Journal of
Applied & Computational Mathematics

Essa and Elsaid, J Appl Computat Math 2015, 4:4
DOI: 10.4172/2168-9679.1000225

Commentary Open Access

Comparison between Gaussian and Non-Gaussian using Different Schemes of Dispersion Parameters for I_{131} and Cs_{137}

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Abstract

In this work, Gaussian and non-Gaussian schemes are used to calculate the concentration for isotopes Iodine (I_{131}) and Cesium (Cs_{137}), using average value for wind speed and different schemes of dispersion parameters. The statistical technique is used to know the best model for calculating isotopes. The most points of the Gaussian and non-Gaussian schemes lie inside a factor of two with observed concentrations.

Keywords: Gaussian and non-gaussian schemes; Different schemes of dispersion parameters; Statistical technique

Introduction

A simple model for atmospheric dispersion in short range in Gaussian plume model is derived by Curtiss and Rabl [1]. One of the most important parameters in plume dispersion modeling is the plume growth as dispersion coefficients (σ) [2]. These plume width parameters depend on the meteorological variables [3]. Various parameterizations exist for the vertical and lateral plume dispersion are expressed as functions of downwind distance and wind speed as a power law function of the vertical height above the ground [4].

Since the Gaussian plume model is expressed in terms of the dispersion parameters σ_y and σ_z, the appropriate selection of lateral and vertical dispersion parameters is much targeted. We select the four different methods namely, power law, Briggs, Irwin and standard methods, for calculating σ_y and σ_z to select the most accurate one [5].

Atmospheric dispersion scientists and modelers seek to characterize air pollution spread in terms of important parameters representing the actual state of the atmospheric turbulence. The methods for dispersion parametersation based on synoptic classification schemes as time of day, cloud cover and mean wind speed is studied by Torben [6]. Atmospheric dispersion modeling refers to the mathematical description of contaminant transport in the atmosphere; the term dispersion in this context is used to describe the combination of diffusion and advection that occurs within the air the earth's surface. The concentration of a contaminant released into the air may therefore be described by the advection - diffusion equation, which is a second order partial differential equation (PDE) of parabolic type [7].

Analytical and approximate solutions for the atmospheric dispersion problem have been derived under wide range of simplifying assumptions, as well as various boundary conditions and parameter dependencies. These analytical solutions are especially useful to engineers and environmental scientists who study pollutant transport, since they allow parameter sensitivity and source estimation studies to be performed [7]. Both our scientific understanding the technical developments have been greatly increased by the use of empirical, analytical and numerical models to predict the air pollution concentration in atmosphere. For this purposed, the advection-diffusion equation has been largely applied in operational atmospheric dispersion models. In principal, from this equation it is possible to obtain the dispersion from a source given appropriate boundary and initial conditions plus knowledge of the mean wind velocity and concentration turbulent fluxes [8].

The advection diffusion equation has been largely in operational atmospheric dispersion models to predict mean concentrations of contaminants in the planetary boundary dispersion from a continuous point source under appropriate boundary and initial conditions as well as knowledge of the mean wind velocity and concentration turbulent fluxes.

Many turbulent dispersion studies are related to the specification of these turbulent fluxes to allow the solution of the averaged advection–diffusion equation. This procedure used to know the closure of the turbulent diffusion problem. Isotopes concentration using different schemes of dispersion parameters are calculated by Abdel-Wahab and et al. [9].

In this work, the relation between the Gaussian plume and non-Gaussian formulas for concentration from a continuous point source of strength Q at a mean wind speed using different schemes of dispersion parameters are calculated. Also statistical technique is used to know the best model with observed data.

Gaussian Distributions

The Gaussian plume formula for concentration from a continuous point source of strength "Q" with interference from the ground at a mean wind speed "U" taking the dilution factors is presented [5].

$$\chi(x,y,z)=\left[Q/\left(\left\{2\pi\sigma_y\sigma_z+C_w A\right\}/U\right)\right]*\exp\left(-\lambda x/U\right)*\left[\exp\int_0^x \frac{dx}{\sigma_z \exp\frac{H^2}{2\sigma_z^2}}\right]^{-\left(\left(\sqrt{2/\pi}\right)*\left(v_d/U\right)\right)}$$

$$*\exp\left(-y^2/2\sigma_y^2\right)*\left\{\exp\left[-(z-H)^2/2\sigma_z^2\right]+\exp\left[-(z+H)^2/2\sigma_z^2\right]\right\}$$

where:

χ is the mean concentration of the effluent at a point (x, y, z) (Bq m^{-3}).

