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Axisymmetric Magneto Dynamic (MHD) Stability of a Compressible Fluid Cylinder

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Abstract

The axisymmetric magneto dynamic (MHD) stability of a compressible fluid cylinder under the action of inertia, and electromagnetic forces is developed. A general eigenvalue relation is derived studied analytically and the results are confirmed numerically. In absence of the effect of the electromagnetic forces interior and exterior the fluid, so the model is only subjected to the capillary force. It is found that the model is unstable in the region 0<x<1. While it is stable in the region 1<x<∞. This means that model is just unstable in small domains of axisymmetric perturbation but it stable in all other domains. For very high intensity of magnetic field the model is completely stable for all values of wave lengths. The compressibility has a stabilizing tendency.

Keywords: Hydro magnetic stability; Compressible fluid cylinder; Capillary force; Magneto hydrodynamic

Introduction

The stability of a liquid column has been studied; the authors derived the dispersion relation. The examined the type of perturbation on the boundary of the capillary instability of liquid jet [1-4]. Such works have been extended [5-14]. The present work is different from those studied before since the velocity is not solenoidal any more (i.e., ∇*u* 6=0) and also that the density of the fluid is not uniform. In this chapter we study the stability of a full fluid cylinder of radius R0 endowed with surface tension and pervaded by axial magnetic field for all modes of perturbation. The fluid is assumed to be compressible, inviscid and perfectly conducting. The fluid (of density ρ) is pervaded by the magnetic field

$$\underline{H}_0=(0,0,H_0) \tag{1}$$

The surrounding tenuous medium around the cylinder is assumed to be pervaded by the magnetic field

$$\underline{H}_0^{vac}=(0,0,\alpha H_0) \tag{2}$$

where α is parameter that must satisfy some physical conditions. We shall use the cylinder coordinates (*r*, *ϕ*, *z*) system with the z-axis coinciding with the axis of the cylinder. The fluid is acting upon the capillary, gradient pressure and electromagnetic forces. The capillary force which is due to the curvature pressure is acting along the fluid-tenuous interface.

Formulation of the Problem

The basic equations which required for studying the problem under consideration are the combination of the ordinary hydrodynamic equations together with those of Maxwell concerning the electromagnetic theory. In addition we have to add the equation concerning the curvature pressure.

Under the present circumstances, the MHD basic equations in the fluid are given be

$$\rho\frac{d\underline{u}}{dt}=-\nabla P+\mu(\nabla\times\underline{H})\times\underline{H} \tag{3}$$

$$\frac{\partial\rho}{\partial t}+\nabla.(\rho\underline{u})=0 \tag{4}$$

$$P=C\rho^{\gamma} \tag{5}$$

$$\frac{\partial\underline{H}}{\partial t}=\nabla(\underline{u}\times\underline{H}) \tag{6}$$

$$\nabla\cdot\underline{H}=0. \tag{7}$$

In the vacuum region, the basic equations are

$$\nabla\cdot\underline{H}^{vac}=0 \tag{8}$$

$$\nabla\times\underline{H}^{vac}=0. \tag{9}$$

Along the fluid vacuum interface, we have

$$P_s=S(\nabla\cdot\underline{n}_s). \tag{10}$$

Here $\underline{u}$ and *P* are the fluid velocity vector and kinetic pressure, μ and $\underline{H}$ are the magnetic field permeability coefficient and intensity of magnetic field, respectively, $\underline{H}^{vac}$ is the magnetic field intensity in the vacuum region, P_s is the curvature pressure, γ and *C* are constants, S is the surface tension coefficient and $\underline{n}_s$ is a unit outward vector normal to the cylinder interface and points like the cylinder coordinate, does.

The unit normal vector $\underline{n}_s$ is given by

$$\underline{n}_s=\frac{\nabla F(\gamma,\varphi,z,t)}{|\nabla F(\gamma,\varphi,z,t)|}, \tag{11}$$

Where $F(\gamma,\varphi,z,t)=0$ (12)

is the equation of the boundary surface.

