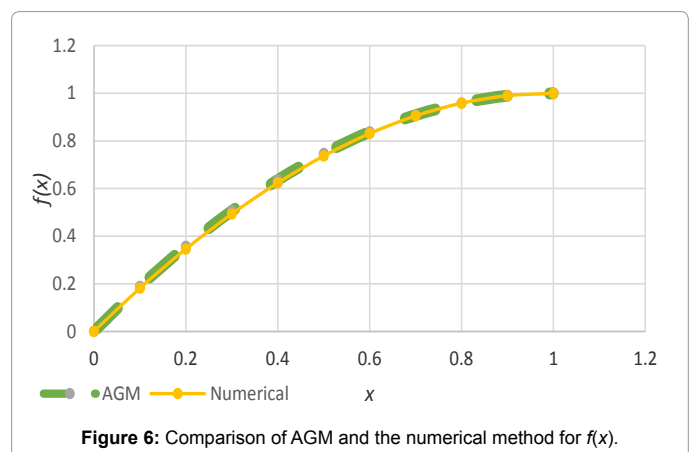


Figure 6: Comparison of AGM and the numerical method for  $f(x)$ .

The boundary conditions are:

$$B \left( u, \frac{\partial u}{\partial n} \right) = 0; r \in \Gamma \quad (44)$$

A: General differential operator, B: Boundary operator,  $f(r)$ : Known analytical function

$\Gamma$ : Boundary of the domain  $\Omega$

The operator A consists of linear and nonlinear parts, so the Eq. 1 can be rewritten in the form below [3,5]:

$$L(u) + N(u) - f(r) = 0; r \in \Omega \quad (45)$$

$$L(u) + N(u) - f(r) = 0; r \in \Omega$$

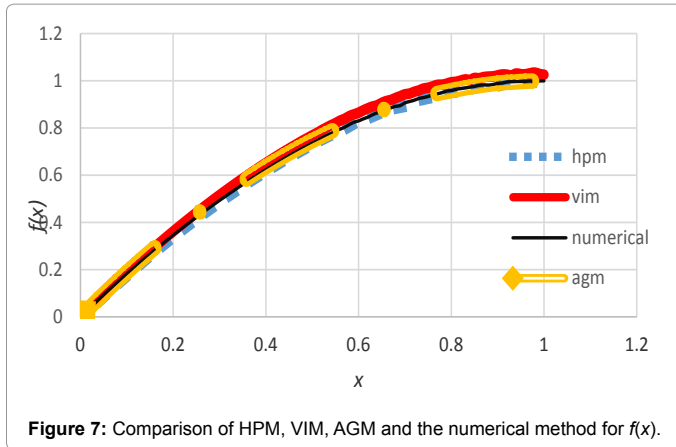


Figure 7: Comparison of HPM, VIM, AGM and the numerical method for  $f(x)$ .

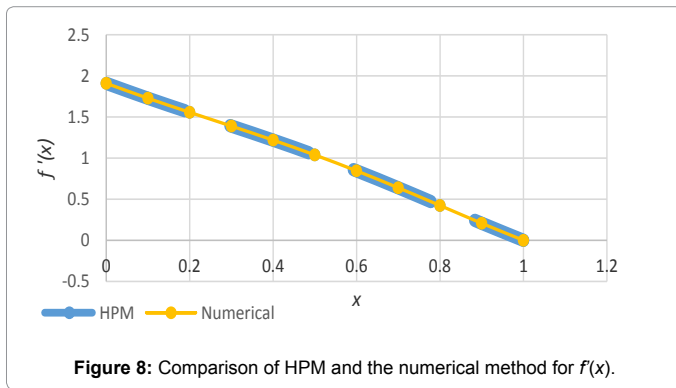


Figure 8: Comparison of HPM and the numerical method for  $f'(x)$ .

