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Counterfeit Coins and the Geometric Distribution: A Note

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Introduction

Currency debasement has been practiced since time immemorial. Such abuse took many forms, ranging from the alloying of gold and silver with other base metals by corrupt heads of state to the filing or clipping of coins by petty criminals. During the period 1285 to 1490, for example, the silver currency of France was debased 123 times and its gold currency 64 times [1]. Today, coins are no longer minted from precious metals, except for a limited number of collectors'/investors' pieces, but they are still being counterfeited. An interesting case in point is the British £1 coin. The Royal Mint estimates that 1 in 30 of these coins is a fake[1]. The problem is so acute that, in 2017, a more complex design and metallic composition will be officially introduced to counteract the fraudster.

The Geometric Distribution: Theory

The geometric distribution rests upon the following assumptions:

(a) The phenomenon being modelled is a series of identical, independent trials

(b) Each trial can result in only two possible outcomes, namely 'success' or 'failure'

(c) The probability of a success is p, the probability of failure is (1-p) and these probabilities are constant in every trial.

If these assumptions hold, then the geometric random variable may be defined in two marginally different ways:-

(1) If X is the number of trials up to and including the first success, the probability mass function for X is given by $P(X=x)=(1-p)^{x-1}.p$, x=1, 2, 3,… with mean 1/p and variance $(1-p)/p^2$.

The cumulative distribution function is given by $F(x)=1-(1-p)^{x}$, x=1, 2, 3,…

A proof of this result for F(x) is provided in the Appendix.

(2) If Y=(X−1) is the number of trials before the first success, then the probability mass function for Y is given by $P(Y= x)=(1-p)^{x}.\ p$, x=0, 1, 2, 3,… with mean (1-p)/p and variance $(1-p)/p^2$.

The cumulative distribution function is given by $F(x)=1-(1-p)^{x+1}$, x=0, 1, 2, 3, Note that since X and Y differ by a constant, the properties of both versions 1 and 2 are almost identical. However, version 1 is employed in the following empirical application.

Geometric Distribution: Empirical Application

A box contained 30 £1 coins, 1 of which was counterfeit. Coins were drawn at random, with replacement, until the counterfeit appeared and the number of trials until this first 'success' was noted. Clearly, the probability of a 'success' that is, drawing a counterfeit, was 0.3333 on each trial. This simple experiment was conducted 100 times and the results are recorded in Table 1.

58	76	61	11	18	32	25	3	31	19
143	86	14	82	60	11	32	32	12	70
38	25	50	8	15	9	3	94	7	52
26	3	15	15	13	68	23	3	17	2
55	22	5	36	9	15	51	32	7	36
29	20	35	51	1	10	132	38	69	30
2	58	67	65	10	81	8	3	29	37
2	5	24	46	76	17	35	8	22	17
34	12	34	18	3	48	24	5	15	24
2	33	4	31	19	7	15	54	48	4

Table 1: Counterfeit appears on trial x=1, 2, 3,…

x	F(x)	Actual
10	0.29	0.26
20	0.49	0.46
30	0.64	0.58
40	0.74	0.74
50	0.82	0.78
60	0.87	0.86
70	0.91	0.92
80	0.93	0.94
90	0.95	0.97
100	0.97	0.98
>100	0.03	0.02

Table 2: Cumulative distribution function: Theoretical *vs.* actual results.

The minimum and maximum number of trials required to locate the counterfeit coin were 1 and 143, respectively, while the mean number of trials was 30.91 with a standard deviation of 27.52. This compares very favourably with a theoretical mean of 1/p=1/0.0333=30.03 and standard deviation of $[(1-p)/p^2]^{0.5}=29.65$.

The cumulative distribution function, $F(x)=1-(1-p)^{x}$, provides the probability that it will take x or fewer trials to produce the first success. A set of these calculations is shown in Table 2. Thus, for example, the theoretical probability that it will require 60 trials or fewer to pick the counterfeit from the box is given by $F(60)=1-(1-0.3333)^{60}=0.87$. The actual proportion of trials necessary is 86/100=0.86.

A cursory glance at the table shows that the geometric distribution fits the empirical data extremely well.

[1]By way of experiment, a random sample of 60 £1 coins were collected and delivered to a bank's cash counting machine. As expected, it rejected 2 counterfeit coins.

References

1. Rolnick AJ, Velde FR, Weber WE (1997). The Debasement Puzzle, Federal Reserve Bank of Minneapolis. Quarterly Review 21: 8-20.

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