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Received April 25, 2015; **Accepted** June 18, 2015; **Published** June 25, 2015

Citation: Essa KSM, Elsaid SEM (2015) Comparison between Gaussian and Non-Gaussian using Different Schemes of Dispersion Parameters for I_{131} and Cs_{137}. J Appl Computat Math 4: 225. doi:10.4172/2168-9679.1000225

Q is the source strength (Bq).

U is the mean wind speed (m s^{-1}).

x,y,z refer to a downwind, crosswind and vertical coordinates respectively at the center of the moving cloud.

σ_i (i=x, y, z) are the plume dispersion coefficients in the x, y and z directions respectively (m) [10-13].

Exp (-x λ /U) is the radioactive decay for the specified nuclide.

"A" is the cross- sectional area of the building normal to the wind.

"C_w" is the 'shape factor' that represents the fraction of 'A' over which the plume is dispersed; C_w =0.5 is a conservative value which is commonly used.

"V_d" is deposition velocity (m/s).

H is the effective stack height {h_s (stack height) +Δ h (plume rise)} (m).

The non-Gaussian plume formula for concentration from a continuous point source of strength Q with interference from the ground at a mean wind speed U as follows [14]

$$\bar{C}_n(x,y,z,t)=\left\{\frac{1}{U\pi^2 xt}-\frac{h_s\sqrt{t}}{\sqrt{K_n\pi}}+\frac{1}{2\sqrt{K_n\pi t}}\left(\exp\left(\frac{(z-2h+h_s)}{(z+h_s)}\right)\right)\right\}Q\exp(-\lambda x/U)\frac{\exp\left(-y^2/2\sigma_y^2\right)}{\sigma_y\sqrt{2\pi}}$$

Dispersion Parameters Schemes

We select four different methods namely, power law , Briggs, Irwin and standard methods for calculating σ_y and σ_z to select the most accurate one [15], as follows.

Power law method

In this method, σ_y and σ_z can be calculated from the following formula:

$$\sigma_y = c\,x^m$$

$$\sigma_z = d\,x^n$$

where c, d, m, n values [3] differ according to stability classes, as follows (Table 1):

Stability	σ_y (m)		σ_z (m)	
	C	m	d	n
A-B	1.46	0.71	0.01	1.54
C	1.52	0.69	0.04	1.17
D	1.36	0.67	0.09	0.95
E-F	0.79	0.70	0.40	0.67

Table 1: Values of the dispersion parameters for the Pasquill stability classes.

Stability classes	A	B	C	D	E	F
r (m/km)	250	202	134	78.7	65.6	37
s (m/km)	102	96.2	72.2	47.5	33.5	2
a (km)	0.927	0.370	0.283	0.707	1.07	1.17
p	0.189	0.162	0.134	0.135	0.137	0.134
q	-1.918	-0.101	0.102	0.465	0.624	0.70

Table 2: Values of the dispersion parameters for the Pasquill stability classes.

Standard method

In this method, σ_y and σ_z can be analytically expressed, based on Passquil - Gifford [16] (P-G) curves, using the following forms:

$$\sigma_y = \frac{r\,x}{\left(1+\frac{x}{a}\right)^p}$$

$$\sigma_z = \frac{s\,x}{\left(1+\frac{x}{a}\right)^q}$$

where r, s, p and q are constants depending on the atmospheric stability. These values are given in the following Table 2.

Briggs method

In this method, σ_y and σ_z can be calculated from the following Table 3 according to Briggs [17].

Irwin method

In this method, σ_y and σ_z are calculated using the following formula:

$$\sigma_y(x) = \frac{\sigma_\theta\, x}{1+0.9\sqrt{\frac{x}{1000U}}}$$

$$\sigma_z(x) = \sigma_\phi\, x$$

where σ_θ and σ_φ are the standard deviation of the wind direction in the horizontal and vertical directions, respectively. Specification of σ_θ and σ_φ can be found in Gifford [16] and Hanna et al. [18] based on the Pasquill stability classes from A to F (Table 4).

Table 5 shows comparison between observed and predicated concentrations for Gaussian and non- Gaussian under using different schemes of dispersion parameters for I_{131}.

In Figure 1 shows that the comparison between observed and predicated concentrations in Gaussian and non-Gaussian cases using different schemes of dispersion parameters for I_{131} as shown in (a) and (b) respectively. This figure shows some points of the predicted concentrations lie in a factor of two with the observed data. Figure 2 shows the comparison between predicated concentration of Gaussian and non-Gaussian cases using different schemes of dispersion parameters for I_{131} via downwind distance as shown in (c) and (d).