$\zeta$	$\beta=0, \psi=1$		
	AGM	Numerical	Error
0	0.886827894	0.886818904	8.99E-06
0.1	0.887936656	0.887927655	9.00E-06
0.2	0.891265668	0.891256693	8.98E-06
0.3	0.896823156	0.896814335	8.82E-06
0.4	0.904622907	0.904614478	8.43E-06
0.5	0.914684345	0.914676629	7.72E-06
0.6	0.927032599	0.927025948	6.65E-06
0.7	0.941698575	0.941693312	5.26E-06
0.8	0.958719023	0.958715403	3.62E-06
0.9	0.978136613	0.978134783	1.83E-06
1	1	1	9.99E-16

Table 2: The results of AGM and the fourth-order Runge-Kutta for  $\Theta(\zeta)$ .

$x$	$f(x)$			
	HPM	VIM	AGM	Numerical
0	0	0	0	0
0.1	0.181728983	0.190293926	0.1886	0.181730225
0.2	0.34588862	0.36205851	0.357	0.345891022
0.3	0.493193118	0.515992529	0.5059	0.493196085
0.4	0.623637046	0.651986985	0.6358	0.623639949
0.5	0.736564905	0.769194539	0.7464	0.736567288
0.6	0.830836697	0.866221325	0.8376	0.830838346
0.7	0.905118449	0.941472686	0.9083	0.90511938
0.8	0.958272246	0.993612536	0.9587	0.958272631
0.9	0.989767158	1.022026176	0.9897	0.989767241
1	1.000000004	1.027139019	1	1

Table 3: Comparing the obtained charts by HPM, VIM, AGM and the numerical approach for  $f(x)$ .

L: Linear part, N: Nonlinear part

The introduced structure of homotopy perturbation method is as below:

$$H(v, p) = (1 - p)[L(v) - L(v_0)] + p[A(v) - f(r)] = 0 \quad (46)$$

While,

$$v(r, p) : \Omega \times [0, 1] \rightarrow R \quad (47)$$

$p \in [0, 1]$ : An embedding parameter,  $u_0$ : First approximation satisfying the boundary condition

If the equation (4) be rewritten as a power series in  $p$  as below:

$$v = v_0 + pv_1 + p^2v_2 \quad (48)$$

The best approximation of the solution is considered as below:

$$u = \lim_{p \rightarrow 1} v = v_0 + v_1 + v_2 \quad (49)$$

Answer terms of the HPM approach to the Problem 2:

$$f_3(x) = -\frac{45391772903}{23717560741875}x + \frac{24547}{5542425}x^{23} - \frac{215183}{10319400}x^{22} + \frac{30009089}{438574500}x^{21} - \frac{266716803}{16665831000}x^{20} + \frac{2486051657}{9166207050}x^{19} - \frac{1503854581}{4341887550}x^{18} + \frac{182146189}{482431950}x^{17} - \frac{15942559}{37837800} + \frac{19616641}{425675250}x^{15} - \frac{61279531}{158918760}x^{14} + \frac{377021}{1719900}x^{13} - \frac{336601}{2619540}x^{12} + \frac{206711}{1559250}x^{11} - \frac{1467612833}{14472958500}x^{10} + \frac{10777231}{868377510}x^9 - \frac{204740047}{4502698200}x^8 - \frac{8932307}{241215975}x^7 + \frac{133508}{19348875}x^6 + \frac{1028171}{321621300}x^5 + \frac{1016179}{192972780}x^4 + \frac{481504330859}{60371972797500}x^2 - \frac{456325}{38594556}x^3 - \frac{7}{3442500}x^{26} + \frac{91}{1721250}x^{25} - \frac{9239}{14779800}x^{24} \quad (50)$$

