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CONTROL OF DC MOTOR USING DIFFERENT CONTROL STRATEGIES

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Abstract
The ultimate goal of this paper is to control the angular speed ω, in a model of a DC motor driving an inertial load has the angular speed, ω, as the output and applied voltage, v_{app}, as the input, by varying the applied voltage using different control strategies for comparison purpose. The comparison is made between the proportional controller, integral controller, proportional and integral controller, phase lag compensator, derivative controller, lead integral compensator, lead lag compensator, PID controller and the linear quadratic tracker design based on the optimal control theory. It has been realized that the design based on the linear quadratic tracker will give the best steady state and transient system behavior, mainly because, the other compensator designs are mostly based on trial and error while the linear quadratic tracker design is based on the optimal control theory which can give best dynamic performance for the controlled system.

Keywords: DC motor, lead compensator, lag compensator, PI compensator, optimal control, tracking

1. Introduction

The term control system design refers to the process of selecting feedback gains that meet design specifications in a closed-loop control system. Most design methods are iterative, combining parameter selection with analysis, simulation, and insight into the dynamics of the plant. Ref [3] covered how it is possible to improve the system performance, along with various examples of the technique for applying casecade and feedback compensators, using the methodes root locus and frequency response. It also covered some methods of optimal linear system design and presentation of eigenvalues assignments for MIMO system by state feedback. In [2] and [4], good descrbtion of the optimat control design, including llinear state regulator control, the output regulator control and linear quadratic tracker

The matlab SISO Design Tool [1] can be used to design compensators by root locus, Bode diagram, and Nichols plot design techniques, and to analyze the resulting designs. In addition to the SISO Design Tool in Matlab, the Control System Toolbox [2] provides a set of commands that you can be used for a broader range of control applications, including Classical SISO design Modern and MIMO design techniques, such as pole placement and linear quadratic Gaussian (LQG) methods.
A simple model of a DC motor driving an inertial load has the angular speed of the load, ω, as the output and applied voltage, v_{app}, as the input. The system was used as an example in [1]. The ultimate goal of this paper is to control the angular rate by varying the applied voltage using different control strategies for comparison purpose. The comparision is made between the proptional controller, integral controller, propotional and integral controller, phase lag compensator, derivitive controller, lead integral compensator, lead lag compensator, PID controller and the the linear quadratic tracker design based on the optimal control theory.

2. Mathematical model of a DC motor

The resistance of the armature is denoted by R (ohm) and the self-inductance of the armature by L (H). The torque (N.m) seen at the shaft of the motor is proportional to the current i (A) induced by the applied voltage (V),

$$\tau = K_m i \qquad (1)$$

where Km, the armature constant, is related to physical properties of the motor. The back (induced) electromotive force, v_{emf} (V), is a voltage proportional to the angular rate seen at the shaft,

$$v_{emf} = K_b \omega \qquad (2)$$

where Kb, the emf constant, also depends on certain physical properties of the motor.
The mechanical part of the motor equations is derived using Newton's law, which states that the inertial load J (kg·m²) times the derivative of angular rate ω (rad/sec) equals the sum of all the torques (N.m) about the motor shaft. The result is this equation,

$$J\frac{d\omega}{dt} = -K_f \omega + K_m i \qquad (3)$$

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where, $K_f \omega$ is a linear approximation for viscous friction.

The electrical part of the motor equations can be described by

$$\frac{di}{dt} = -\frac{R}{L} i - \frac{K_b}{L}\omega + \frac{1}{L} v_{app} \qquad (4)$$

Given the two differential equations, you can develop a state-space representation of the DC motor as a dynamic system. The current i and the angular rate are the two states of the system. The applied voltage, v_{app}, is the input to the system, and the angular velocity ω is the output.

$$\frac{d}{dt}\begin{bmatrix} i \\ \omega \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_b}{L} \\ \frac{K_m}{J} & \frac{-K_f}{J} \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_{app} \qquad (5)$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} v_{app} \qquad (6)$$

3. Controlling DC Motor Angular Velocity through Different Compensation Techniques

In this paper the DC motor model was used in order to compare different control strategies and compensation techniques. The proposed control schemes were designed in order to derive the angular velocity ω to unity with best design criteria's that can be achieved, i.e, rise time of less than 0.5 second, overshoot of less than 10%, gain margin greater than 20 dB, phase margin greater than 40 degrees

The following nominal values for the various parameters of a DC motor used: R= 2.0 Ohm, L= 0.5 Henrys, Km = .015, Kb = .015, Kf = 0.2 , J= 0.02 kg·m², so the transfer function of the DC motor

$$\frac{\omega(s)}{v_{app}(s)} = \frac{1}{s^2 + 14s + 40.02} = \frac{1}{(s+9.996)(s+4.004)} \qquad (7)$$

$$\omega_{ss} = .035(rad/\sec) \qquad t_s \approx 1.1(\sec)$$

Fig1 shows the open loop response of the dc motor angular speed due to step input, $v_{app} = 1, v_{ap}(s) = \frac{1}{s}$.