Unperturbed State

The Unperturbed state is studied by considering the basic equations

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system in eqns.(3) and (10) by taking into account eqns. (1) and (2), the hydromagnetics equation (3)yields

$$\nabla\left(P_0+\frac{\mu}{2}\left(\underline{H}_0\cdot\underline{H}_0\right)\right)=0 \tag{13}$$

from which, we get

$$P_0+\frac{\mu}{2}H_0^2=const. \tag{14}$$

In order to identify this constant of integration we have to apply the balance of the pressure across the boundary surface at $r=R_0$. Taking into account that eqn. (10) in the initial state gives

$$P_{0s}=\frac{S}{R_0}. \tag{15}$$

Consequently, the unperturbed pressure distribution is finally given by

$$P_0=\frac{S}{R_0}+\frac{\mu H_0^2}{2}\left(\alpha^2-1\right) \tag{16}$$

where the first term $\frac{S}{R_0}$ in the right side eqn. (16) represents the capillary force contribution while $\frac{\mu H_0^2}{2}\left(\alpha^2-1\right)$ is due to the electromagnetic forces influence internal and external the fluid cylinder.

In the unperturbed state the pressure P_0 must be positive and so, using eqn. (16), we find that α satisfies the restriction

$$\alpha\geq\sqrt{1-\frac{2S}{\mu H_0^2 R_0}} \tag{17}$$

where the equality is pertaining to the limiting case of zero fluid pressure.

Perturbation Analysis

We consider a small disturbance to the basic state. A perturbed quantity Q(r,0,z;t) may be expressed as

$$Q=Q_{0+}Q_{1+}.......,|Q_1|<<Q_0 \tag{18}$$

with,

$$Q_1(r,0,z,t)=Q_(r)\delta(t)\exp(i(kz)), \tag{19}$$

where Q(r,0,z,t) stands for P, $\underline{u}$, $\underline{n}_s$, P_s, $\underline{H}$ and while the subscripts (0)and (1) are pertaining to the unperturbed and perturbed quantities respectively and δ(0) is the initial amplitude of perturbation.

$$\delta(t)=\delta_0 exp(\sigma t), \tag{20}$$

and hence,

$$Q_1(r,z;t)=Q(r)\delta_0\exp(\sigma t+ikz), \tag{21}$$

where σ is the growth rate. If ($\sigma=iw$, $i=\sqrt{-1}$) is imaginary then $\frac{w}{2\pi}$ is the wave oscillation frequency. The perturbed radial distance of the fluid cylinder may be expressed as

$$r=R_{0+}R_1,\ R_{1=}\delta_0\exp(ikz+\sigma t) \tag{22}$$

Here R_1 is the elevation of the surface wave measured from the unperturbed position, k is the longitudinal wave number and σ is the growth rate. By the use of the expansion (22), the perturbed equations are given by

$$\rho_0\frac{\partial\underline{u}_1}{\partial t}+\nabla P_1=\mu\left[\left(\underline{H}_0\cdot\nabla\right)\underline{H}_1-\frac{1}{2}\nabla\left(\underline{H}_0\cdot\underline{H}_1\right)\right] \tag{23}$$

$$\frac{\partial\rho_1}{\partial t}+\nabla\cdot\left(\rho_0\underline{u}_1\right)=0 \tag{24}$$

$$p_1=a^2\rho_1 \tag{25}$$

$$\frac{\partial\underline{H}_1}{\partial t}=\nabla\times\left(\underline{u}_1\times\underline{H}_0\right) \tag{26}$$

$$\nabla\cdot\underline{H}_1=0 \tag{27}$$

while those equations in the region surrounding the fluid cylinder are

$$\nabla\cdot\underline{H}_1^{vac}=0 \tag{28}$$

$$\nabla\times\underline{\ }^{vac}= \tag{29}$$

and along the fluid-vacuum interface we have

$$P_{1s}=\frac{-S}{R_0}\left(R_1+R_0^2\frac{\partial^2 R_1}{\partial z^2}\right). \tag{30}$$