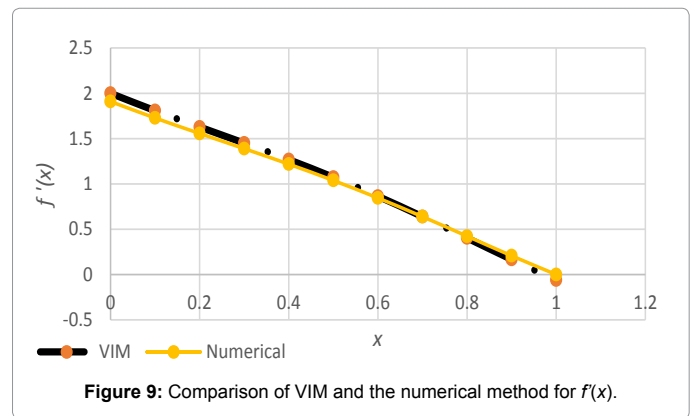


Figure 9: Comparison of VIM and the numerical method for  $f(x)$ .

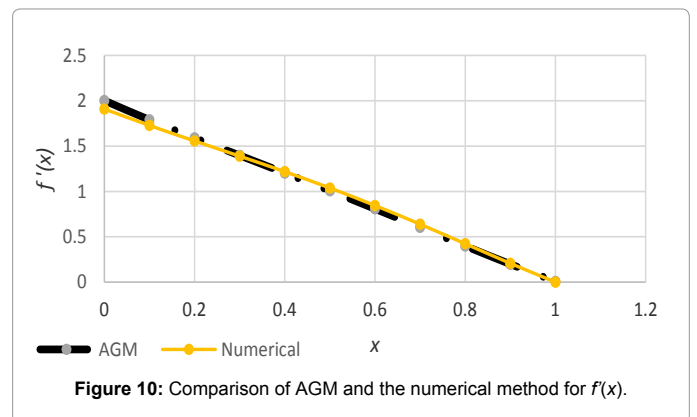


Figure 10: Comparison of AGM and the numerical method for  $f(x)$ .

$$f_3(x) = -\frac{405658497511479907}{3312868296642072918750}x + \frac{39836300049677}{116880139335960}x^{23} - \frac{51789939743231}{127043629713000}x^{22} + \frac{1768611653621}{3849806961000}x^{21} - \frac{193670612903}{434188755000}x^{20} + \frac{109811128723}{303932128500}x^{19} - \frac{320226806923}{1148188041000}x^{18} + \frac{61472796976}{258342309225}x^{17} - \frac{1845420119}{10131070950}x^{16} + \frac{83412542789}{1013107095000}x^{15} - \frac{26719578911}{13828911846750}x^{14} + \frac{3499052681}{263407844700}x^{13} - \frac{24226459}{17144889300}x^{12} - \frac{375595963}{39800635875}x^{11} - \frac{1673831481319}{90557959196250}x^{28} + \frac{103051068431}{1990284817500}x^{27} - \frac{2554138}{16567891875}x^{31} - \frac{204178760771}{197457204262500}x^{30} + \frac{33212328043}{6581906808750}x^{29} - \frac{199036157989699}{6640917007725000}x^{10} - \frac{1748074812487}{51083976982500}x^9 - \frac{21217737858113}{1033031534535000}x^8 - \frac{19566395921389}{2789185143244500}x^7 + \frac{44325301833991}{19922751023175000}x^6 - \frac{1552062491257}{603719727975000}x^5 + \frac{27132296319721}{7969100409270000}x^4 - \frac{511}{17381182500}x^{34} + \frac{43992766081460126129}{43540554755867244075000}x^2 - \frac{481504330859}{18111591832500}x^3 + \frac{511}{511211250}x^{33} - \frac{465757}{29454030000}x^{32} - \frac{176523923566853}{1569671292735000}x^{26} + \frac{37694149682417}{195321088462500}x^{25} - \frac{463766994431}{1702799232750}x^{24}$$

(51)

