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Coupled Fixed Points of α-Ψ-Contractive Type Multi Functions

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Abstract

Recently, Samet, Vetro and Vetro introduced α-ψ-contractive mappings and gave some results on fixed points of the mappings . In fact, their technique generalized some ordered fixed point results . Also they have proved some results on coupled fixed points of α-ψ-contractive mappings. In 1974 Ciric introduced quasicontractive mappings and obtained an important generalization of Banach's contraction principle. Recently Mohammadi, Rezapour and Shahzad have proved some fixed point results on α-ψ-contractive and α-ψ-quasicontractive multifunction's. By using the main idea of, we give some new results for coupled fixed points of α-ψ-contractive multifunction.

Keywords: Coupled fixed point; α-ψ-Contractive; Multifunction

Introduction

Denote by Ψ the family of non-decreasing functions $\psi: [0, +\infty) \to [0, +\infty)$ such that $\sum_{n=1}^{+\infty}\psi^n(t) < +\infty$ for all $t > 0$. It is well known that $\psi(t) < t$ for all $t > 0$.

Definition 1.1: Let (X, d) be a metric space and $\alpha: X \times X \to [0, \infty)$ be a map. We say that X satisfies condition (C_α), if for any sequence $\{x_n\}$ in X, that $x_n \to x$ and $\alpha(x_n, x_{n+1}) \geq 1$ for all n, then $\alpha(x_n, x) \geq 1$ for all n. Also, we say that the selfmap T on X is α-admissible whenever $\alpha(x, y) \geq 1$ implies $\alpha(T_x, T_y) \geq 1$.

Definition 1.2: Let X is an arbitrary space and $\alpha: X^2 \times X^2 \to [0, +\infty)$ a map. A mapping $F: X^2 \to X$ is said to be α-admissible whenever $\alpha((x, y), (u, v)) \geq 1$ implies

$$\alpha((F(x, y), F(y, x)), (F(u, v), F(v, u))) \geq 1.$$

Definition 1.3: Let (X, d) be a metric space and $\alpha: X^2 \times X^2 \to [0, \infty)$. We say that X satisfies condition (C_α) if for any two sequences $\{x_n\}$ and $\{y_n\}$ in X, that $x_n \to x$, $y_n \to y$ and

$$\alpha((x_n, y_n), (x_{n+1}, y_{n+1})) \geq 1, \alpha((y_{n+1}, x_{n+1}), (y_n, x_n)) \geq 1$$

For all n, then we have

$$\alpha((x_n, y_n), (x, y)) \geq 1, \alpha((y, x), (y_n, x_n)) \geq 1$$

for all n.

Definition 1.4: Let X is an arbitrary space and $F: X^2 \to X$ is a mapping. We say that $(x^*, y^*) \in X^2$ is a coupled fixed point of F, if we have $F(x^*, y^*)=x^*$ and $F(y^*, x^*)=y^*$.

In 2011, Samet, Vetro and Vetro have proved the following theorem [1].

Theorem 1.1: Let (X, d) be complete a metric space, $\alpha: X^2 \times X^2 \to [0, \infty)$ a function, $\psi \in \Psi$ and $F: X^2 \to X$ an α-admissible mapping such that

$$\alpha((x, y), (u, v))d(F(x, y), F(u, v)) \leq \frac{1}{2}\psi(d(x, u) + d(y, v)).$$

for all $(x, y), (u, v) \in X^2$. Assume that the following assertions hold. (i) There exists $(x_0, y_0) \in X^2$ such that

$$\alpha((x_0, y_0), (F(x_0, y_0), F(y_0, x_0))) \geq 1,$$

$$\alpha((F(y_0, x_0), F(x_0, y_0)), (y_0, x_0)) \geq 1.$$

(ii) Either F is continuous or X satisfies condition (C_α^*). Then F has a coupled fixed point in X^2.

