Buoyancy Explains Terminal Velocity in Skydiving
Landell-Mills N*
Edinburgh University, 75 Chemin Sous Mollards, Argentiere 74400, France

Abstract
Estimates show that skydivers in free-fall displace a mass of air downwards equal to their own mass every second, in order to maintain a constant terminal velocity. This is also demonstrated at indoor skydiving centers where air blown upwards can suspend skydivers in mid-air. Like a boat floating in water, the skydiver is floating on air. Consequently, Archimedes principle of buoyancy can be used to explain the physics of terminal velocity in skydiving. Conventional physics explains that drag, the force needed to push air out of a skydiver’s path, sets a limit to a skydiver’s velocity. Which is correct but incomplete. It is more accurate to add that according to buoyancy, the skydiver’s velocity will increase until a mass of air equal to his own mass is displaced each second.

Keywords: Physics; Buoyancy; Skydiving; Archimedes; Terminal; Velocity; Airborne; Fly; Float

Method
This work was completed after extensive research, as well as numerous discussions with academics, engineer and skydivers. Experiments were conducted skydiving to test the validity of the assertions documented in this paper. The findings from the experiments were consistent with the assertions made in this paper (Figure 1).

Definitions
Terminal velocity
Terminal velocity is the highest velocity attainable by an object as it falls through a fluid (e.g. water or air) and there is no acceleration.

Free-fall
Free-fall is the downward movement of an object towards the ground under the force of gravity only.

Key Equations
Equation A
The equation for the drag on a skydiver at terminal velocity:

\[ \text{Drag} = 0.5 \times (\text{Velocity}^2 \times \text{Air Density} \times \text{Surface Area} \times \text{Drag Coefficient}) \]

Equation B
Weight = Mass × Gravity
Force = Mass × Acceleration

Equation C
Fluid Pressure = (Volume of Air × Air Density × Acceleration) / Area

Equation D
Upward Buoyancy Force = Net Air Pressure × Surface Area
= (Volume of Air × Air Density × Acceleration) / Surface Area
= Volume of Air × Air Density × Acceleration
= Mass of Air × Gravity

where the buoyancy force is measured every second.

*Corresponding author: Landell-Mills N, Edinburgh University, 75 Chemin Sous Mollards, Argentiere 74400; France, Tel: 0033-638773940; E-mail: nicklm@gmx.com

Received February 11, 2017; Accepted April 19, 2017; Published April 24, 2017


Copyright: © 2017 Landell-Mills N. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.
**Physics, Philosophy and Logic**

Buoyancy is consistent with the laws of physics

This paper claims that Archimedes, 2,300 years-old principle of buoyancy is a fundamental principle of physics that applies to all objects at all times. Therefore, buoyancy should be applied moving objects, such as a skydiver in free-fall.

As a skydiver falls, energy is used to displace the air mass in his path, as well as indirectly displacing air. There is no net loss or gain of mass, energy or momentum. Energy is transferred from the falling skydiver to the air to generate a buoyancy force. In a vacuum the lack of air means that a buoyancy force is impossible; so, objects will fall at the same terminal velocity.

Philosophy and logic

This argument for buoyancy is consistent with what is observed in reality and the factors that affect the terminal velocity of a skydiver, as explained later. This paper also takes note of Occam’s razor: The simplest explanation given the evidence is often true. Current explanations of drag tend to be complex and abstract.

The logic of applying buoyancy to moving objects is straightforward:

a. Gravity applies universally to all stationary and moving objects.

b. Buoyancy is a product of gravity.

c. Therefore, buoyancy should apply universally to all stationary and moving objects at all times, such as skydivers. Gravity and buoyancy are universal.

The laws of physics should be applied universally, not selectively: Currently Archimedes principle of buoyancy is selectively applied only to static objects, such as boats. It is generally not applied to objects moving through fluids, such as speed boats, skydivers or planes. It has never been proved, or disproved, whether Archimedes’ principle of buoyancy applies to objects that move. Boats and hot air balloons float due to buoyancy. There is no reason that moving objects should be exempt from the need to maintain buoyancy. Buoyancy doesn’t stop acting on the object just because it’s moving.