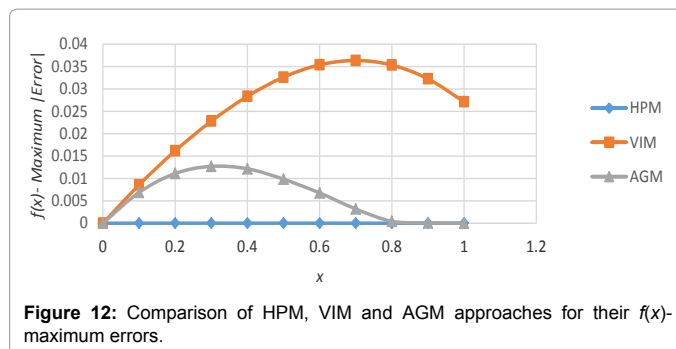


Figure 12: Comparison of HPM, VIM and AGM approaches for their  $f(x)$ -maximum errors.

$$f(x) = \sum_{i=0}^{i=4} p^i f_i(x) = \frac{72744466563256185043862}{38097985411383838565625}x + \frac{201769768723177}{584400696679800}x^{23} - \frac{27219544428883}{63521814856500}x^{22} + \frac{2032031436863}{3849806961000}x^{21} - \frac{2500130731321}{4124793172500}x^{20} + \frac{3652623989647}{5774710441500}x^{19} - \frac{6462716064487}{10333692369000}x^{18} + \frac{319374971831}{516684618450}x^{17} - \frac{25285944119}{40524283800}x^{16} - \frac{642057235969}{43540554755867244075000}x^{15} + \frac{3183909058579}{2789185143244500}x^{14} - \frac{40978926958}{19922751023175000}x^{13} + \frac{1013107095000}{5028695217000}x^{12} - \frac{65851961175}{2461033684}{x^{11}} + \frac{3904847398}{7960127175}x^{10} - \frac{1673831481319}{90557959196250}x^9 - \frac{4286222325}{1990284817500}x^8 - \frac{2554138}{16567891875}x^7 - \frac{204178760771}{197457204262500}x^6 + \frac{33212328043}{6581906808750}x^5 - \frac{3171738108384851}{6640917007725000}x^4 + \frac{21022637795657}{39064217692500}x^3 - \frac{329398467503081}{2114611134289421}x^2 - \frac{11097224706015809}{11097224706015809}x - \frac{516515767267500}{2789185143244500} + \frac{19922751023175000}{19922751023175000} - \frac{1718146300726423}{6640917007725000}x^5 - \frac{3057404630291279}{7969100409270000}x^4 - \frac{511}{17381182500}x^{34} - \frac{4132843788240228922691}{315511458093463}x^2 - \frac{511}{315511458093463}x^3 - \frac{511211250}{511211250}x^{33} - \frac{465757}{29454030000}x^{32} - \frac{176527115345807}{1569671292735000}x^{26} + \frac{640976092443559}{3320458503862500}x^{25} - \frac{29454030000}{1859325724679}x^{24} - \frac{6811196931000}{6811196931000}x^{24}$$

(52)

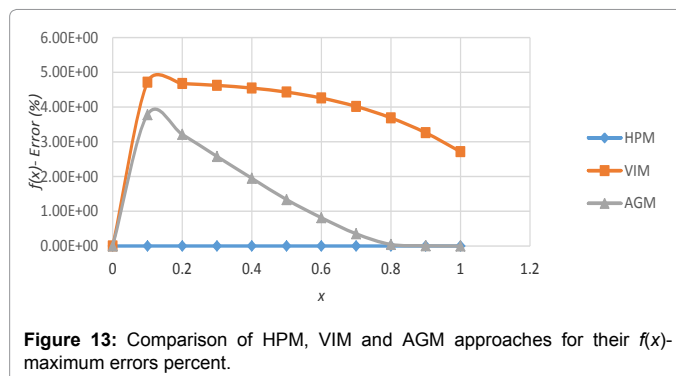


Figure 13: Comparison of HPM, VIM and AGM approaches for their  $f(x)$ -maximum errors percent.