Let (X, d) be a metric space. Define the metric δ on X^2 by $\delta((x, y), (u, v))=d(x, u) + d(y, v)$, for all $(x, y), (u, v) \in X^2$. Also if $F: X^2 \to X$ then put [2].

$$m((x, y), (u, v))=\max\{\delta((x, y), (u, v)), \delta((x, y), (F(x, y), F(y, x))), \delta((u, v), (F(u, v), F(v, u)))$$

$$\frac{1}{2}[\delta((x,y),(F(u,v),F(v,u)))+\delta((u,v),(F(x,y),F(y,x)))]\},$$

for all $(x, y), (u, v) \in X^2$. It is easy to see that $m((x, y), (u, v))=m((v, u), (y, x))$. Recently Rezapour and H. Asl have extended theorem 1.1 to quasi-contractions as follow [3].

Theorem 1.2: Let (X, d) be a complete metric space, $\alpha: X^2 \times X^2 \to [0, +\infty)$ a function, $\psi \in \Psi$ and $F: X^2 \to X$ an α-admissible mapping such that [4]

$$\alpha((x, y),(u, v))d(F(x, y), F(u, v)) \leq \frac{1}{2}\psi(m((x, y), (u, v))).$$

for all $(x, y), (u, v) \in X^2$. Assume that the following assertions hold. (i) There exists $(x_0, y_0) \in X^2$ such that

$$\alpha((x_0, y_0), (F(x_0, y_0), F(y_0, x_0))) \geq 1,$$

$$\alpha((F(y_0, x_0), F(x_0, y_0)), (y_0, x_0)) \geq 1.$$

(ii) Either F is continuous or ψ is right upper semi continuous and X satisfies condition (C_α^*).

Then F has a coupled fixed point in X^2.

Definition 1.5: Let (X, d) be a metric space and $F: X^2 \to X$. We say that F is orbitally continuous if for any two sequences $\{x_n\}$ and $\{y_n\}$ in X, that $x_n \to x$, $y_n \to y$ and $x_{n+1}=F(x_n, y_n)$, $y_{n+1}=F(y_n, x_n)$, then $F(x_n, y_n) \to F(x, y)$.

Obviously theorem 1.2 is true if F be orbitally continuous instead of continuity. Now we give the following example to show that theorem 1.2 is a real generalization of theorem 1.1, which is there exist mappings

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that we can use theorem 1.2, but we can't use theorem 1.1 for them [5].

Example 1.6:

Let $M1=\{\frac{m}{n}: m=0, 1, 3, 9, \ldots$ and $n=3k+1$ $(k \ge 0)\}$, $M2=\{\frac{m}{n}: m=1, 3, 9, 27, \ldots$ and $n=3k+2$ $(k \ge 0)\}$. Set $M=M1 \cup M2$, $d(x, y)=|x-y|$ and define $F: M^2 \to M$ by

$$F(x,y)=\begin{cases} 3x/13 & x,y\in M_1, x\ge y \\ x/8 & x,y\in M_2, x\ge y \\ 0 & otherwise \end{cases} \tag{1}$$

and $\alpha : M^2 \times M^2 \to [0, +\infty)$ by

$$\alpha((x,y),(u,v))=\begin{cases} 1 & (x,y),(u,v)\in M_1^2\cup M_2^2\ ,x\ge y,u\ge v \\ 0 & otherwise. \end{cases} \tag{2}$$

If $(x, y) \in M_1^2$, $x \ge y$ and $(u, v) \in M_2^2$, $u \ge v$, then

$$d(F(x,y),F(u,v))=\left|\frac{3x}{13}-\frac{u}{8}\right|=\frac{3}{13}\left|x-\frac{13u}{24}\right|.$$

Now we consider two cases. If $x>\frac{13}{24}u$, then we have

$$\frac{3}{13}\left|x-\frac{13u}{24}\right|=\frac{3}{13}(x-\frac{13u}{24})\le\frac{3}{13}(x-\frac{u}{8})=\frac{1}{2}(\frac{12}{13})\frac{d(x,F(u,v))}{8}.$$

$$\le\frac{1}{2}(\frac{12}{13})\frac{\delta(x,y,F(u,v),F(u,v)))}{2}\le\frac{1}{2}(\frac{12}{13})m(x,y),(u,v)).$$