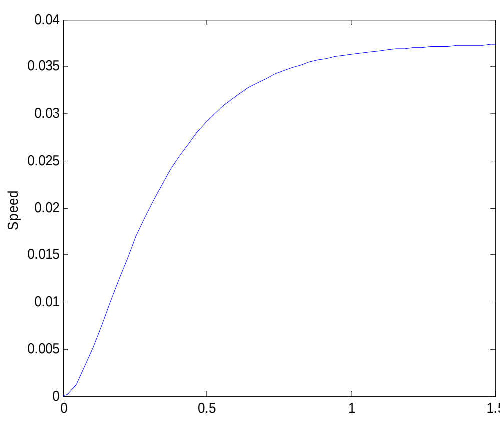

Fig. 1. The open loop step response of the DC motor angular velocity ω (rad/sec)

As the open loop step response has a large steady state error, $\omega_{ss} = 0.037, e_{ss} = .963(rad/\sec)$, different closed loop control strategies and compensator designs were compared in this paper in order to eliminate the steady state error and enhance the system transient response. Fig. 2 shows the DC motor with negative unity feed back, and a feed forward compensator C added in series with the DC motor so it will control the applied voltage to DC motor. The main objective is to design a feed forward compensator C that will derive the DC motor angular velocity to unity.

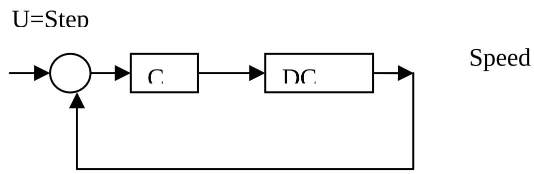

Fig. 2. Closed loop control of DC motor, C is the compensator

3.1. Proportional Controller C=1

The motor was controlled with the feed forward proportional compensator $C = 1$. The overall closed loop transfer function of the controlled system

$$\frac{\omega(s)}{U(s)} = \frac{1.5(s+10)(s+4)}{(s+10)(s+4)(s+4.26)(s+9.73)} \qquad (8)$$

$$\omega_{ss} = .036(rad/\sec) \qquad t_s \approx 1.05(\sec)$$

Fig. 3 shows the closed loop step response of the DC motor angular velocity ω, the root locus and bode plots of the controlled system. It can be noted that the proportional compensator of C=1 could not reduce the steady state error in the angular speed.

\3.2. Proportional Controller C=100:

The motor was controlled with the feed forward proportional compensator $C = 100$. The overall closed loop transfer function of the controlled system

$$\frac{\omega(s)}{U(s)} = \frac{1.5(s+10)(s+4)}{(s+10)(s+4)(s+4.26)(s+9.73)} \qquad (9)$$

$$\omega_{ss} = .789(rad/\sec) \qquad t_s \approx .58(\sec)$$

Fig. 4 shows the closed loop step response of the DC motor angular velocity ω, the root locus and bode plots of the controlled system. It can be noted that the increasing the proportional compensator gain, C=100, reduced the steady state error in angular speed but could not eliminate it. The system will be stable as the propotional gain increased.

3.3. Integral Controller $C = 100/s$:

The motor was controlled with the feed forward integral compensator $C = 100/s$. The overall closed loop transfer function of the controlled system

$$\frac{\omega(s)}{U(s)} = \frac{150(s+10)(s+4)}{s(s+10)(s+4)(s+11)(s+1.16+3.4i)(s+1.16-3.4i)} \qquad (10)$$

$$\omega_{ss} = 1(rad/\sec) \qquad t_{ss} \approx 5(\sec)$$

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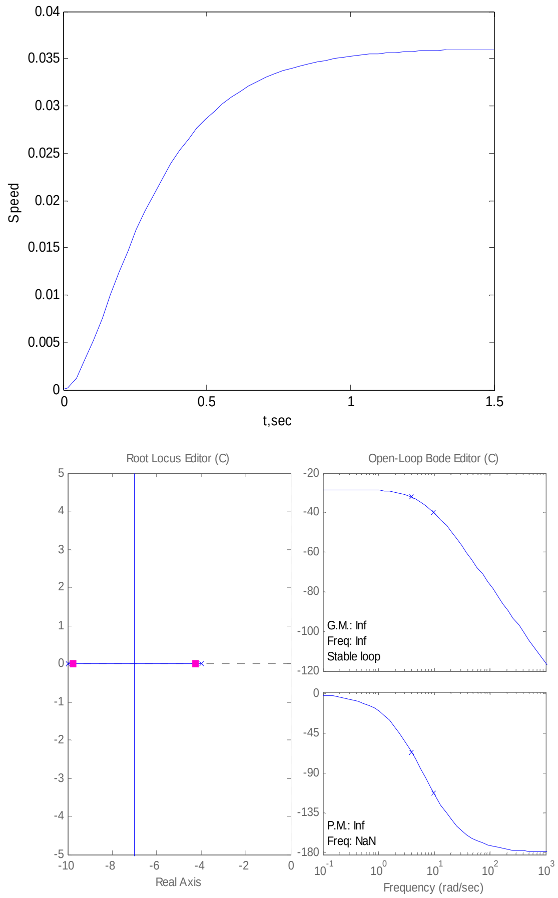

Fig. 3. The closed loop step response of the DC motor angular velocity ω, the root locus and bode plot of the controlled system when C=1.