In view of the expansion in eqns. (21) and (22), the relevant perturbation eqns. (23-30) may be simplified and solved. Under the present circumstances the non-singular solution is given by

$$\rho_0\alpha^2\left(\nabla\cdot\underline{u}_1\right)=-\sigma P_1 \tag{31}$$

$$\left(\sigma+\frac{\Omega_A^2}{\sigma}\right)\underline{u}_1=-\nabla\Pi_1+\frac{i\Omega_A^2}{ka^2\rho_0}P_1\underline{e}_z \tag{32}$$

$$\sigma\underline{H}_1=ikH_0\underline{u}_1+\frac{\sigma H_0}{\rho_0 a^2}P_1 e_z \tag{33}$$

$$\Omega_A^2=\frac{\mu H_0^2 k^2}{\rho_0} \tag{34}$$

$$\Pi_1=\frac{P_1}{\rho_0}+\frac{\mu}{\rho_0}\left(\underline{H}_0\cdot\underline{H}_1\right) \tag{35}$$

$$\underline{H}_{1z}=\frac{H_0}{\rho_0 a^2\sigma}\left(\sigma+\frac{k^2a^2}{\sigma}\right)P_1 \tag{36}$$

$$\Pi_1=\frac{P_1}{\rho_0}\varsigma\ \ \varsigma=1+\frac{\mu H_0^2}{\rho_0 a^2\sigma}\left[\sigma+\frac{k^2a^2}{\sigma}\right]. \tag{37}$$

By substituting from eqn. (32) into eqn. (31), we get

$$\frac{1}{\gamma}\frac{\partial}{\partial\gamma}\left(\gamma\frac{\partial\Pi_1}{\partial\gamma}\right)-\eta^2\Pi_1=0 \tag{38}$$

with

$$\eta^2=k^2+\frac{\sigma^2}{a^2\varsigma}. \tag{39}$$

By using eqn. (38) taking into account the dependence in eqn. (22), the solution of eqn. (38) is given in terms of Bessel functions in the form

$$\Pi_{1=}AI_0(\eta r)\exp[ikz+\sigma t], \tag{40}$$

where $I_0(\eta r)$ is the modified Bessel function of the first kind of order zero and A is an arbitrary constant to be determined .

The perturbed magnetic field in the region surrounding the fluid cylinder is obtained by solving the relevant perturbation eqns. (28) and (29).

Eqn. (28) means that the perturbed magnetic field in the region surrounding the field can be derived from scalar function ψ_1, say

$$\underline{H}_1^{vac}=\nabla_{\psi 1}. \tag{41}$$

By combining eqns. (28) and (41), we get

$$\nabla^2_{\psi 1}=0. \tag{42}$$

By using expansion in eqn. (21) and taking into account the space dependence in eqn. (22), the solution in eqn. (42) is given in terms of Bessel functions. Under the present circumstances, we have

$$\psi_1 = BK_0(\eta r)\exp[ikz+\sigma t] \tag{43}$$

and consequently

$$\underline{H}_1^{vac} = B\nabla\left[k_0(\eta r)\exp[ikz+\sigma t]\right], \tag{44}$$

where $K_0(kr)$ is the modified Bessel function of the second kind of order zero and B is an arbitrary constant. Finally inserting eqn. (22) into eqn. (30), yields

$$P_{1s} = \frac{-S}{R_0^2}\left(1-k^2R_0^2\right)R_1 \tag{45}$$

Boundary Conditions

The solution gives by eqns. (31,32,33, 34 and 45) must satisfy some boundary conditions appropriate to the problem under consideration. these conditions may be written as follows.

1. The normal component of the velocity vector u must be compatible with the velocity of the fluid particles across the boundary surface at $r=R_0$.

$$u_{1r} = \frac{d\gamma}{dt}. \tag{46}$$

By use of the eqns. (22) and (32) for the conditions in eqn. (46), we get

$$A = \frac{-\left(\sigma^2+\Omega_A^2\right)\delta_0}{\eta I_0'(y)} \tag{47}$$

where $y=\eta R_0$ is the dimensionless longitudinal wave number. then, we get

$$\left(\sigma+\frac{\Omega_A^2}{\sigma}\right)\underline{u}_1 = \frac{\left(\sigma+\Omega_A^2\right)\delta_0}{\eta I_0'(y)}\nabla\left[I_0(y)R_1\right]+\frac{i\Omega_A^2}{ka^2p_0}P_1e_z \tag{48}$$

from which

$$\sigma\underline{H}_1 = \frac{ikH_0}{\sigma+\frac{\Omega_A^2}{\sigma}}\left[\frac{\left(\sigma+\Omega_A^2\right)\delta_0}{\eta I_0'(y)}\nabla\left[I_0(y)R_1\right]+\frac{i\Omega_A^2}{ka^2p_0}P_1e_z\right]+\frac{\sigma H_0}{a^2p_0}p_1e_z \tag{49}$$

with

$$P_1 = \rho_0\Pi_1 - \mu H_0H_{1z} \tag{50}$$

2. The normal component of the magnetic field H must be continuous across the perturbed boundary (22) at $r=R_0$. This condition may be written as

$$\underline{n}\cdot(\underline{H}^{vac} - \underline{H}^{fluid}) = 0. \tag{51}$$