Hence

$$d(F(x,y),F(u,v))\le\frac{1}{2}(\frac{12}{13})m(x,y),(u,v)).$$

If $x<\frac{13}{24}u$, then

$$\frac{3}{13}\left|x-\frac{13u}{24}\right|=\frac{3}{13}(\frac{13u}{24}-x)\le\frac{3}{13}(u-x)$$

$$\le\frac{1}{2}(\frac{6}{13})\delta(x,y),(u,v))\le\frac{1}{2}(\frac{12}{13})m(x,y),(u,v)).$$

Hence

$$d(F(x,y),F(u,v))\le\frac{1}{2}(\frac{12}{13})m(x,y),(u,v)).$$

If $(x, y),(u, v)\in M_1^2$, $x \ge y$, $u \ge v$, then

$$d(F(x,y),F(u,v))=\left|\frac{3x}{13}-\frac{3u}{13}\right|=\frac{3}{13}d(x,u)\le\frac{1}{2}(\frac{12}{13})m(x,y),(u,v)).$$

If $(x, y),(u, v)\in M_2^2$, then

$$d(F(x,y),F(u,v))=\left|\frac{x}{8}-\frac{u}{8}\right|=\frac{1}{8}d(x,u)\le\frac{1}{2}(\frac{12}{13})m(x,y),(u,v)).$$

Now put $\psi(t)=\frac{12t}{13}$, for all $t \ge 0$, and then we see that

$$\alpha((x,y),(u,v))d(F(x,y),F(u,v))\le\frac{1}{2}\psi(m((x,y),(u,v))),$$

for all $(x, y), (u, v) \in M^2$. Also

$\alpha((1, 1), (F(1, 1), F(1, 1)))=\alpha((1, 1), (3/13, 3/13))=1$,

$\alpha((F(1, 1), F(1, 1)), (1, 1))=\alpha((3/13, 3/13), (1, 1))=1$.

To show that F is α-admissible, assume $\alpha((x, y), (u, v)) \ge 1$. Then $(x, y),(u, v)\in M_1^2\cup M_2^2, x \ge y, u \ge v$. Hence either $x > y$ and $F(y, x)=0$ or $x=y$ and $F(x, y)$ $F(y, x)$. However $F(x, y) \ge F(y, x)$. Similarly $F(u, v) \ge F(v, u)$. On the other hand $F(x, y)$, $F(y, x)$, $F(u, v)$, $F(v, u) \in M_1$. Hence

$\alpha((F(x, y), F(y, x)), (F(u, v), F(v, u))) \ge 1.$

It is easy to check that F is orbitally continuous. Now by theorem 1.2 we can say that F has a coupled fixed point in X^2. In fact (0, 0) is a coupled fixed point of F [6].

Now, we show that we cannot apply theorem 1.1 in this example. To see this put [7]

$x=y=9,\ u=\frac{243}{26}, v=\frac{243}{29}$. Then

$$\alpha((x,y),(u,v))d(F(x,y),F(u,v))=\left|\frac{27}{13}-\frac{243}{26\times8}\right|=\frac{189}{208}=0.9.$$

On the other hand

$$\frac{1}{2}\psi(\delta((x,y),(u,v)))\le\frac{1}{2}\delta((x,y),(u,v))=\frac{1}{2}(|x-u|+|y-v|)$$

$$\frac{1}{2}(\left|9-\frac{243}{26}\right|+\left|9-\frac{243}{26}\right|)=\frac{729}{1508}=0.48.$$

Hence

$$\alpha((x,y),(u,v))d(F(x,y),F(u,v))>\frac{1}{2}\psi(\delta((x,y),(u,v))).$$

Let (X, d) be a metric space and $C B(X)$ is the set of all nonempty closed bounded subsets of X, $\alpha : X \times X \to [0, \infty)$ a mapping and $T : X \to C B(X)$ a multifunction. We say that T is α-admissible whenever for each $x \in X$ and $y \in T x$ with $\alpha(x, y) \ge 1$ we have $\alpha(y, z) \ge 1$ for all $z \in Ty$ ([4]). Recall that T is continuous whenever $H (Tx_n,Tx) \to 0$ for all sequence $\{x_n\}$ in X with $x_n \to x$, where H is the Hausdorff metric on $CB(X)$ defined by $H (A, B)=max\{sup_x\in_A d(x, B), sup_y\in_B d(y, A)\}$, for all $A, B \in C B(X)$. Also we say that T is orbitally continuous whenever $H (Tx_n, T_x) \to 0$ for all sequence $\{x_n\}$ in X with $x_{n+1} \in Tx_n$ for all n and $x_n \to x$. Recently Mohammadi, Rezapour and Shahzad have proved the following lemma [8].