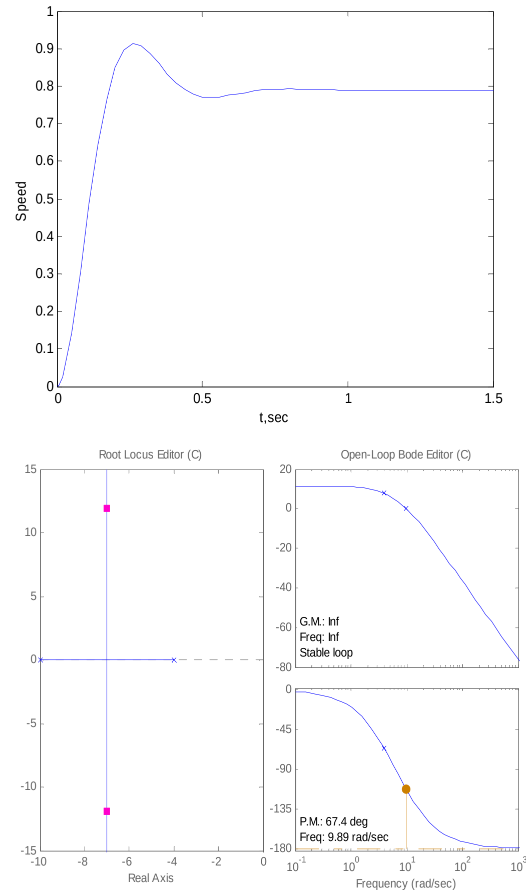

Fig. 4. The closed loop step response of the DC motor angular velocity ω, the root locus and bode plot of the controlled system when C=100.

Fig. 5 shows the closed loop step response of the DC motor angular velocity ω, the root locus and bode plots of the controlled system. It can be noted that the integral controller eliminated the steady state error in angular speed so the steady state response is improved, but the settling time and amount of the overshoot are large, also the system is subject to instability problems as the integral gain increased, so a compensator consisting of an integrator is not enough to satisfy the design requirements.

3.4. Proportional Integral Controller $C = 100*(1+s)/s$:

The motor was controlled with the feed forward proportional integral compensator $C = 100*(1+s)/s$. The overall closed loop transfer function of the controlled system

$$\frac{\omega(s)}{U(s)} = \frac{150s(s+10)(s+4)(s+1)}{s(s+10)(s+4)(s+0.8)(s+6.58+11i)(s+6.58-11i)} \qquad (11)$$

$$\omega_{ss} = 1(rad/\sec) \qquad t_{ss} \approx 5(\sec)$$

Fig. 6 shows the closed loop step response of the DC motor angular velocity ω, the root locus and bode plots of the controlled system. It can be noted hat the proportional and integral controller eliminated the steady state error, the system is stable as the controller gain increased, the amount of overshoot is reduced, but the system settling time still high.

3.5. Phase Lag Compensator $C = 100*(s+0.1)/(s+0.01)$:

Generally the lag compensator has the following form, $C = \frac{A}{\alpha}\frac{s+1/T}{s+1/(\alpha T)}$, is known to result in large increase in gain which means a much smaller steady state error, and a decrease in ω_n and so has the disadvantage of producing an increase in settling time. The zero $s=-1/T$ and the pole $s=-1/(\alpha T)$ are selected close together with α is chosen large value such as 10. The pole and zero are located to the left and close to origin, these results in increased gain.

The motor was controlled with the feed forward phase compensator, $C = 100*(s+0.1)/(s+0.01)$. The overall closed loop transfer function of the controlled system

$$\frac{\omega(s)}{U(s)} = \frac{150(s+10)(s+4)(s+0.1)(s+0.01)}{(s+10)(s+4)(s+0.08)(s+0.01)(s+6.9+11.8i)(s+6.9-11.8i)}$$

$$\omega_{ss} = .974(rad/\sec) \qquad t_s \approx 27.4(\sec) \qquad (12)$$

Fig. 7 shows the closed loop step response of the DC motor angular velocity ω, the root locus and bode plots of the controlled system. It can be noted that the steady state error is reduced but not fully eliminated while the settling time is large. The controlled