**Archimedes principle of buoyancy**

Archimedes principle of buoyancy [1,2] states that boats float due to an upward buoyancy force equaling the downward weight of the boat, as shown by Figure 2. To float, boats must displace a mass of water equal to their own mass. The buoyancy force is just the upward water pressure at the bottom of the boat; which is equal to the amount of water pushed up by the boat. Pressure and weight are both due to gravity.

Archimedes principle of buoyancy:

Weight of boat = Buoyancy Force (Water Pressure).


Mass of Boat = Mass of Water Displaced.

**Example of a stone falling in different fluids**

A stone falling at terminal through water and air demonstrates the principle of buoyancy applied to moving objects. This paper claims that at terminal velocity a stone displaces a mass of air or water downwards equal to its own mass each second, consistent with the principle of buoyancy (Figure 3).

The speed of the stone depends primarily on how far it must fall each second before it displaces a mass equal to its own mass. Water has a much greater density (mass per unit volume) than air. Consequently, the stone falls a much smaller distance in water than air each second. The mass of water or air displaced, includes both the mass directly in the path of the stone, as well as the mass that is indirectly displaced.

Note that water is more viscous than air, so the friction will be greater than in the air. But this remains a relatively minor consideration in this example.

**Results**

The difference speeds of stone falling in water and air at terminal velocity is consistent with the principle of buoyancy. Experimentation needs to be done to verify exactly what the difference is.

**The Physics of Terminal Velocity in Skydiving**

The principle of buoyancy is applied to skydiving to provide insight into how buoyancy affects objects falling through the air.

**Buoyancy force vs. drag at terminal velocity**

According to conventional physics, drag sets a limit to the skydiver’s velocity. Drag is the force (energy) needed to physically push the air out of the path of the skydiver. At terminal velocity drag will equal the weight of the skydiver. Drag does not analyse how much air is displace nor in what direction it is displaced (up or down).

Whereas, according to buoyancy, the skydiver’s velocity will increase until the mass of air displaced each second equals to his own mass. At terminal velocity, the buoyancy force will equal the weight of the skydiver. For example; At terminal velocity, a 80 kg skydiver falls at about 66.7 m³ (240 km/hr). Conventional physics explains that this is the maximum speed possible, where the skydiver has sufficient force to push air out of his way. Drag prevents his velocity from increasing. Which is correct but incomplete.
It is more accurate to add this statement that 80 kg/s of air is supporting the skydiver in the air, due to the buoyancy force from the air being displaced above and below the skydiver. This is demonstrated by the fact that if air is blown upwards at 66.7 m/s at an indoor skydiving centre; it will suspend the same 80 kg skydiver in mid-air. Then there is no drag on the skydiver (the skydiver is the drag on the air blown upwards). But the same buoyancy force exists. Here the skydiver is floating in the air, for the same principles of physics that explain how a boat floats in water: buoyancy.

The equation for buoyancy force

Buoyancy Force = Mass of air displaced (every second) × Gravity

The equation for the buoyancy force is consistent with Newton’s 2nd law of motion (F = ma) [1].

Applying Newton’s 3rd law of motion to skydiving: In stable, free-fall descent at terminal velocity the forces are in balance. Therefore, every force will have an equal and opposite force. The weight of the skydiver will equal the buoyancy force.

Weight of skydiver = Mass of skydiver × Gravity
Mass of skydiver = Buoyancy Force
= Mass of Air displaced × Gravity
= Mass of Air displaced

Consequently, the skydiver will displace a mass of air equal to his own mass each second. For example, to maintain buoyancy an 80 kg skydiver will displace 80 kg/s of air downwards; in order to create a sufficient equal & opposite force to push the skydiver upwards.

Jet packs example

To put it another way. If the skydiver could hypothetically (somehow) blow or displace 80 kg of air downwards each second, then he could remain suspended in mid-air. By comparison, water jet packs work on this principle (albeit, displacing water downwards, not air) (Figure 4).