The unit vector n normal to the cylinder interface and the magnetic fields H and H^{vac} are given by

$$\underline{n} = \underline{n}_0 + \delta_0\underline{n}_1 \tag{52}$$

then

$$n=(1,0,0)+(0,0,-ik)R_1 \tag{53}$$

$$\underline{H} = \underline{H}_0+\delta_0\underline{H}_1 \qquad \underline{H}^{vac} = \underline{H}_0^{vac}+\delta_0\underline{H}_0^{vac}. \tag{54}$$

By substituting from eqns. (33) and (44) into eqn. (52), we get

$$B = \frac{i\delta_0H_0\alpha}{K_0'(x)}. \tag{55}$$

Therefore, the perturbed magnetic field external the fluid cylinder is given by

$$\underline{H}_1^{vac} = \frac{i\delta_0\underline{H}_0\alpha}{K_0'(x)}\nabla\left[k_0(\eta r)\exp[ikz+\sigma t]\right] \tag{56}$$

3. The normal component of the stresses due to the kinetic pressure of the fluid and the magnetic pressure of electromagnetic forces acting inside and outside column must be discontinuous by the curvature pressure. This may be written as

$$P_{1s}+\mu H_0H_{1z} = P_{1s}+\mu H_0H_{1z}^{vac}. \tag{57}$$

Therefore, the condition in eqn. (57) may be rewritten in the form

$$P_{1s}+\mu H_0H_{1z} = P_{1s}+\mu H_0H_{1z}^{vac}. \tag{58}$$

Substituting from eqns. (35, 44 and 45) into eqn. (58), after lengthy calculations, we finally arrive to

$$\sigma^2 = \frac{S}{p_0R_0^3}\left(1-x^2\right)\frac{yI_0'(y)}{I_0(y)}+\frac{\mu H_0^2}{P_0R_0^2}\left(-x^2+\left(xy\alpha^2\right)\frac{I_0'(y)k_0(x)}{I_0(y)K_0'(x)}\right). \tag{59}$$

This dispersion relation is valid for all axisymmetric m=0 and non-axisymmetric (m ≥ 1) modes of perturbation.

General Discussions

Eqn. (59) is the capillary dispersion relation of a fluid cylinder acting upon Lorentz force with a homogeneous magnetic field. It is a simple linear combination of the dispersion relation of a fluid cylinder subject to the capillary force only and that one subject upon electromagnetic force with uniform magnetic fields.

The relation in eqn. (59) is valid for all axisymmetric and non-axisymmetric modes of perturbation. By means of that relation the hydrodynamic, magnetodynamic and magnetohydrodynamic (MHD) characteristics of the present model may be determined. The eigenvalue relation in eqn. (59) relates the growth rate σ, or rather the oscillation frequency ω as σ=iω is imaginary with transverse wave number.

Some reported works may be obtained from relation in eqn. (59) under appropriate choices.

If we impose $H_0=0$, eqn. (59) reduce to

$$\sigma^2 = \frac{S}{p_0R_0^3}\left(1-x^2\right)\frac{yI_0'(y)}{I_0(y)}. \tag{60}$$

As the fluid is incompressible as a → ∞ and in such case y → x, therefore eqn. (59) degenerates to

$$\sigma^2 = \frac{S}{p_0R_0^3}\left(1-x^2\right)\frac{yI_0'(y)}{I_0(y)}+\frac{\mu H_0^2}{P_0R_0^2}\left(-x^2+x^2\alpha^2\frac{I_0'(y)k_0(x)}{I_0(y)K_0'(x)}\right). \tag{61}$$

If we impose $H_0=0$ eqn. (61) reduce to that derived by Rayleigh [15]

$$\sigma^2 = \frac{S}{p_0R_0^3}\left(1-x^2\right)\frac{xI_0'(x)}{I_0(x)}. \tag{62}$$

Hydrodynamic Instability

If the model under consideration is acting upon the capillary force only and all other forces are neglected, the stability criterion for such a case is given by the relation in eqn.(60). The discussions of this relation showed that the model is capillary stable in the nonaxisymmetric modes for all short and long wave lengths. In the axisymmetric mode the incompressible model is stable as long as x ≥ 1 and only unstable if 0<x<1 where x=1 corresponding to the marginally stability state.

