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Application of Variational Iteration Method to the Solution of Convection-Diffusion Equation

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Abstract

In this paper, an algorithm is constructed based on Variational Iterational Method (VIM) to solve Convection Diffusion equation. The algorithm converges faster and proved elegant. Numerical examples are presented to show the efficiency of the method.

Keywords: Convection equation; Diffusion equation; Algorithm; Differential equation

Introduction

Convection-Diffusion equation describes the physical phenomenon where particles, energy and other physical quantities are transferred inside a system due to diffusion or convection. This equation is of the form

$$\frac{\partial u}{\partial t}+\varepsilon\frac{\partial u}{\partial x}=\gamma\frac{\partial\ u}{\partial x}\quad 0\le x\le l,\ \ t\ge 0 \tag{1}$$

Subject to the initial condition

$u(x,0)=g(x)$, $0 \le x \le 1$ and boundary conditions $u(0,t)=0$, $t \ge 0$, $u(l,t)=0$, $t \ge 0$ where the parameters γ is the Viscosity Coefficient and ε is the phase speed and both are assumed to be positive. g is a given function of sufficient smoothness.

In EL-Wakil [1], Yee [2], Adomian decomposition method was used to solve Convection-Diffusion (CD) equation, in Ghasemi [3], Porshokouhi et al. [4], He's homotopy perturbation method was used and in Fallahzadeh [5], Homotopy analysis method was used to solve convection diffusion equations. In this paper, the equation was solved by Variational Iterational method [6-11]. To illustrate the efficiency, applicability and reliability of the method, some examples are presented.

Variational Iteration Method

The basic idea of the He's Variational Iteration Method (VIM) [6-11], can be explained by considering the following nonlinear partial differential equations

$$Lu+Nu=g(x) \tag{2}$$

Where L is the linear operator, N is the nonlinear operator and $g(x)$ is the inhomogeneous term. According to the method, we can construct a correction functional as follows

The corresponding variational iteration method for solving (2) is given as

$$u_{n}\ \ (x)=u_n(x)+\int\ \ (s)\left[Lu_n(s)+N\bar{u}_n(s)-g(s)\right]ds\ , \tag{3}$$

Where λ is a Lagrange multiplier which can be identified optimally by variational iteration method. The subscript n denote the nth approximation, $\bar{u}_n$ is considered as a restricted variation i.e $\delta\bar{u}_n=0$. The successive approximation $u_{n+1,}$ $n \ge 0$ of the solution u can be easily obtained by determine the Lagrange multiplier and the initial guess u_0, consequently, the solution is given by $u=\lim_{n\to\infty}u_n$. λ=-1 for problems under consideration.

Numerical Examples

In this section, examples of convection diffusion equation and results will be compared with the exact solutions. Three examples are solved with the VIM algorithm and the results have been generated by Maple 18.

Example 1: Consider the CD equation in [5].

$$u_t\text{-}0.02u_{xx}\text{+}0.1ux\text{=}0 \tag{4}$$

With the initial condition $u(x,0)=e^{1.17712434446770x}$. The exact solution of this equation is $u(x,t)=e^{1.17712434446770x-0.09t}$

Applying (3) to obtain the following:

$$u_1(x,t)=e^{1.177124344x}-0.08999999998e^{1.177124344x}t \tag{5}$$

$$u_2(x,t)=e^{1.177124344x}-0.0899999999e^{1.177124344x}t+0.004049999997e^{1.177124344x}t^2 \tag{6}$$

$$\begin{aligned}u_3(x,t)&=e^{1.77124344x}-0.08999999998e^{1.177124344x}t+0.004049999997e^{1.177124344x}t^2\\&-0.0001214999999e^{1.177124344x}t^3\end{aligned} \tag{7}$$

$$\begin{aligned}u_4(x,t)&=e^{1.177124344x}-0.08999999998e^{1.177124344x}t+0.004049999997e^{1.17724344x}t^2\\&-0.0001214999999\ell^{1.177124344x}t^3+0.000002733749998e^{1.177124344x}t^4\end{aligned} \tag{8}$$

$$\begin{aligned}u_5(x,t)&=e^{1.177124344x}-0.08999999998e^{1.177124344x}t+0.004049999997e^{1.177124344x}t^2\\&-0.0001214999999e^{1.177124344x}t^3+0.000002733749998e^{1.177124344x}t^4\\&-4.92074999410^{-8}e^{1.177124344x}t^5\end{aligned} \tag{9}$$