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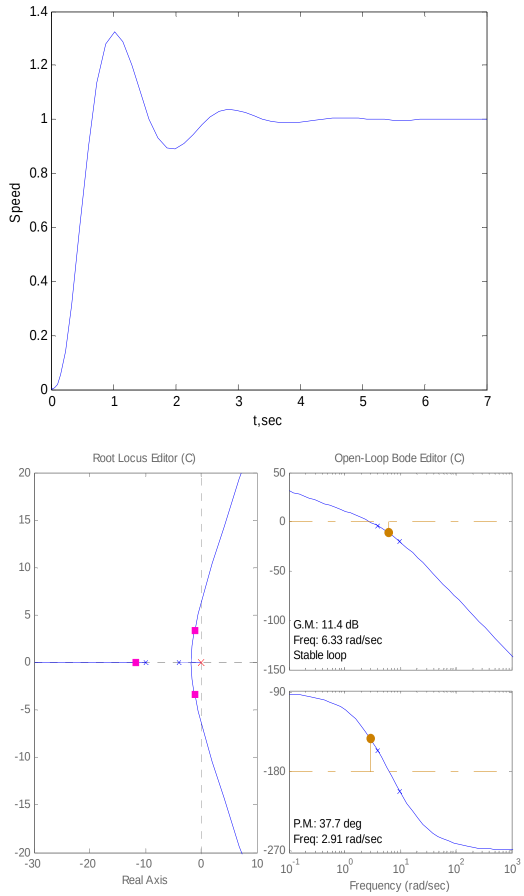

Fig. 5. The closed loop step response of the DC motor angular velocity ω, the root locus and bode plot of the controlled system when $C = 100/s$.

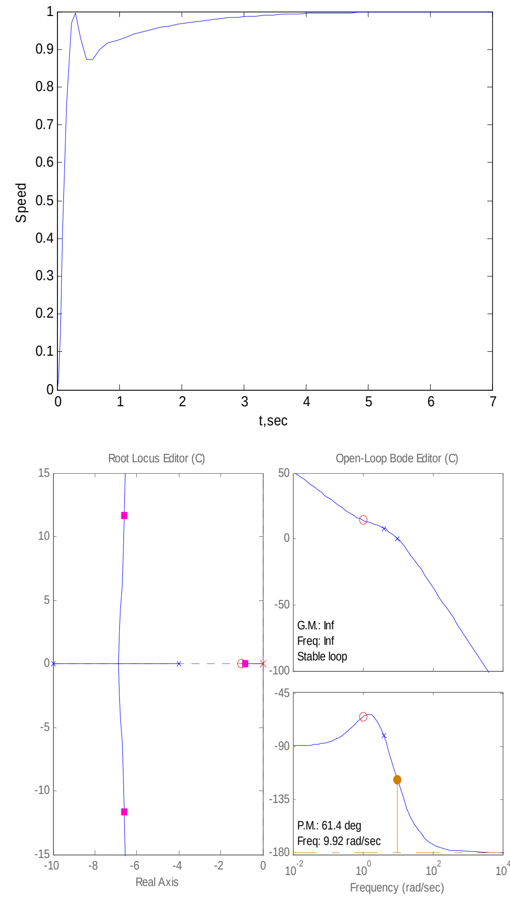

Fig. 6. The closed loop step response of the DC motor angular velocity ω, the root locus and bode plot of the controlled system when $C = 100*(1+s)/s$.

system is not subject to instability problem as the controller gain increased.

3.6. Derivative Compensator $C = s$:

The motor was controlled with the feed forward derivative compensator $C = s$. The overall closed loop transfer function of the controlled system

$$\frac{\omega(s)}{U(s)} = \frac{0.5s(s+10)(s+4)}{(s+10)(s+4)\,(s+12.23)(s+3.273)} \qquad (13)$$

$$\omega_{ss} = 0 \qquad t_s \approx 1.44$$

Fig. 8 shows the closed loop step response of the DC motor angular velocity ω, the root locus and bode plots of the controlled system..It can be noted that derivative compensator will derive the motor angular speed to zero and so the steady state error is not acceptable.

3.7. Lead Integral Compensator $C = 100*(s+10)/(s(s+100))$

settling time. The zero $s = -1/T$ is superimposed on a pole of the original system, and that results in moving the root locus to left and thus increasing the undamped natural frequency. $\alpha = 0.1$ is a common choice.

Generally the lead compensator has the following form:

$C = A\dfrac{s+1/T}{s+1/\alpha T}$. The lead compensator results in moderate increase in gain and thereby improving the steady state error. It also results in large increase in ω_n and therefore reduces the

The motor was controlled with the feed forward lead and integral compensator $C = 100*(s+10)/(s(s+100))$. The overall closed loop transfer function of the controlled system

$$\frac{\omega(s)}{U(s)} = \frac{150s(s+10)^2(s+4)(s+100)}{s(s+10)^2(s+4)\,(s+3.56)(s+3.27)(s+100)^2(s+0.42)}$$

$$\omega_{ss} = 1(rad/\sec) \qquad t_{ss} \approx 15(\sec) \qquad (14)$$

Fig. 9 shows the closed loop step response of the DC motor angular velocity ω, the root locus and bode plots of the controlled system. It can be noted that the lead integral compensator will eliminate the steady state error, but the transient response settling time is large, also the system is subject to instability problems as the controller gain increased.