Table 6 shows that comparison between observed and predicated concentrations for Gaussian and non-Gaussian under using different schemes of dispersion parameters for Cs_{137}. Figure 3 shows the comparison between predicated and observed concentration in Gaussian and non-Gaussian cases using different schemes of dispersion parameters for Cs_{137}, as shown in (a) and (b) respectively. This figure shows that most points of the predicted concentrations lie in factor of two with the observed data.

Figure 4 shows that comparison between predicated and observed concentrations in Gaussian and non-Gaussian cases using different schemes of dispersion parameters for Cs_{137} via downwind distance as

Stability classes	A	B	C	D	E	F
σ_y (x)	.32x $(1+0.0004x)^{-1/2}$	0.32x $(1+0.0004x)^{-1/2}$	0. 32x $(1+0.0004x)^{-1/2}$	0.16x $(1+0.0004x)^{-1/2}$	0.11x $(1+0.0004x)^{-1/2}$	0.11x $(1+0.0004x)^{-1/2}$
σ_z (x)	0.24x $(1+0.001x)^{-1/2}$	0.24x $(1+0.00x)^{-1/2}$	0.20x	0.14x $(1+0.0003x)^{-1/2}$	0.08x $(1+0.00015x)^{-1/2}$	0.08x $(1+0.00015x)^{-1//2}$

Table 3: Formulas produced by Briggs (1973) for σy (x) and σz (x).

Experiment	Downwind distance 'x' (m)	U (m/s)	Stability classes	H (m)
1	92	4	A	49
2	96	4	A	48
3	97	6	B	45
4	98	4	C	46
5	99	4	A	45
6	100	4	D	45
7	115	4	E	47
8	132	4	C	46
9	134	4	A	47
10	165	3	D	28
11	184	2	B	28.3
12	200	3	A	30.8
13	300	3	A	30.6

Table 4: Meteorological data (downwind distance 'x', wind speed 'U', stability classes and different effective heights).

Observed concentration (Bq/m^3)	predicated concentration Gaussian (Bq/m^3)				Predicated concentration nonGuassian (Bq/m^3)			
	Briggs method	Standard method	Irwin method	Power low	Power law	standard method	Briggs method	Irwin method
0.025	0.05	0.01	0.05	0.06	0.011	0.054	0.010	0.046
0.037	0.04	0.02	0.04	0.03	0.021	0.129	0.030	0.061
0.090	0.06	0.03	0.04	0.07	0.018	0.035	0.050	0.045
0.200	0.17	0.03	0.31	0.10	0.011	0.011	0.006	0.676
0.270	0.14	0.04	0.31	0.11	0.069	0.039	0.008	0.795
0.190	0.20	0.05	0.13	0.14	0.015	0.043	0.009	0.196
0.450	0.23	0.07	0.26	0.16	0.018	0.078	0.101	0.429
0.120	0.26	0.11	0.16	0.34	0.023	0.096	0.211	0.610
0.030	0.02	0.20	0.04	0.02	0.120	0.113	0.056	0.564
0.420	0.37	0.28	0.63	0.51	0.357	0.060	0.807	0.135
0.420	0.41	0.37	0.27	0.63	0.016	0.055	0.250	0.650
0.670	0.44	0.47	0.57	0.53	0.530	0.022	0.347	0.570
0.670	0.52	0.50	0.32	0.43	0.394	0.068	0.443	0.700

Table 5: Comparison between observed and predicated concentrations for Gaussian and non- Gaussian under using different schemes of dispersion parameters for I_{131}.

Observed concentration (Bq/m^3)	Predicated concentration Gaussian (Bq/m^3)				Predicated concentration nonGuassian (Bq/m^3)			
	Briggs method	Standard method	Irwin method	Power low	Power law	standard method	Briggs method	Irwin method
0.002	0.003	0.001	0.0028	0.002	0.004	0.0039	0.0314	0.012
0.004	0.009	0.001	0.0080	0.006	0.006	0.0034	0.0229	0.033
0.005	0.022	0.002	0.0070	0.001	0.056	0.0061	0.0035	0.004
0.007	0.004	0.007	0.0034	0.006	0.001	0.0038	0.0021	0.001
0.009	0.004	0.007	0.0048	0.007	0.007	0.0058	0.0030	0.002
0.007	0.005	0.004	0.0055	0.004	0.003	0.0071	0.0040	0.004
0.007	0.004	0.006	0.0056	0.006	0.005	0.0123	0.0061	0.002
0.019	0.005	0.012	0.0481	0.024	0.034	0.0371	0.0010	0.020
0.006	0.004	0.004	0.0119	0.004	0.029	0.0070	0.0051	0.005
0.002	0.009	0.004	0.0055	0.004	0.004	0.0012	0.0037	0.002
0.004	0.005	0.005	0.0169	0.001	0.008	0.0046	0.0082	0.004
0.008	0.005	0.005	0.0014	0.005	0.004	0.0021	0.0058	0.005
0.009	0.008	0.040	0.042	0.008	0.0006	0.005	0.003	0.004