Lemma 1.3: ([4] Let (X, d) be a complete metric space, $\alpha : X \times X \to [0, \infty)$ a function, $\psi \in \Psi$ a strictly increasing map and $T : X \to C B(X)$ an α-admissible multifunction such that $\alpha(x, y)H (Tx, Ty) \le \psi(d(x, y))$ for all $x, y \in X$ and there exist $x_0 \in X$ and $x_1 \in Tx_0$ with $\alpha(x_0, x_1) \ge 1$. If T is continuous or X satisfies the condition (C_α), then T has a fixed point. Note that if T be orbitally continuous instead of continuity, then the lemma1.3 is also true [2].

Main Results

Now, we are ready to state and prove our main results.

Definition 2.1: Let X is an arbitrary space and $\alpha: X^2 \times X^2 \to [0, +\infty)$ a map. A multifunction $F : X^2 \to C B(X)$ is said to be α-admissible whenever if $(x, y) \in X^2$,$(u, v) \in F(x, y) \times F (y, x)$ and $\alpha((x, y), (u, v)) \ge 1$, $\alpha((v, u), (y, x)) \ge 1$, then $\alpha((u, v), (w, z)) \ge 1$, $\alpha((z, w), (v, u)) \ge 1$, for all $(w, z) \in F(u, v) \times F (v, u)$.

Definition 2.2: Let (X, d) be a metric space and $F: X^2 \to CB(X)$ is a multifunction. We say that $(x^*, y^*) \in X^2$ is a coupled fixed point of F if we have $x^* \in F (x^*, y^{**})$ and $y^* \in F (y^*, x^*)$.

Theorem 2.1: Let (X, d) be a complete metric space, $\alpha: X^2 \times X^2 \to [0,$

$+\infty)$ a function, $\psi \in \Psi$ a strictly increasing map and F: $X_2 \to CB(X)$ an α-admissible multifunction such that [7]

$$\alpha((x,y),(u,v))H\left(F(x,y),F(u,v)\right) \le \frac{1}{2}\psi\left(d(x,u)+d(y,v)\right).$$

for all $(x, y), (u, v) \in X^2$. Assume that the following assertions hold.

(i) There exists $(x_0, y_0) \in X_2$ and $(x_1, y_1) \in F(x_0, y_0) \times F(y_0, x_0)$ such that

$$\alpha((x_0, y_0),(x_1, y_1)) \ge 1, \quad \alpha((y_1, x_1),(y_0, x_0)) \ge 1.$$

(ii) Either F is continuous or X satisfies condition (C^*). Then F has a coupled fixed point in X^2.

Proof Define $\beta: X_2 \times X_2 \to [0, +\infty)$ by

$$\beta((\xi_1, \xi_2), (\eta_1, \eta_2))=\min\{\alpha((\xi_1, \xi_2), (\eta 1, \eta 2)), \alpha((\eta_1, \eta_2), (\xi_1, \xi_2))\} \tag{3}$$

for all

$\xi=(\xi_1, \xi_2) \in X^2, \eta=(\eta_1, \eta_2) \in X^2.$

Also suppose $T: X^2 \to C\,B(X^2)$ is defined by $T(x, y)=F(x, y) \times F(y, x)$. Obviously the metric space (X^2, δ) is complete. Now since F is α-admissible, it is easy to see that T is β-admissible. Also by assumption

$$\alpha((x,y),(u,v))H\left(F(x,y),F(u,v)\right) \le \frac{1}{2}\psi\left(d(x,u)+d(y,v)\right),$$

$$\alpha((v,u),(y,x))H\left(F(v,u),F(y,x)\right) \le \frac{1}{2}\psi\left(d(v,y)+d(u,x)\right).$$

for all $(x, y), (u, v) \in X^2$. By adding the above relations we obtain.