The skydiver directly and indirectly displaces air

Critically, the total mass of air displaced downwards by the skydiver includes the air directly in his path and air indirectly displaced (Figure 5). Substantial energy is being transferred from the skydiver to the air. This energy has to go somewhere. It is used to directly and indirectly displace air.

Two airflows and two Newtonian forces

At terminal velocity, there are two separate forces that keep the skydiver’s velocity from increasing. These force result from the skydiver pushing the air below him downwards, and pulling air above him down (Figures 6 and 7).

Applying Newton’s 3rd law of motion [1] it is shown that there are two separate equal and opposite reactions to the forces acting on the skydiver. As the skydiver falls at terminal velocity:

1. A vacuum of air develops immediately above (behind) the skydiver. This causes the air above the skydiver to expand creating LOW air pressure. This PULLS air above the skydiver down. The equal & opposite force PULLS the skydiver UP.

2. The skydiver compresses the air below him as he falls; creating HIGH air pressure. This air is PUSHED down. The equal & opposite force PUSHES the skydiver UP.

At terminal velocity the forces are in balance. The buoyancy force (and net air pressure) equals the weight of the skydiver. Consequently, the mass of the skydiver will equal the mass of air displaced downwards.

Weight of skydiver = Mass of skydiver × Gravity
Mass of skydiver = Buoyancy Force
Key determinants of terminal velocity

This demonstrates that the key determinants of a skydiver's terminal velocity are consistent with buoyancy and empirical observations. These key determinants the skydiver’s mass and the mass of air that he displaces, i.e., This is how far the skydiver falls each second before he displaces a mass of air equal to his own mass.

The skydiver’s mass: Heavier skydivers fall faster: Heavier skydivers achieve a higher terminal velocities (assuming a constant surface area). A bigger mass of a skydiver will fall a bigger distance each second, in order to displace a correspondingly bigger air mass. Also, a bigger mass provides greater momentum and energy to displace more air each second. This is easiest shown by tandem skydivers, who fall a lot faster than solo skydivers (while having approximately the same surface area exposed to the direction of descent). Tandem skydivers typically deploy mini-parachutes to slow themselves down in free-fall (and to provide extra stability); so as to fall at the same velocity as solo skydivers (Figure 7).

Result: This observation and logic is consistent with buoyancy.

The mass of air displaced by the skydiver (surface area and air density): The amount of air mass displaced by the skydiver each second depends on his surface area exposed to the direction of travel (downwards); and the air density. The evidence for this is that skydivers go faster when head first and when the air is thinner. (Figure 8). In fact, the skydiver in the 2012 Red Bull Sky Jump reportedly broke the sound barrier when falling through the stratosphere, and slowed down as he came closer to the earth.

In a vacuum, no buoyancy force is possible. There is no air to provide any resistance to the object as it falls. So, objects in a vacuum will fall at the same velocity.

Result: This observation and logic is consistent with buoyancy.

Conclusions: Empirical observations of the factors that affect the terminal velocity of a skydiver, is consistent with buoyancy. Consequently, a skydiver in free-fall will have a higher terminal velocity if:

1. The object is heavier (i.e., bigger mass); so, will travel farther in one second to displace more air. Heavier people or tandem skydivers will fall faster in free-fall; if the surface area is constant.

2. The surface area of the object exposed to the downward direction is smaller; or the lower the air density. In these cases, the object will go a farther distance down each second, before displacing a given mass of air each second.

The one-second time period

This paper estimates that Archimedes principle of buoyancy acts over a one second time period. i.e., An object must displace a mass of air equal to its own mass each second to maintain buoyancy. This one second time period is just an initial estimate.

Experimentation needs to be done to verify if this one second is accurate. The actual time frame may be slightly shorter or longer, and not be exactly one second. The core idea proposed by this paper does not alter if experimentation shows that the time period is different to one second.