It is deduced that the fluid cylinder is capillary unstable only in the axisymmetric mode as long as the perturbed wave length is longer than the circumference of the fluid jet.

Magnetodynamic Stability

In the absence of the capillary force and the fluid cylinder is only subjected to the electromagnetic forces interior and exterior the fluid, the dispersion relation in eqn. (59) yields

$$\sigma^2 = \frac{\mu H_0^2}{P_0 R_0^2}\left(-x^2 + x^2\alpha^2 \frac{I_0^{\cdot}(y)k_0(x)}{I_0(y)K_0'(x)}\right). \qquad (63)$$

The influence of the longitudinal magnetic field interior the fluid in eqn. (1) is represented by the term $-x^2$ following the fundamental quantity $\frac{\mu H_0^2}{\rho_0 R_0^3}$ in eqn. (63).

It has a strong stabilizing influence for all short and long wave lengths.

Now we prove that the exterior magnetic field in eqn. (2) is also stabilizing.

By the use of the recurrence relations of the modified Bessel functions

$$I_0'(y) = \frac{1}{2}\left(I_{m-1}(y) + I_{m+1}(y)\right), \quad K_0'(y) = \frac{1}{2}\left(K_{m-1}(y) + k_{m+1}(y)\right) \qquad (64)$$

and the fact that

$$I_m(y) > 0 \text{ and } K_m(y) > 0 \qquad (65)$$

For every non-zero value of x and y, we may prove that

$$I'_m(y) > 0 \quad \text{and } k'_m(y) < 0 \qquad (66)$$

For all $x \neq 0$, $y \neq 0$ and $m \geq 0$.

From the point of view of the relation in eqn. (64) and the inequalities in eqn. (66) it can be shown that the magnetic field acting exterior the fluid cylinder is stabilizing. That character is valid not only in the non-axisymmetric modes but also in the axisymmetric mode for all short and long wave lengths.

We conclude that the electromagnetic forces acting interior and exterior the fluid cylinder are stabilizing. This may be due to the fact that the applicable magnetic fields are axial and uniform.

Magneto Hydrodynamic Stability

In order to discuss this general case in which the fluid cylinder is acting upon the capillary and electromagnetic forces we have to discuss the relation in eqn. (59) in its general form we have the following

(i) The capillary force is stabilizing if $0 \leq x < \infty$, $1 \leq y < \infty$ where the equality corresponds to the neutral stability, while it is destabilizing as long as $0 \leq x < 1$, $0 \leq y < 1$ (ii) The lorentz force is stabilizing for all values of x 6=0, y 6=0 i.e., for all short and long wave lengths.

We deduce that the model under consideration is MHD stabilizing for small wave length.

Numerical Discussions

In order to verify the results which obtained analytical concerning the acting different forces effect on the present model. It is found very important to discuss the dispersion relation in eqn. (59) numerically for axisymmetric perturbation mode. In order to do that we have to rewrite this dispersion relation in non-dimensional form, so we may insert this relation in the computer for making the numerical computation.

Based on the input data, if the values of $\frac{\sigma^2}{\left(\frac{S}{\rho_0 R_0^3}\right)}$ are positive then we have unstable regions. As $\frac{\sigma^2}{\left(\frac{S}{\rho_0 R_0^3}\right)}$ values are negative we put $\acute{o} = iw$ so $\frac{w}{\sqrt{\frac{S}{\rho_0 R_0^3}}}$ are the values which concerning the stable domains.

In the transition from the negative values to positive values of σ^2 we have to path with the values of $\sigma=0$ which means marginal stability states. The points at which the transition from stability regions to those of instability called the critical points.