3.8. Lead Lag Compensator

$C = 100(s+10)(s+0.1)/((s+100)(s+0.01))$

Lead lag compensator shall combine the desirable characteristic of the lead and lag compensators. It shall result in large increase

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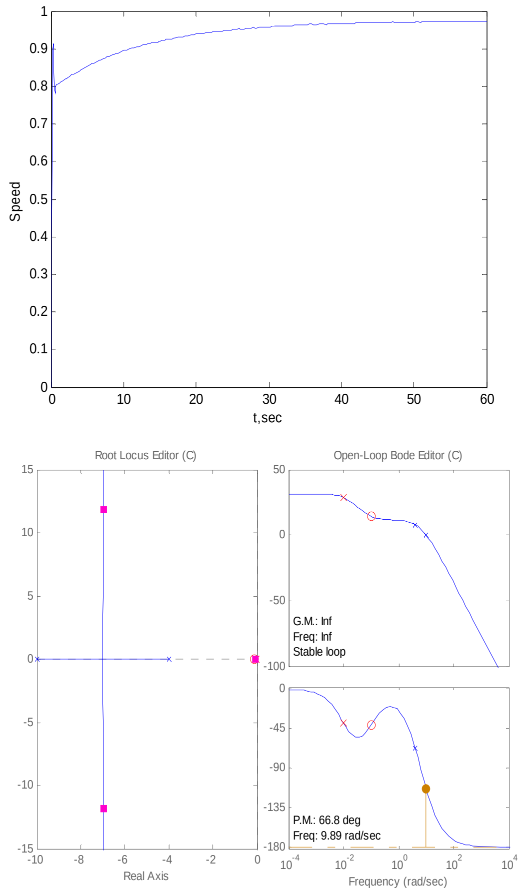

Fig. 7. The closed loop step response of the DC motor angular velocity ω, the root locus and bode plot of the controlled system when $C = 100*(s+0.1)/(s+0.01)$.

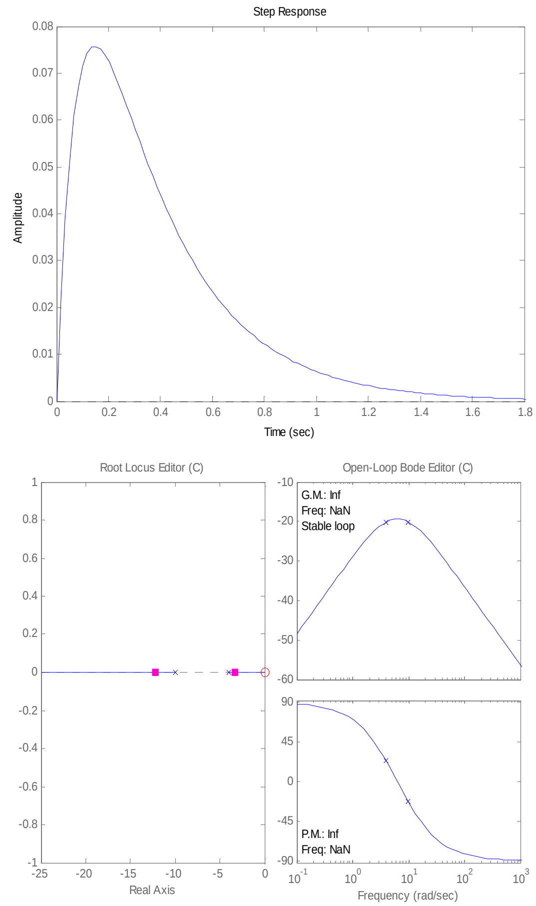

Fig. 8. The closed loop step response of the DC motor angular velocity ω, the root locus and bode plot of the controlled system when $C = s$

in gain which improves the steady state response, and it shall result in an increase in ω_n, which improves the transient settling time. The motor was controlled with the feed forward lead and integral compensator $C = 100(s+10)(s+0.1)/((s+100)(s+0.01))$. The overall closed loop transfer function of the controlled system

$$\frac{\omega(s)}{U(s)} = \frac{150(s+100)\ (s+10)(s+10)(s+4)(s+0.1)(s+0.01)}{(s+100)\ (s+98.4)(s+10)(s+10)\ (s+5.57)(s+4)(s+0.035)\ (s+0.01)}$$

$$\omega_{ss} = 0.78(rad/\sec) \qquad t_{ss} \approx 101(\sec) \qquad (15)$$

Fig. 10 shows the closed loop step response of the DC motor angular velocity ω, the root locus and bode plots of the controlled system. It can be noted that the lead lag compensator will only reduce the steady state error, while the transient response settling time is very large. The controlled system is not subject to instability problem as the controller gain increased.

3.9. Proportional Integral Derivative Compensator (PID) $C = 100 + 100/s + 100*s$

The motor was controlled with the feed forward PID compensator $C = 100 + 100/s + 100*s$. The overall closed loop transfer function of the controlled system

$$\frac{\omega(s)}{U(s)} = \frac{150s(s+10)(s+4)(s^2+s+1)}{s(s+162.8)(s+10)(s+4)\,(s^2+1.16s+0.92)}$$

$$\omega_{ss} = 1(rad/\sec) \qquad t_{ss} \approx 8(\sec) \qquad (16)$$

Fig. 11 shows the closed loop step response of the DC motor angular velocity ω, the root locus and bode plots of the controlled system. It can be noted that the PID compensator can eliminate the steady state error, but still the transient response settling time is quite large. The controlled system will have poles in the imaginary axis as the controlled gain increased.