Table 1 shows the errors index of the approximate solution at different points (x,t). Also the graph of u(exact) with u(approx.) is shown in Figure 1 and 2 when t=0.1 and t=1 respectively. Figures 3 and 4 show the 3-D graph of u(exact) and u(approx.) respectively.

Example 2: Consider the CD equation [4,5].

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X	0	1	2	3	4	5	6	7	8	9	10
Error	0	0	10^{-8}	0	0	0	10^{-6}	0	0	0	10^{-4}

Table 1: The errors index of the approximate solution at the points (x,t), x=1,2,3..10, t=0.1 for example 1.

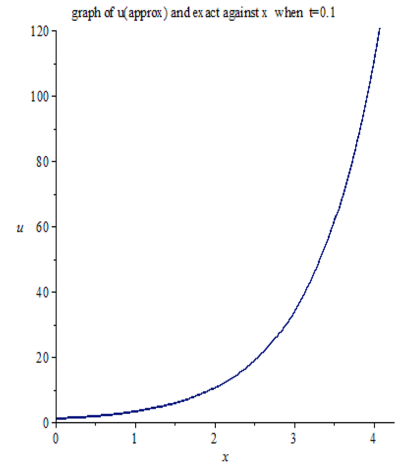

Figure 1: u(exact) with u(approx.) when t=0.1.

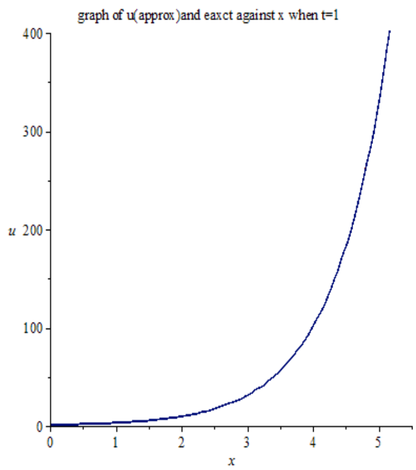

Figure 2: u(exact) with u(approx.) whent=1.

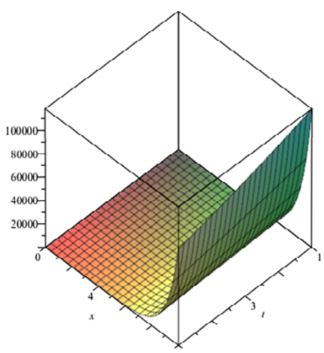

Figure 3: 3-D graph of u(exact).

$$u_t+0.22u_{xx}-0.5u_x=0 \quad (10)$$

With initial condition $u(x,0)=e^{0.22x}\sin(\pi x)$. The exact solution is $u(x,t)=e^{0.22x-(0.024+0.5\pi 2)t}\sin(\pi x)$

Applying (3) to obtain the following:

$$u_1(x,t)=e^{0.22x}\sin(3.141592652x)+2.270664968e^{0.2200000000x}\sin(3.141592654x)t +1.266690158e^{0.2200000000x}\cos(3.141592654x)t \quad (11)$$

$$u_2(x,t)=e^{0.22x}\sin(3.141592652x)+2.270664969e^{0.2200000000}\sin(3.141592654x)t +1.266690158e^{0.2200000000x}\cos(3.141592654x)t+1.775707721t^2e^{0.2200000000x}\sin(3.141592654x) +2.876228968t^2e^{0.2200000000x}\cos(3.141592654) \quad (12)$$