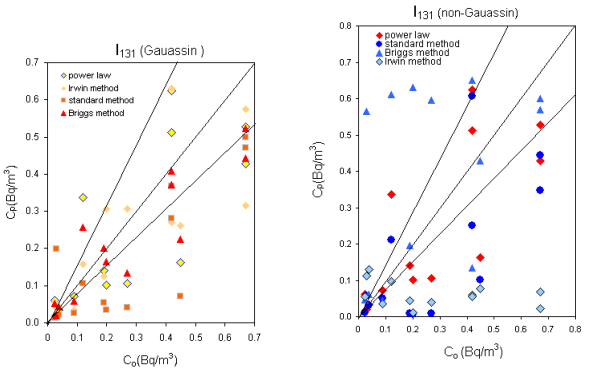
Table 6: Comparison between observed and predicated concentrations for Gaussian and non- Gaussian under using different schemes of dispersion parameters for Cs_{137}.

shown in (c) and (d) respectively.

Statistical Method

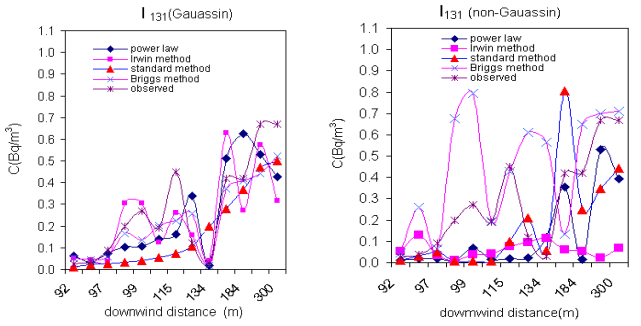
Now, the statistical method is presented and comparison among analytical, statically and observed results will be made [18]. The following standard statistical performance measures characterize the agreement between model prediction (Cp=Cpred/Q) and observations (Co=Cobs/Q):

$$\text{Normalized Mean square Error (NMSE)} = \frac{\overline{(C_p - C_0)^2}}{\overline{(C_p C_0)}}$$



a b

Figure 1: Comparison between predicated and observed concentration in Gaussian and non-Gaussian cases using different schemes of dispersion parameters for I_{131}, as shown in (a) and (b) respectively.



c d

Figure 2: Comparison between predicated and observed concentrations in Gaussian and non-Gaussian cases using different schemes of dispersion parameters for I_{131} via downwind distance as shown in (c) and (d) respectively.

$$\text{Fractional Bias (FB)} = \frac{(\overline{C_0} - \overline{C_p})}{[0.5(\overline{C_0} + \overline{C_p})]}$$

$$\text{Correlation Coefficient (COR)} = \frac{1}{N_m}\sum_{i=1}^{N_m}(C_{pi} - C_p) \times \frac{(C_{0i} - \overline{C_0})}{(\sigma_p \sigma_0)}$$

$$\text{Factor of Two (FAC2)} = 0.5 \le \frac{C_p}{C_0} \le 2.0$$

Where σ_p and σ_o are the standard deviations of C_p and C_o respectively. Here the over bars indicate the average over all measurements (Nm). A perfect model would have the following idealized performance:

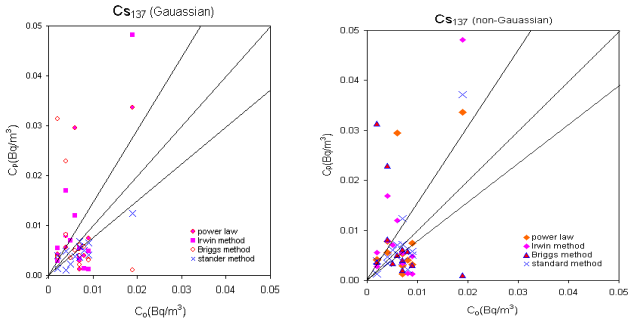
NMSE=FB=0 and COR=FAC2=1.0

Table 7 shows that comparison between Gaussian and non-Gaussian using four methods for I_{131} according to standard statistical performance measure. It was found in the Gaussian for I_{131} lie inside a factor of 2 with observed data. Regarding to NMSE, FB and correlation coefficient, it is found that all methods are better with observed data. While in the non-Gaussian case the correlation coefficient of all methods are better with observed data. Regarding to NMSE, it is found that Briggs, power methods are better than Irwin and Standard methods. One finds that Gaussian formula is better than non-Gaussian with respect to observed data with respect to I_{131}.