$$\beta((x, y), (u, v))[H(F(x, y), F(u, v)) + H(F(v, u), F(y, x))] \le \psi(\delta((x, y), (u, v))). \tag{4}$$

Assume that H_δ is the Hausdorff metric on (X^2, δ). We should show that

$$H_\delta(T(x, y), T(u, v)) \le H(F(x, y), F(u, v)) + H(F(v, u), F(y, x)). \tag{5}$$

For this we have

$$H_\delta(T(x, y), T(u, v))=H_\delta(F(x, y) \times F(y, x), F(u, v) \times F(v, u))$$

$$= \max\{ \sup_{(\xi_1,\xi_2)\in F(x,y)\times F(y,x)} \delta((\xi_1,\xi_2), F(u,v)\times F(v,u)),$$

$$\sup_{(\eta_1,\eta_2)\in F(u,v)\times F(v,u)} \delta((\eta_1,\eta_2), F(x,y)\times F(y,x))\}.$$

Let $(\xi_1, \xi_2) \in F(x, y) \times F(y, x)$. Then

$$\delta\left((\xi_1,\xi_2), F(u,v)\times F(v,u)\right) = \inf_{(\eta_1,\eta_2)\in F(u,v)\times F(v,u)} \delta((\xi_1,\xi_2),(\eta_1,\eta_2)),$$

$$= \inf_{(\eta_1,\eta_2)\in F(u,v)\times F(v,u)} [d(\xi_1,\eta_1) + d(\xi_2,\eta_2)].$$

$$= \inf_{\eta 1\in F(u,v)} d(\xi_1,\eta_1) + \inf_{\eta_2\in F(v,u)} d(\xi_2,\eta_2)$$

$$= d(\xi_1, F(u,v)) + d(\xi_2, F(v,u)) \le H\left(F(x,y),F(u,v)\right) + H(F(y,x),F(v,u)).$$

Similarly for $(\eta_1, \eta_2) \in F(u, v) \times F(v, u)$ we have

$$\delta((\eta_1, \eta_2), F(x, y) \times F(y, x)) \le H(F(x, y), F(u, v)) + H(F(y, x), F(v, u)).$$

Hence (5) holds. By (4) and (5) we have

$$\beta((x, y), (u, v))H_\delta(T(x, y), T(u, v)) \le \psi(\delta((x, y), (u, v))). \tag{6}$$

Hence for any $\xi=(\xi_1, \xi_2) \in X^2$, $\eta=(\eta_1, \eta_2) \in X^2$ we have

$$\beta(\xi, \eta)H_\delta(T\xi, T\eta) \le \psi(\delta(\xi, \eta)).$$

So $T: X^2 \to C\,B(X^2)$ is a β-ψ-contractive multifunction. Put $P_0=(x_0, y_0)$, $P_1=(x_1, y_1)$. Then $P1 \in T\,P_0$, $\beta(P_0, P_1) \ge 1$. Now let F be continuous. We show that T is continuous. To see this let $\{(x_n, y_n)\}$ be a sequence in X^2 such that $\delta((x_n, y_n), (x, y)) \to 0$. Then $d(x_n, x)+d(y_n, y) \to 0$ and hence $d(x_n, x) \to 0$, $d(y_n, y) \to 0$. Now since F is continuous, hence

$$H(F(x_n, y_n), F(x, y)) \to 0, H(F(y_n, x_n), F(y, x)) \to 0.$$

Therefore

$$H_\delta(T(x_n, y_n), T(x, y)) \le H(F(x_n, y_n), F(x, y)) + H(F(y_n, x_n), F(y, x)) \to 0.$$

This shows that $T: X^2 \to C(X^2)$ is continuous.