4. Linear quadratic tracker design:

The continuous linear quadratic tracker problem [2] is summarized as follows. The system model,

$$\dot{x} = f(x,y) = Ax + Bu + Ed \qquad (17)$$

$$y = Cx + Du + Fd \qquad (18)$$

To keep a specified linear combination of the states $y = Cx + Du + Fd$ close to given reference track $r(t)$, let us prescribe the quadratic cost index,

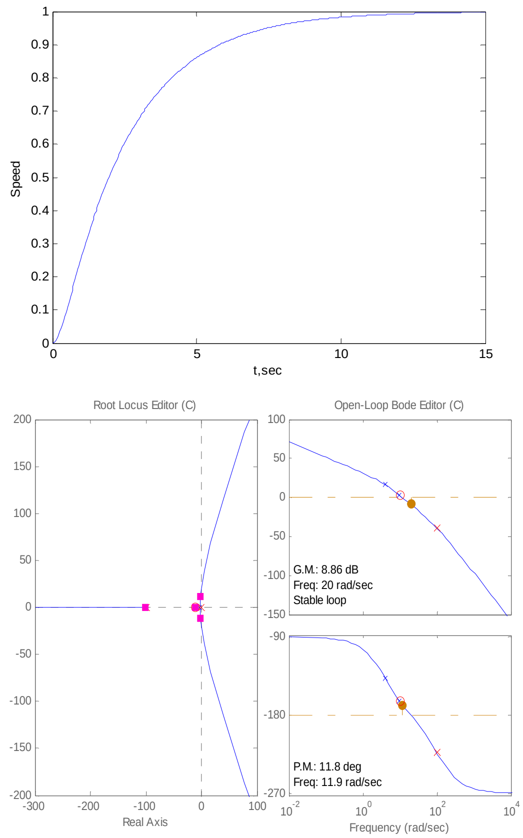

Fig. 9. The closed loop step response of the DC motor angular velocity ω, the root locus and bode plot of the controlled system when $C = 100*(s+10)/(s(s+100))$

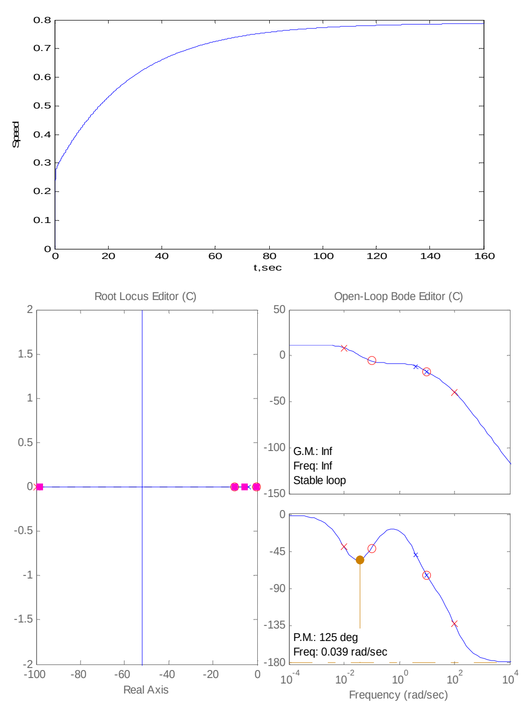

Fig. 10. The closed loop step response of the DC motor angular velocity ω, the root locus and bode plot of the controlled system when $C = 100(s+10)(s+0.1)/((s+100)(s+0.01))$

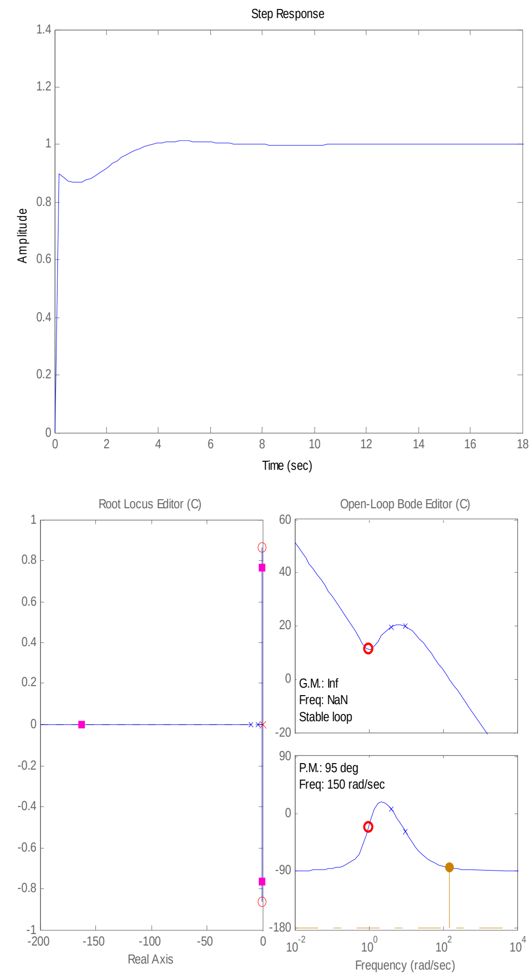

Fig. 11. The closed loop step response of the DC motor angular velocity ω, the root locus and bode plot of the controlled system when $C = 100 + 100/s + 100*s$

$$J = \vartheta(x(T),T) + \int_{to}^{T} [L(x,u,t)dt$$

$$J = \frac{1}{2}[(Cx(T) + Du(T) + Fd(T) - r(T))^{T} \quad (19)$$

$$F(Cx(T) + Du(T + Fd(T) - r(T))]$$

$$+\frac{1}{2}\int_{to}^{T}[(Cx + Du + Fd - r)^{T} Q(Cx + Du + Fd - r)] + [u^{T} Ru]dt$$