$$u_3(x,t)=e^{0.22x}\sin(3.141592654x)+2.270664969e^{0.2200000000x}\sin(3.141592654)t +1.266690158e^{0.2200000000x}\cos(3.141592654x)t+1.775707722t^2e^{0.2200000000x}\sin(3.141592654x) +2.876228968t^2e^{0.2200000000x}\cos(3.141592654x)+0.1295821303t^3e^{0.2200000000x}\sin(3.141592654x) +2.926741284t^3e^{0.2200000000x}\cos(3.141592654x) \quad (13)$$

$$u_4(x,t)=e^{0.22x}\sin(3.141592654x)+2.270664969e^{0.2200000000x}\sin(3.141592654)t +1.266690158e^{0.2200000000x}\cos(3.141592654x)t+1.775707722t^2e^{0.2200000000x}\sin(3.141592654x) +2.876228968t^2e^{0.2200000000x}\cos(3.141592654x)+0.1295821303t^3e^{0.2200000000x}\sin(3.141592654x) +2.926741284t^3e^{0.2200000000x}\cos(3.141592654x)-0.8532591938t^4e^{0.2200000000x}\sin(3.141592654x) +1.702447328t^4e^{0.2200000000x}\cos(3.141592654x) \quad (14)$$

$$u_5(x,t)=e^{0.22x}\sin(3.141592654x)+2.270664969e^{0.2200000000x}\sin(3.141592654)t +1.266690158e^{0.2200000000x}\cos(3.141592654x)t+1.775707722t^2e^{0.2200000000x}\sin(3.141592654x) +2.876228968t^2e^{0.2200000000x}\cos(3.141592654x)+0.1295821303t^3e^{0.2200000000x}\sin(3.141592654x) +2.926741284t^3e^{0.2200000000x}\cos(3.141592654x)-0.8532591938t^4e^{0.2200000000x}\sin(3.141592654x) +1.702447328t^4e^{0.2200000000x}\cos(3.141592654x)-0.8187878072t^5e^{0.2200000000x}\sin(3.141592654x) +0.5569744970t^5e^{0.2200000000}\cos(3.141592654x) \quad (15)$$

The graph of u(exact) with u(approx.) is shown in Figures 5 and 6 when t=0.1 and t=1 respectively. Figures 7 and 8 show the 3-D graph of u(exact) and u(approx.) respectively.

Example 3: Consider the CD equation [4,5].

$$u_t-0.2u_{xx}+0.1u_x=0,\ 0\le x\le 1,\ t\ge 0 \quad (16)$$

With initial condition $u(x,0)=e^{0.25x}\sin(\pi x)$. The exact solution of the problem is $u(x,t)=e^{0.25x-(0.0125+0.2\pi 2)t}\sin(\pi x)$

Applying (3) to obtain the following:

$$u_1(x,t)=e^{0.25x}\sin(3.141592654x)-0.2211420880e.^{0.2500000000x}\sin(3.141592654x)t -0.2827433388e^{0.2500000000x}\cos(3.141592654x)t \quad (17)$$

$$u_2(x,t)=e^{0.25x}\sin(3.141592654x)-0.2211420880e^{0.2500000000x}\sin(3.141592654x)t -0.2827433388e^{0.2500000000x}\cos(3.141592654x)t-0.01551998626t^2e^{0.2500000000x}\sin(3.141592654x) +0.06252645235t^2e^{0.2500000000x}\cos(3.141592654x) \quad (18)$$

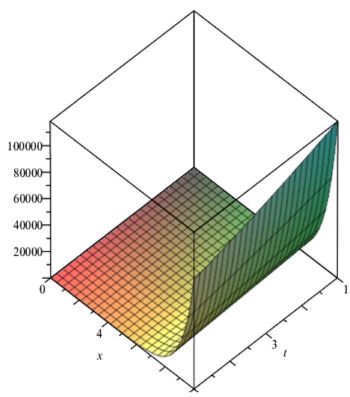

Figure 4: 3-D graph of u(approx.).