A skydiver displaces air downwards as he falls. It is unclear over exactly what time period the skydiver will displace a mass of air downwards equal to his own mass.
The theoretical explanation for this one second time period is uncertain. Why one second; rather than 1.2 seconds or 0.8 seconds (for example), is unclear. There does not appear to be anything intrinsically or fundamentally special about this one second time period.

**Conclusions:** These arguments demonstrate that buoyancy applies to moving objects. That it is logically feasible for a skydiver to displace a mass of air downwards equal to his own mass each second (at terminal velocity, in stable free-fall descent).

### Skydiving Calculation Example

**Objective:** This is to demonstrate via an example calculation, that it is theoretically feasible for a man to displace a mass of air equal to his own mass each second, at terminal velocity while in a free-fall descent (Figure 9). Experimentation needs to be done to confirm this is actually what happens.

**Calculation of buoyancy**

**Assumptions:** A skydiver is in a stable free-fall descent at terminal velocity.

- a. An 80-kg skydiver descends at a terminal velocity of 66.7 m/s (or 240 km/hr).
- b. This assumption is an estimate is based on anecdotal evidence from
  - c. Skydivers and skydiving organizations.
  - d. This assumption is also consistent with the terminal velocity provided by the conventional equation (A) for drag, based on a drag coefficient of 0.294. See
  - e. Section 6.4 (Example calculation of drag), for the detailed calculation of this.
  - f. The 80 kg includes his clothes and equipment.
  - g. Lower surface area of the man = 1.0 m².

This is about half the estimated total surface skin area of a 80 kg man with a height of 180 cm (5 ft. 10 inches); of about 2.0 m², based on the body surface area (BSA) calculator; (using the Motseller formula). In respect to this assumption of surface area; Note that the skydiver’s lower legs are partially obscured from the direction of descent, but this is compensated for by the skydiver’s clothes and equipment adding to the total surface area [2,3].

- i. Standard air density = 1.2 kg/m³.
- ii. These calculations exclude any significant friction that would materially slow the skydiver’s descent. The viscosity of air is low, so air friction is considered to be negligible. Evidence for this is the lack of any heating due to friction during a free-fall descent. However, note that as velocity increases, friction becomes increasingly significant.
- iii. The skydiver only displaces air that is directly in his path.
- iv. For simplicity purposes, there is no air that is indirectly displaced by the skydiver. This parameter is extremely difficult to estimate. Therefore, the estimate of the total air displaced by the skydiver is under-estimated by this assumption.
- v. These estimates and assumptions are approximate, as they are only used to demonstrate the feasibility and reasonableness of the argument for buoyancy.

**Calculations**

First, the volume of air displaced by the skydiver is estimated:

Volume of air displaced by the skydiver = Velocity of Skydiver × Lower Surface Area of Skydiver

= 66.7 m/s × 1.0 m²
= 66.7 m³/s

As Mass = Volume × Density; then,

Mass of Air Directly Displaced by Skydiver = Volume of Air Displaced × Air Density

= 66.7 m³/s × 1.2 kg/m³
= 80 kg/s

Each 1 kg/s of air displaced must be displaced down by the skydiver about 0.83 meters; given that the density of the air is 1.2 kg/m³ (0.83 m = 1 kg / 1.2 kg/m³). So, 80 kg of air must be displaced down about 0.83 meters, each second (by the 80 kg skydiver).

Note that these results are consistent with the data quoted by most skydiving organizations.

These estimate the terminal velocity of a 70-80 kg skydiver at 55-67 m/s (200-240 km/hr).

**Conclusion**

Based on the assumptions above, an 80 kg skydiver will directly displace 80 kg of air downwards each second, at terminal velocity. Therefore, at terminal velocity a skydiver will displace a mass of air equal to his own mass each second. Skydiving centres use large fans used to blow air upwards, to suspend skydivers in mid-air, to practice free-fall (Figure 10). This provides a reasonableness check that the estimate of terminal velocity for an 80-kg skydiver is about 66.7 m/s. In physics, all movement is relative. So to demonstrate buoyancy, it does not matter if the skydiver is falling through stationary air, or if a stationary skydiver is having air blown upwards. The physics is the same, assuming that air is Galilean invariant.