If we define the Hamiltonion function,

$$H = L(x,u,t) + \lambda^{T} f(x,u,t) \quad (20)$$

$$L = [Cx + Du - r]^{T} Q[Cx + Du - r] + u^{T} Ru$$

$$L = (Cx)^{T} Q(Cx) + (Cx)^{T} Q(Du) + (Cx)^{T} Q(Fd) - (Cx)^{T} Qr + (Du)^{T} Q(Cx) + (Du)^{T} Q(Du) + (Du)^{T} Q(Fd)$$

$$-(Du)^{T} Qr - r^{T} Q(Cx) - r^{T} Q(Fd) - r^{T} Q(Du) + r^{T} Qr + u^{T} Ru + (Fd)^{T} Q(Cx) + (Fd)^{T} Q(Du)$$

$$-(Fd)^{T} Qr + (Fd)^{T} Q(Fd)$$

The optimal control is given by solving,
State system,

$$\dot{x} = \frac{\partial H}{\partial \lambda} = f(x,u,t) = Ax + Bu + Ed \qquad t \geq t_0 \quad (21)$$

$$y = Cx + Du + Fd \quad (22)$$

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Costate system,

$$-\dot{\lambda} = \frac{\partial H}{\partial x} = (\frac{\partial f}{\partial x})^T \lambda + \frac{\partial L}{\partial x} \qquad t \le T \tag{23}$$

$$-\dot{\lambda} = A^T \lambda + (C^T QC)x + (C^T QD)u + (C^T QF)d - C^T Qr \qquad t \le T$$

Stationary conditions,

$$0 = (\frac{\partial f}{\partial u})^T \lambda + (\frac{\partial L}{\partial u}) \tag{24}$$

$$\frac{\partial L}{\partial u} = Ru + D^T QCx - D^T Qr + D^T QDu + D^T QFd \tag{25}$$

$$0 = B^T \lambda + Ru + D^T QCx - D^T Qr + D^T QDu + D^T QFd \tag{26}$$

$$u = -(R + D^T QD)^{-1}(D^T QCx - D^T Qr + D^T QFd + B^T \lambda) \tag{27}$$

Then, the optimal controller becomes,

$$\dot{x} = Ax + Bu + Ed \qquad t \ge t_0 \tag{28}$$

$$-\dot{\lambda} = A^T \lambda + (C^T QC)x + (C^T QD)u - C^T Qr \tag{29}$$

$$u = -(R + D^T QD)^{-1}(D^T QCx - D^T Qr + B^T \lambda + D^T QFd) \tag{30}$$

$$-\dot{\lambda} = A^T \lambda + (C^T QC)x + (C^T QD)u - C^T Qr + C^T QFd$$
$$= (A^T - C^T QDR_o^{-1} B^T)\lambda + (C^T QC - C^T QDR_o^{-1} D^T QC)x$$
$$+(C^T QDR_o^{-1} D^T Q - C^T Q)r + (-C^T QDR_o^{-1} D^T QF + C^T QF)d$$

(31)

If we considered,

$$H_1 = A - BR_o^{-1} D^T QC \tag{32}$$

$$H_2 = -BR_o^{-1} B^T \tag{33}$$

$$H_3 = -(C^T QC - C^T QDR_o^{-1} D^T QC) \tag{34}$$

$$H_4 = -(A^T - C^T QDR_o^{-1} B^T) \tag{35}$$

$$H_5 = BR_o^{-1} D^T Q \tag{36}$$

$$H_6 = -(C^T QDR_o^{-1} D^T Q - C^T Q) \tag{37}$$

$$H_7 = E - BR_o^{-1} D^T QF \tag{38}$$

$$H_8 = -C^T QDR_o^{-1} D^T QF + C^T QF \tag{39}$$

Then

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} H_1 & H_2 \\ H_3 & H_4 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} + \begin{bmatrix} H_5 \\ H_6 \end{bmatrix} r + \begin{bmatrix} H_7 \\ H_8 \end{bmatrix} d \tag{40}$$