Table 8 shows that comparison between Gaussian and non-Gaussian using four methods for Cs_{137} according to standard statistical performance measure. It was found in the Gaussian for Cs_{137} using Briggs method lie inside factor of 2 with observed data than other methods. For NMSE and FB, the predicted concentrations for Cs_{137} using Briggs method are closer to the observed data than the other methods. The correlation coefficient of the predicted concentration for Cs_{137} using power law is good agreement to the observed data than other methods. While in the non-Gaussian case using standard method lie inside factor of 2 with observed data than other methods, the correlation coefficient of standard method better than other methods with respect to observed data. Regarding to NMSE, FB, it is found that Standard method are better than other methods. One finds that non-Gaussian formula is better than Gaussian with respect to observed data.

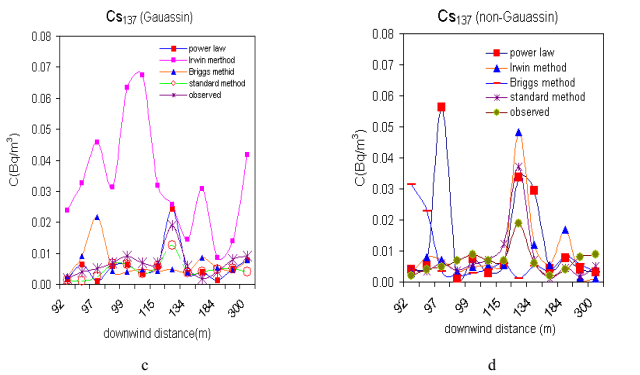
Conclusion

Gaussian and non-Gaussian schemes are used to calculate the concentration for isotopes iodine (I_{131}) and cesium (Cs_{137}), using average value for wind speed and different schemes of dispersion parameters. The statistical technique is used to know the best model for calculating isotopes. One finds that most points of The Gaussian and non-Gaussian schemes lie inside a factor of two. One finds that



a b

Figure 3: Comparison between predicated and observed concentration in Gaussian and non-Gaussian cases using different schemes of dispersion parameters for Cs_{137}, as shown in (a) and (b) respectively.



c d

Figure 4: Comparison between predicated and observed concentrations in Gaussian and non-Gaussian cases using different schemes of dispersion parameters for Cs_{137} via downwind distance as shown in (c) and (d) respectively.

Predicated Concentration model	Gaussian				NonGaussian			
	NMSE	FB	COR	FAC2	NMSE	FB	COR	FAC2
Briggs method	0.29	0.06	0.78	1.94	0.30	0.14	0.79	1.05
Standard method	0.21	-0. 02	0.83	1.0	0.61	0.49	0.81	0.97
Power law	0.16	-0.15	0.92	1.49	0.35	0.14	0.76	1.05
Irwin method	0.86	-0.08	0.66	1.29	0.46	0.35	0.89	0.92

Table 7: Comparison between Gaussian and non-Guassian under four method for I_{131} according to standard statistical performance measure.

Predicated Concentration model	Gaussian				NonGaussian			
	NMSE	FB	COR	FAC2	NMSE	FB	COR	FAC2
Briggs method	1.00	0.08	-0.20	1.37	2.46	-0.11	0.50	2.31
Standard method	21.73	-1.79	0.03	25.97	0.64	-0.11	0.86	1.05
Power law	4.06	-1.41	0.42	7.62	3.19	-. 58	0.26	2.15
Irwin method	4.36	-1.32	0.12	6.35	2.64	-0.10	0.16	1.63

Table 8: Comparison between Gaussian and non-Gaussian under four method for Cs_{137} according to standard statistical performance measure.

Gaussian formula is better than non-Gaussian with respect to observed data I_{131}. It was found in the Gaussian for Cs_{137} using Briggs method lie inside factor of 2 with observed data than other methods. While in the non-Gaussian case using standard and Irwin methods lie inside a factor of two with observed data than other methods, the correlation coefficient of standard method better than other methods with respect to observed data. Regarding to NMSE, FB, it is found that Standard method are better than other methods. One found that non-Gaussian formula is better than Gaussian with respect to observed data.

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