**Results**

Anecdotal evidence shows that the skydiver remains suspended mid-air if the fan blows the air up at the same speed as the man’s terminal velocity when actually skydiving. So a fan blowing with a 80 kg/s force (with air blown up at 66.7 m/s), can suspend a 80 kg man mid-air. This speed of 66.7 m/s is well within the range that indoor skydiving centers advertise for the capacities of their fans. The skydiver is floating in the air based on the same principles of physics that explain how boats float on water.

![Figure 10: Skydiving practice.](image-url)
Sensitivity analysis

Using similar assumptions to those as above in Section 8.1:

a) Air Density = 1.2 kg/m³

b) Lower surface area of the skydiver is based on half the estimated body surface area (BSA) for a ma 180 cm tall; using the Motseller formula.

c) Air friction is not included.

d) The skydiver only displaces air that is directly in his path.

Using similar methodology to that above in Section 5.1; Based on the principle of buoyancy, the terminal velocity of the skydiver is estimated based on his mass and the corresponding mass of air he is estimated to displace (Table 1).

Results

Terminal velocity for skydivers with a mass on 60-90 kg is 208-255 km/hr. Each additional 1 kg on the skydiver adds about 0.4 m/s (1.6 km/hr) to the skydiver’s terminal velocity. The terminal velocities estimated are consistent with what is observed in reality, albeit slightly higher; based on evidence from skydiving institutions and skydivers [4].

Conclusion

Sensitivity analysis demonstrates it is feasible for buoyancy to explain the differences in terminal velocity of skydivers with a different mass. Buoyancy is consistent with what is observed in reality.

The Limitations of the Equation for Drag

The equation for drag

Conventional physics states that the for formula for drag is (Equation A):

\[
\text{Drag} = 0.5 \times \text{Velocity}^2 \times \text{Air Density} \times \text{Area} \times \text{Drag Coefficient}
\]

Drag is defined in conventional physics as the physical force needed to push the air out of the way of the falling skydiver. This sounds a lot like the kinetic energy transferred from the skydiver to the air. Conventional physics claims that at terminal velocity, the weight of the skydiver will equal his drag (Figure 11). “The drag coefficient then expresses the ratio of the drag force to the force produced by the dynamic pressure times the area”. The drag coefficient is typical a number between zero and one; but is rarely above two.

Problems with the equation for drag

This paper claims that the conventional equation for drag has limited usefulness due to the problems with the drag coefficient, as explained below. In short, the drag coefficient appears to be the residual factor that is used to make the drag equation work.

Problems with the drag coefficient include:

a. The drag coefficient can only be indirectly observed. It cannot be directly measured. Whereas parameters like velocity and area can be directly measured. This is a significant problem because it means that the terminal velocity of an object in free-fall cannot be estimated unless its drag coefficient is already known. But to accurately calculate the drag coefficient, the terminal velocity must be known.

b. The drag coefficient is not a fixed amount for an object, varies between objects and can change constantly with circumstances.

c. The drag coefficient varies with the speed and type of airflow. Laminar airflow can produce a different drag coefficient to disrupted or turbulent airflow. The drag coefficient varies with the Reynolds number.

d. Similar objects (similar shapes) can have different drag coefficients. A heavier skydiver has a different drag coefficient to a lighter skydiver.

e. The drag coefficient can be different between water and air for the same object.

f. The drag coefficient changes with the amount of surface area. So the same object can have a different drag coefficient based on different amounts of surface area being exposed to the direction of travel. A skydiver falling head-first down, has a different drag coefficient to a skydiver in a standard flat alignment.

g. The appropriate drag coefficient for the skydiver is uncertain. But the calculation for terminal velocity is extremely sensitive to the drag coefficient.

Example calculation of drag

The conventional equation for drag is used to demonstrate that calculation for the terminal velocity of the skydiver above of 66.7 m/s is reasonable, but nothing more (Figure 9).