Substituting,

$$\lambda = Sx + v \tag{41}$$

$$\dot{\lambda} = \dot{S}x + S\dot{x} + \dot{v} \tag{42}$$

From that, we have

$$\dot{v} = [H_4 - SH_2]v + [H_6 - SH_5]r + [H_8 - SH_7]d \tag{43}$$

$$\dot{S} = H_3 - SH_1 + H_4 S - SH_2 S \tag{44}$$

$$-\dot{S} = -H_3 + SH_1 - H_4 S + SH_2 S \tag{45}$$

$$A_o^T K + KA_o + Q_o - KBR_o^{-1} B^T K = 0 \tag{46}$$

Where,

$$H_1 = A_o = A - BR_o^{-1} D^T Q \ C \tag{47}$$

$$A_o^T = -H_4 = A^T - C^T QDR_o^{-1} B^T \tag{48}$$

$$Q_0 = -H_3 = C^T QC - C^T QDR_o^{-1} D^T QC \tag{49}$$

$$H_2 = -BR_o^{-1} B^T \tag{50}$$

In steady state, $\dot{v} = 0$

$$v = -[H_4 - SH_2]^{-1}[H_6 - SH_5]r + [H_4 - SH_2]^{-1}[H_8 - SH_7]d \tag{51}$$
$$v = K_r r + K_d d$$

Thus,

$$u = -R_o^{-1}[B^T(Sx + v) + D^T QCx - D^T Qr + D^T QFd]$$
$$u = -R_o^{-1}(B^T S + D^T QC)x +$$
$$(-R_o^{-1} B^T K_r + R_o^{-1} D^T Q)r - (R_0^{-1} B^T K_d + R_o^{-1} D^T QF)d$$

(52)

We can summarise that continuous linear quadratic tracker optimal control as follows,

$$u = F_x x + F_r r + F_d d \tag{53}$$

$$F_x = -R_o^{-1}(B^T S + D^T QC)$$
$$F_r = -R_o^{-1} B^T K_r + R_o^{-1} D^T Q \tag{54}$$
$$F_d = -R_o^{-1} B^T K_d - R_o^{-1} D^T QF$$

Where, S is the solution of the Riccati equation

$$A_o^T S + SA_o + Q_o - SBR_o^{-1} B^T S = 0 \tag{55}$$

$$H_1 = A_o = A - BR_o^{-1} D^T Q \ C \tag{56}$$

$$A_o^T = -H_4 = A^T - C^T QDR_o^{-1} B^T \tag{57}$$

$$Q_0 = -H_3 = C^T QC - C^T QDR_o^{-1} D^T QC \tag{58}$$

$$R_o = R + D^T QD \tag{59}$$

$$H_1 = A - BR_o^{-1} D^T QC \tag{60}$$

$$H_2 = -BR_o^{-1} B^T \tag{61}$$

$$H_3 = -(C^T QC - C^T QDR_o^{-1} D^T QC) \tag{62}$$

$$H_4 = -(A^T - C^T QDR_o^{-1} B^T) \tag{63}$$

$$H_5 = BR_o^{-1} D^T Q \tag{64}$$

$$H_6 = -(C^T QDR_o^{-1} D^T Q - C^T Q) \tag{65}$$

$$H_7 = E - BR_o^{-1} D^T QF \tag{66}$$

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$$H_8 = -C^T Q D R_o^{-1} D^T Q F + C^T Q F \qquad (67)$$

After the solution of the linear quadratic tracker problem, the following control scheme is applied

$$V_{app} = k_1 i + k_2 \omega + k_3 * 1$$

Fig. 12 shows the closed loop step response of the DC motor angular velocity ω. $\omega_{ss} = 1(rad/\sec)$ $t_{ss} \approx 1(\sec)$. So, the designed linear quadratic has the best steady state and transient responses. It fully eliminated the steady state error with small transient settling time. There is no overshoot and the system is completely stable.

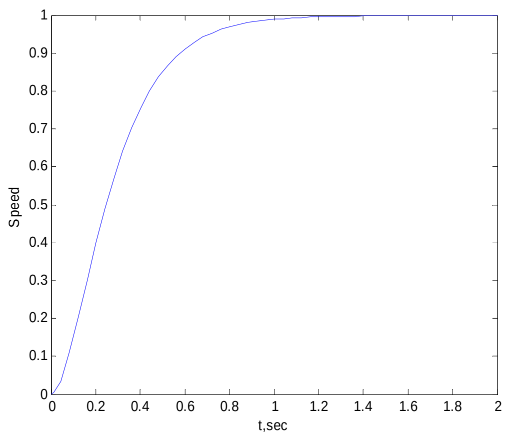

Fig. 12. The closed loop step response of the DC motor angular velocity ω when the optimal output tracker applied

5. Conclusion:

A simple model of a DC motor driving an inertial load has the angular rate of the load, ω, as the output and applied voltage, v_{app}, as the input. Different control strategies and compensator designs with the objective to control the angular speed to be unity with the best steady state and transient performance. The comparision was made between the proptional controller, integral controller, propotional and integral controller, phase lag compensator, derivitive controller, lead integral compensator, lead lag compensator, PID controller and the linear quadratic tracker design based on the optimal control theory. It was found that the designed linear quadratic gave the best steady state and transient responses performances. It fully eliminated the steady state error with the least transient settling time. There is no overshoot and the system is completely stable. The reason is that the other compensator designs are mostely based on trial and error while the linear quadratic tracker design is based on the optimal control theory which can give best dynamic performance for the controlled system.

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