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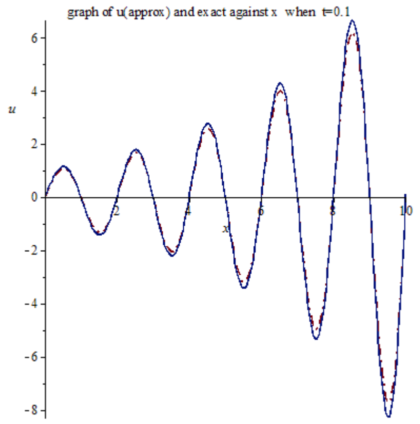

Figure 5: u(exact) with u(approx.) when t=0.1.

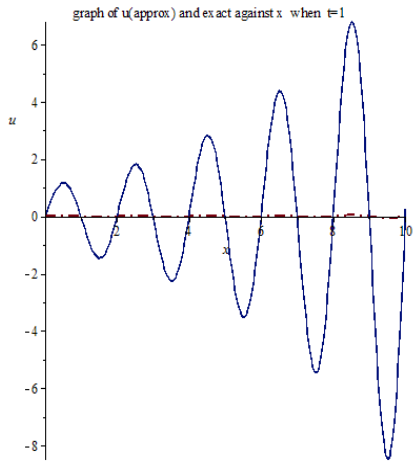

Figure 6: u(exact) with u(approx.) when t=1.

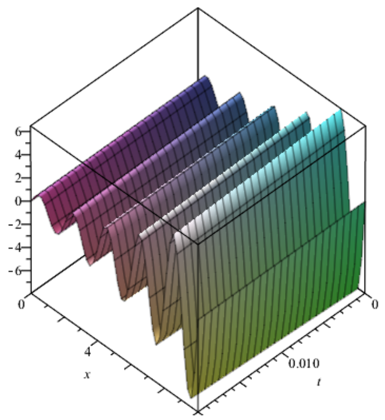

Figure 7: 3-D graph of u(exact).

$$
\begin{aligned}
u_3(x,t) &= e^{0.25x}\sin(3.141592654x) - 0.2211420880e^{0.2500000000x}\sin(3.141592654x)t \\
&-0.2827433388e^{0.2500000000x}\cos(3.141592654x)t - 0.01551998626t^2e^{0.2500000000x}\sin(3.141592654x) \\
&+0.06252645235t^2e^{0.2500000000x}\cos(3.141592654x) + 0.007037020026t^3e^{0.2500000000x}\sin(3.141592654x) \\
&-0.003146352499t^3e^{0.2500000000x}\cos(3.141592654x)
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
u_4(x,t) &= e^{0.25x}\sin(3.141592654x) - 0.2211420880e^{0.2500000000x}\sin(3.141592654x)t \\
&-0.2827433388e^{0.2500000000x}\cos(3.141592654x)t - 0.01551998626t^2e^{0.2500000000x}\sin(3.141592654x) \\
&+0.06252645235t^2e^{0.2500000000x}\cos(3.141592654x) + 0.007037020026t^3e^{0.2500000000x}\sin(3.141592654x) \\
&-0.003146352499t^3e^{0.2500000000x}\cos(3.141592654x) - 0.0006114478782t^4e^{0.2500000000x}\sin(3.141592654x) \\
&-0.0003234698942t^4e^{0.2500000000x}\cos(3.141592654x)
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
u_5(x,t) &= e^{0.25x}\sin(3.141592654x) - 0.2211420880e^{0.2500000000x}\sin(3.141592654x)t \\
&-0.2827433388e^{0.2500000000x}\cos(3.141592654x)t - 0.01551998626t^2e^{0.2500000000x}\sin(3.141592654x) \\
&+0.06252645235t^2e^{0.2500000000x}\cos(3.141592654x) + 0.007037020026t^3e^{0.2500000000x}\sin(3.141592654x) \\
&-0.003146352499t^3e^{0.2500000000x}\cos(3.141592654x) - 0.0006114478782t^4e^{0.2500000000x}\sin(3.141592654x) \\
&-0.0003234698942t^4e^{0.2500000000x}\cos(3.141592654x) + 0.000008751580532t^5e^{0.2500000000x}\sin(3.141592654x) \\
&+0.00004888312448t^5e^{0.2500000000x}\cos(3.141592654x)
\end{aligned}
\tag{21}
$$

The graph of u(exact) with u(approx.) is shown in Figures 9 and 10

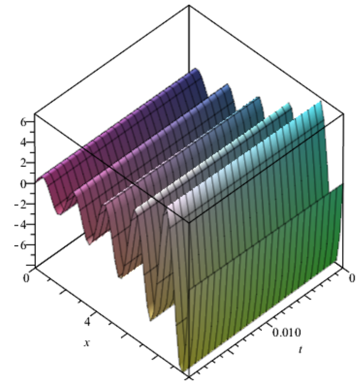

Figure 8: 3-D graph of u(approx.).