Conventional physics states that the for formula for drag on a falling object is (Equation A):

\[
\text{Drag} = 0.5 \times \text{Velocity}^2 \times \text{Air Density} \times \
\]

At terminal velocity, all forces are in balance; so, Weight = Drag

Therefore,

\[
\text{Mass of Skydiver} \times \text{Gravity} = 0.5 \times \text{Terminal Velocity}^2 \times \text{Air Density} \times \
\]

Terminal Velocity km/hr 208 225 240 255

Surface Area m² 0.87 0.94 1.00 1.06

Terminal Velocity m/s 57.8 62.4 66.7 70.8

Mass of Skydiver kg 60.0 70.0 80.0 90.0

Table 1: The terminal velocity of the skydiver based on his mass and the corresponding mass of air he is estimated to displace.
Re-stating these equations:

Terminal Velocity\(^2\) = \frac{(\text{Mass of Skydiver} \times \text{Gravity})}{(0.5 \times \text{Air Density} \times \text{Surface Area} \times \text{Drag Coefficient})}

**Assumptions**
- Air Density = 1.2 kg/m\(^3\)
- Gravity = 9.8 kg m/ s\(^2\)
- Surface area of skydiver = 1.0 m\(^2\)
- Mass of skydiver = 80 kg
- Drag Coefficient for the skydiver = 0.294

This is simply the drag coefficient that makes the equation work so that the mass of the skydiver will equal the mass of air displaced.

**Terminal velocity can be calculated**

\[
\text{Terminal Velocity}^2 = \frac{(80 \text{ kg} \times 9.8 \text{ kg m/s}^2)}{(0.5 \times 1.2 \text{ kg/m}^3 \times 1.0 \text{ m}^2 \times 0.294)} = 784 / 0.176 = 4444.4
\]

Terminal Velocity = 66.7 m/s (or 240 km/hr)

**Results**

Based on these assumptions, a 80 kg skydiver would have a terminal velocity of 66.7 m/s. This is consistent with the calculations above using buoyancy to estimate terminal velocity of the skydiver, in Section 5.

**The assumption for the drag coefficient**

Note that this calculation (using the conventional equation for drag to estimate terminal velocity), is based on selecting a drag coefficient that makes the equation work. A drag coefficient was deliberately selected so that the mass of the skydiver will equal the mass of air displaced. The question therefore is whether the drag coefficient used (of 0.294) is reasonable. As the drag coefficient cannot be directly measured, this is impossible to confirm.

**For comparison purposes**

1. A cube has a drag coefficient of 1.0;
2. A sphere has drag coefficient of 0.5, and
3. A streamlined object (e.g. aircraft wing) has a drag coefficient of about 0.04.

Therefore, a drag coefficient of about 0.3 for a skydiver is only reasonable if the skydiver is more streamlined than a sphere but less than a wing. The appropriate drag coefficient for the skydiver is uncertain. But the calculation for terminal velocity is extremely sensitive to the drag coefficient. However, a drag coefficient of over 0.5, provides estimates for terminal that are inconsistent with what is observed in reality. This would arise if the skydiver was less streamlined than a sphere. For example, a drag coefficient of 0.6 suggest that an 80-kg skydiver would fall at a terminal velocity of 46.7 m/s (168 km/hr); which is well below what is observed in reality [4].

**Conclusion - Drag Coefficient**

The drag coefficient is not a very reliable source it to make calculations for terminal velocity using the equation for drag. There is a substantial amount of variation possible when applying the drag coefficient in the conventional equation for drag to estimate terminal velocity. As the drag coefficient cannot be directly measured it is open to a lot of speculation when using it to estimate terminal velocity of an object such as a skydiver. Consequently, enormous care must be used when relying on the drag coefficient.

**Acknowledgement**

This paper and the related analysis is entirely the work of the author.

**References**

2. NASA (2017) Glenn Research Centre. National Aeronautics and Space Administration, USA.
4. Publically available data from skydiving institutions.