x	f(x)-Maximum  Error		
	HPM	VIM	AGM
0	0	0	0
0.1	1.24E-06	0.008563701	0.006869775
0.2	2.40E-06	0.016167488	0.011108978
0.3	2.97E-06	0.022796445	0.012703915
0.4	2.90E-06	0.028347035	0.012160051
0.5	2.38E-06	0.032627251	0.009832712
0.6	1.65E-06	0.035382979	0.006761654
0.7	9.31E-07	0.036353306	0.00318062
0.8	3.86E-07	0.035339905	0.000427369
0.9	8.31E-08	0.032258935	6.72E-05
1	4.00E-09	0.027139019	0

Table 4: Comparing the obtained charts by HPM, VIM and AGM for their  $f(x)$ -errors in accordance to the numerical approach.

## Appendix 2

**Description of the VIM approach:** To clarify, note the equation below:

$$A(u) = g(x), A(u) = L(u) + N(u) \quad (53)$$

Where L & N represent the linear and nonlinear parts of the general differential operator A.  $g(x)$  is the inhomogeneous term of the

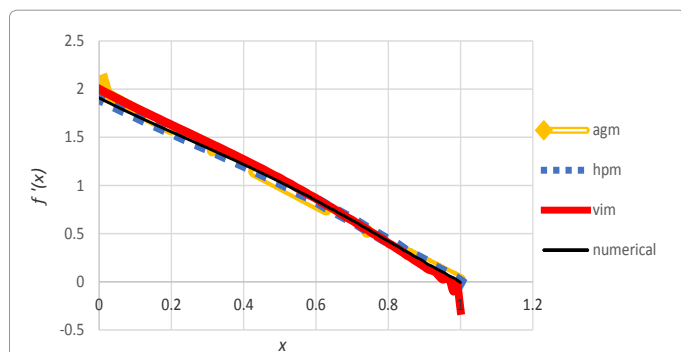


Figure 11: Comparison of HPM, VIM, AGM and the numerical method for  $f(x)$ .

x	f(x)			
	HPM	VIM	AGM	Numerical
0	1.909404547	2	2.0025	1.909413569
0.1	1.727633922	1.808441316	1.7894	1.727647369
0.2	1.556743091	1.628011282	1.5901	1.556752052
0.3	1.389328156	1.450478213	1.3971	1.389330476
0.4	1.218431025	1.267967474	1.2034	1.218427709
0.5	1.03817714	1.0738019	1.0077	1.03817046
0.6	0.844968441	0.863996832	0.80784	0.844968053
0.7	0.638698783	0.638722634	0.6042	0.638692269
0.8	0.423436239	0.402993554	0.3987	0.423431972
0.9	0.207100456	0.165978303	0.19825	0.20709865
1	-0.000000009	-0.061143663	0.0077515	0

Table 5: Comparing the obtained charts by HPM, VIM, AGM and the numerical approach for  $f(x)$ .

equation. The introduced structure of the VIM method is as below [3]:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(Lu_n(\tau) + Nu_n(\tau) - g(\tau)) d\tau \quad (54)$$

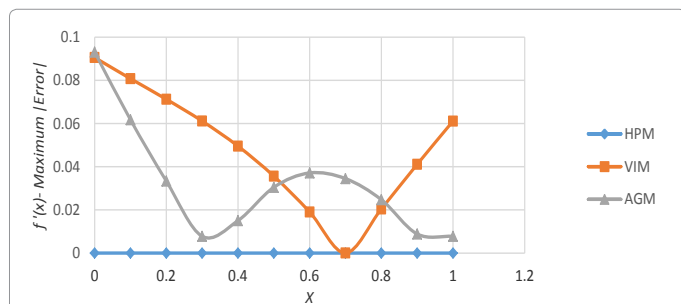
$\lambda$  is the general Lagrange multiplier which can be obtained by disparate ways including variational theory, Laplace method and etc. the subscript  $n$  represents the  $n^{\text{th}}$  approximation of the solution, The appellation of  $\tilde{u}_n$  is justified by the fact that  $\delta u_n \equiv 0$ . The ultimate solution is given as:

$$u(x) = \lim_{n \rightarrow \infty} u_n(x) \quad (55)$$

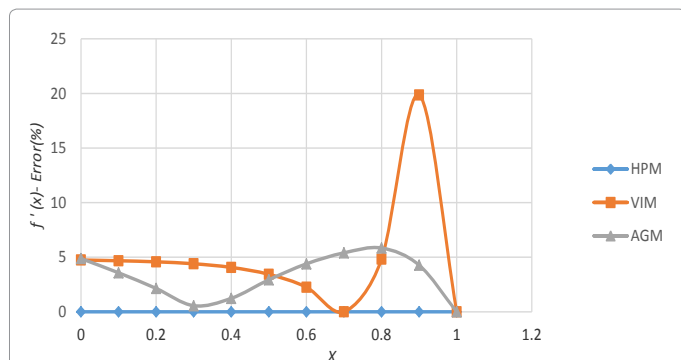
**Answer terms of the VIM approach of Problem 2:**

$$f_2(x) = \frac{1}{3}x^3 + 2x - \frac{170347}{311850}x^{12} + \frac{7706}{17325}x^{11} - \frac{11}{1492627500}x^{40} + \frac{41}{447788250}x^{39} - \frac{902149}{1130033835000}x^{38} + \frac{154157}{29737732500}x^{37} - \frac{153389}{5907360375}x^{36} + \frac{7631233}{74388982500}x^{35} - \frac{3186179767}{9819345690000}x^{34} - \frac{2880696397}{343535142000}x^{33} - \frac{5686447603}{325561194000}x^{32} + \frac{1252554187}{406951492500}x^{31} - \frac{18124598909}{3806965575000}x^{30} + \frac{569117}{81033750}x^{29} - \frac{284583223}{26254935000}x^{28} + \frac{1300956379}{73138747500}x^{27} - \frac{2227520147}{77395500000}x^{26} + \frac{126270101}{29767500000}x^{25} - \frac{34726441}{625117500}x^{24} + \frac{510276677}{7531177500}x^{23} - \frac{43463209}{509355000}x^{22} + \frac{18378074}{156279375}x^{21} - \frac{188379067}{1131165000}x^{20} + \frac{12350041}{56558250}x^{19} - \frac{37887727}{151814250}x^{18} + \frac{1159157}{4498200}x^{17} - \frac{1110959}{3969000}x^{16} + \frac{208499}{595350}x^{15} - \frac{4735399}{10319400}x^{14} + \frac{408601}{737100}x^{13} - \frac{1}{112980420000}x^{12} + \frac{1}{2690010000}x^{11} - \frac{1244}{2835}x^{10} + \frac{1291}{2268}x^9 - \frac{1871}{2520}x^8 + \frac{187}{210}x^7 - \frac{29}{45}x^6 + \frac{4}{15}x^5 - \frac{5}{12}x^4 - x^2$$

$f_3(x), f_3(x)$ ; too many sentences.



**Figure 14:** Comparison of HPM, VIM and AGM approaches for their  $f'(x)$ -maximum errors.



**Figure 15:** Comparison of HPM, VIM and AGM approaches for their  $f'(x)$ -maximum errors percent.

x	F'(x)-Maximum  Error		
	HPM	VIM	AGM
0.1	9.02E-06	0.090586431	0.093086431
0.2	1.34E-05	0.080793947	0.061752631
0.3	8.96E-06	0.07125923	0.033347948
0.4	2.32E-06	0.061147737	0.007769524
0.5	3.32E-06	0.049539765	0.015027709
0.6	6.68E-06	0.03563144	0.03047046
0.7	7.59E-06	0.01903598	0.037120853
0.8	6.51E-06	3.04E-50	0.034492269
0.9	4.27E-06	0.020438418	0.024731972
1	1.81E-06	0.041120346	0.00884865

**Table 6:** Comparing the obtained charts by HPM, VIM and AGM for their  $f'(x)$ -errors in accordance to the numerical approach.

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