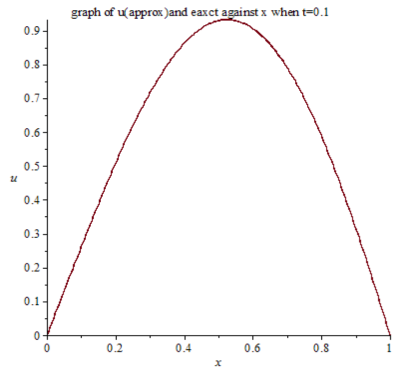

Figure 9: u(exact) with u(approx) when t=0.1.

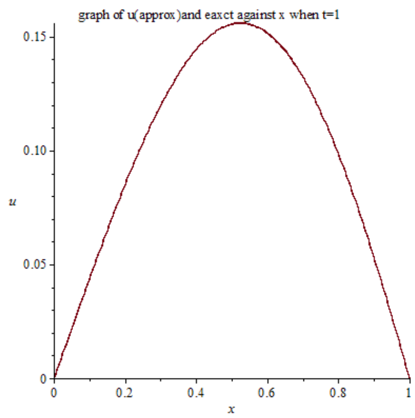

Figure 10: u(exact) with u(approx) t=1.

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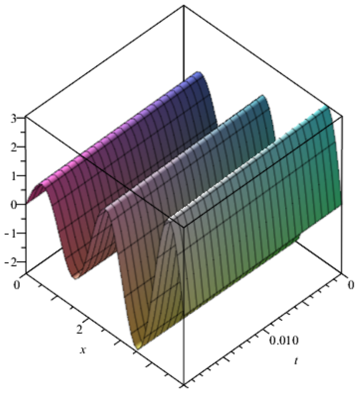

Figure 11: 3-D graph of u(exact).

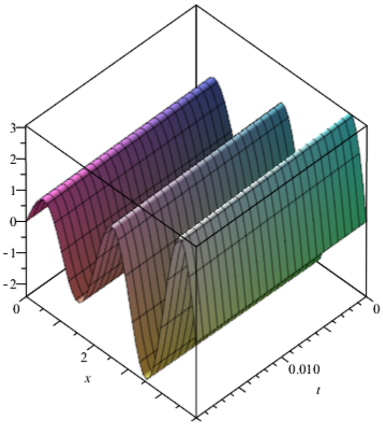

Figure 12: 3-D graph of u(approx).

X	0	1	2	3	4	5	6	7	8	9	10
Error	0	10^{-12}	10^{-12}	0	0	0	10^{-6}	0	0	0	10^{-9}

Table 2: The errors index of the approximate solution at the points (x,t), x=1,2,3..10, t=0.1 for example 2.

X	0	1	2	3	4	5	6	7	8	9	10
Error	10^{-15}	10^{-10}	10^{-9}	10^{-9}	10^{-8}	10^{-9}	10^{-9}	10^{-8}	10^{-9}	10^{-8}	10^{-8}

Table 3: The errors index of the approximate solution at the points (x,t), x=1,2,3..10, t=0.1 for example 3.

when t=0.1 and t=1 respectively. Figures 11 and 12 show the 3-D graph u(exact) and u(approx.) respectively.

Conclusion

In this paper, VIM was used for solving the Convection-Diffusion equations. The obtained result in comparison with exact solution admits a remarkable efficiency. The computations associated with the examples in the paper were performed using Maple 18.

Tables 1-3 and Figures 1-12 justify that the method is reliable and can be applied to nonlinear Convection-Diffusion equations of different parameters.

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