The dispersion relation in eqn. (59) has been formulated in the dimensionless form upon using the quantity $\sqrt{\frac{S}{\rho_0 R_0^3}}$ which has a unit of times as $\frac{1}{\sigma}$ and that the quantity $\sqrt{\frac{S}{\mu R_0}}$ say H_s has a unit of the intensity of magnetic field. Consequently, equation in eqn. (58) takes the form

$$\frac{\sigma^2}{\left(\frac{S}{\rho_0 R_0^3}\right)} = \left(1-x^2\right)\frac{yI_0'(y)}{I_0(y)} + \left(\frac{H_0}{H_s}\right)\left[-x^2 + x^2\alpha^2 \frac{I_0^{\cdot}(y)k_0(x)}{I_0(y)K_0'(x)}\right] \qquad (67)$$

The numerical calculations have been performed by inserting the dispersion relation in eqn. (67) in the computer and computed for the important mode of perturbation. Taking into account that $I_0'(x) = I_1(x)$ and that $k_0'(x) = -k_1(x)$. The calculations have been carried out for all short and long wave lengths as $0.1 \leq x < 4$ for several values of $\left(\frac{H_0}{H_s}\right) = 0, 0.2, 0.4, 0.6, 0.8, 1.0 \text{ and } 2.0$ and 2.0.

For each values of $\left(\frac{H_0}{H_s}\right)$, the calculations performed for several values of (α,a)=(1,5), (2,5), (3,5), (4,5), (1,30), (1,20),(1,10), (2,20), (3,10) and (4,10).

In each case the data are collected, tabulated in tables and presented in figures, (Figures 1-4). In each figures and table it is printed clearly the stable and unstable domains corresponding to a given value of $\left(\frac{H_0}{H_s}\right)$ and several values of (α,a).

For $\left(\frac{H_0}{H_s}\right) = 0.0$, we obtain the same unstable domain which is due to the capillary force. Whatever are the values of (α,a) we found the unstable is $0<x<1$ and the neighboring stable domain $1 \leq x < \infty$. The transition form stability to that of instability is occurred at $x_c=1$. Not that as $H_0=0$ then the values of (α,a) has no effect at all (Figure 1).

For $\left(\frac{H_0}{H_s}\right) = 0.4$ corresponding to (α,a)=(1,30), (2,20), (3,10) and (4,5), it is found that the unstable domain are given by $0.1<x<0.718$, $0.1<x<0.654$ $0.1<x<0.481$ and $0.1<x<0.356$ while the neighboring stable domains are given by $0.718<x<\infty$, $0.654<x<\infty$, $0.481<x<\infty$ and $0.356<x<\infty$. The transition form unstable domains to those of stability occurred at the critical points x_c=0.718, 0.654, 0.481, 0.356 (Figure 2).

For $\left(\frac{H_0}{H_s}\right) = 1.0$, corresponding to (α,a)=(1,30), (2,20), (3,10) and (4,5), it is found that there is no any unstable domain. The model is completely stable (Figure 3).

For $\left(\frac{H_0}{H_s}\right) = 2.0$, corresponding to (α,a)=(1,30), (2,20), (3,10) and (4,5), it is found that there is no any unstable domain. The model is

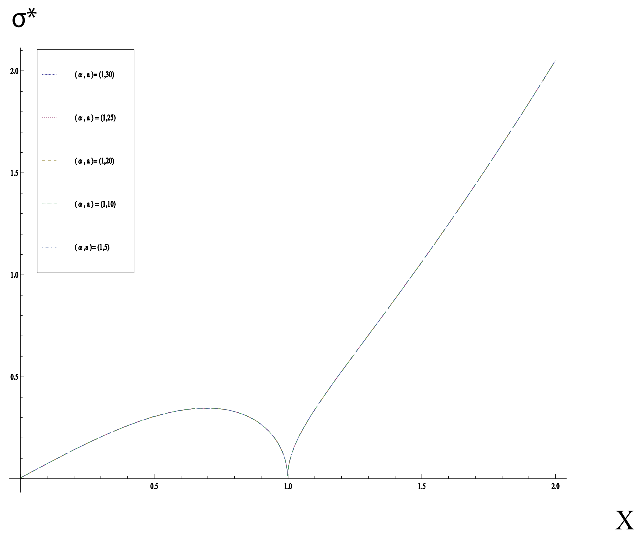

Figure 1: Magnetodynamic stable and unstable domains for a compressible fluid as H_0/H_G=0.

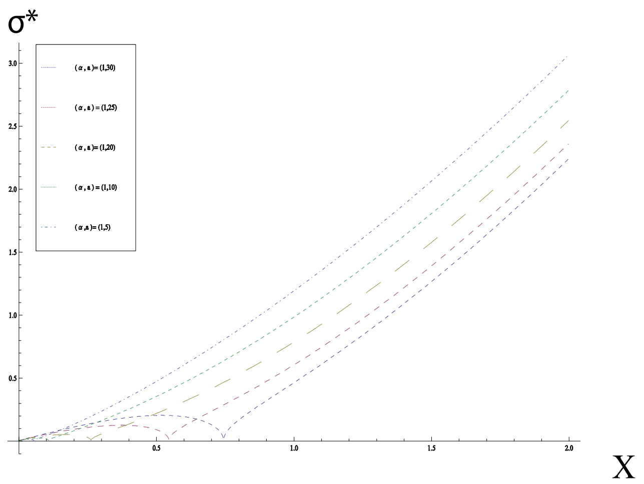

Figure 2: Magnetodynamic stable and unstable domains for a compressible fluid as H_0/H_G=0.4.

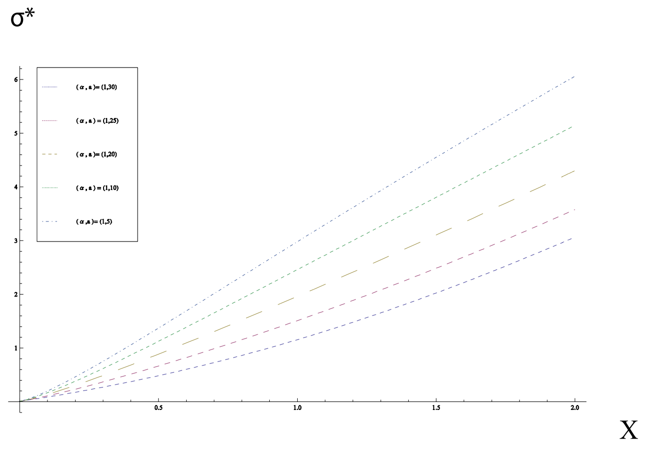

Figure 3: Magnetodynamic stable and unstable domains for a compressible fluid as H_0/H_G=1.0.

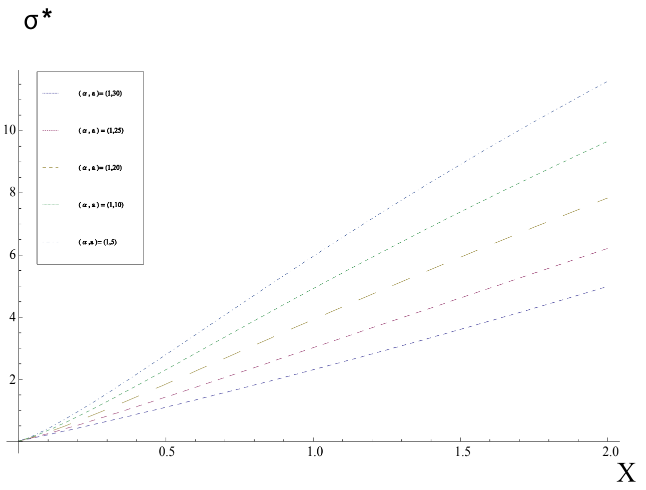

Figure 4: Magnetodynamic stable and unstable domains for compressible fluid as H_0/H_G=2.0

completely stable (Figure 4).

Conclusion

From the forgoing discussions, we may conclude the following results.

1. In absence of the effect of the electromagnetic forces interior and exterior the fluid, so the model is only subjected to the capillary force. It is found that the model is unstable in the region $0<x<1$. While it is stable in the region $1<x<\infty$. This means that model is just unstable in small domains of axisymmetric perturbation but it stable in all other domains.
2. For very high intensity of magnetic field the model is completely stable for all values of wave lengths.
3. The compressibility has a stabilizing tendency.

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