Also if X has (C_α^*) condition, it is easy to see that X^2 has condition C_β. Now all of the conditions of lemma 1.3 hold. Hence by the lemma there exists $(x^*, y^*) \in X^2$ such that $(x^*, y^*) \in T(x^*, y^*)=F(x^*, y^*) \times F(y^*, x^*)$, hence $x^* \in F(x^*, y^*)$, $y^* \in F(y^*, x^*)$. That is (x^*, y^*) is a coupled fixed point of F in X^2. Now by using the following simple definitions we want to prove another version of theorem 2.1.

Definition 2.3: Let (X, d) is a metric space and $\alpha: X^2 \times X^2 \to [0, +\infty)$ is a map. A multifunction $F: X^2 \to C\,B(X)$ is said to be modified α-admissible whenever if $(x, y) \in X^2, (u, v) \in F(x, y) \times F(y, x)$ and $\alpha((x, y), (u, v)) \ge 1$, then $\alpha((u, v), (w, z)) \ge 1$, for all $(w, z) \in F(u, v) \times F(v, u)$.

Definition 2.4: Let (X, d) be a metric space and $\alpha: X\,2 \times X\,2 \to [0, \infty)$. We say that X satisfies condition (B_α^*) if for any two sequences $\{x_n\}$ and $\{y_n\}$ in X, that $x_n \to x$, $y_n \to y$ and $\alpha((x_n, y_n), (x_{n+1}, y_{n+1})) \ge 1$ for all n, then we have $\alpha((x_n, y_n), (x, y)) \ge 1$ for all n.

Theorem 2.2: Let (X, d) be a complete metric space, $\alpha: X^2 \times X^2 \to [0, +\infty)$ a function, $\psi \in \Psi$ a strictly increasing map and $F: X^2 \to C\,B(X)$ a modified α- admissible multifunction such that

$$\alpha((x,y),(u,v))H\left(F(x,y),F(u,v)\right) \le \frac{1}{2}\psi\left(d(x,u)+d(y,v)\right).$$

For all $(x, y), (u, v) \in X^2$. Assume that the following assertions hold.

(i) There exist $(x_0, y_0) \in X^2$ and $(x_1, y_1) \in F(x_0, y_0) \times F(y_0, x_0)$ such that

$$A((x_0, y_0), (x1, y1)) \ge 1, \quad \alpha((y_0, x_0), (y_1, x_1)) \ge 1.$$

(ii) Either F is continuous or X satisfies condition (B_α^*).

Then F has a coupled fixed point in X^2.

Proof Define $\beta: X^2 \times X^2 \to [0, +\infty)$ by

$$\beta((\xi_1, \xi_2), (\eta_1, \eta_2))=\min\{\alpha((\xi_1, \xi_2), (\eta_1, \eta_2)), \alpha((\xi_2, \xi_1), (\eta_2, \eta_1))\} \tag{7}$$

for all

$$\xi = (\xi_1,\xi_2) \in X^2, \eta = (\eta_1,\eta_2) \in X^2$$

The remain of proof is completely similar to proof of theorem 2.1

Example 2.5: Let X be the space of real numbers with the usual metric $d(x, y)=|x - y|$ and define $F: X^2 \to C\,B(X)$ by

$$F(x,y) = \begin{cases} \left[0, \frac{x-y}{3}\right] & x \ge y \\ \{0\} & x < y \end{cases} \tag{8}$$

Also define $\alpha: X^2 \times X^2 \to [0, +\infty)$ by

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$$\alpha((x,y),(u,v)) = \begin{cases} 1 & x \ge y, u \ge v \\ 0 & otherwise \end{cases} \tag{9}$$

If $(x, y), (u, v) \in X^2$ and $x \ge y$, $u \ge v$, then

$$H(F(x,y),F(u,v)) = H([0,\frac{x-y}{3}],[0,\frac{u-v}{3}])$$

$$= \frac{1}{3}|x-y-(u-v)| \le \frac{1}{2}(\frac{2}{3})(d(x,u)+d(y,v)).$$

Now put $\psi(t) = \frac{2t}{3}$. We see that

$$\alpha((x,y),(u,v))H(F(x,y),F(u,v)) \le \frac{1}{2}\psi(d(x,u)+d(y,v)).$$

Also for $(x_0, y_0)=(1, 1)$, $(x_1, y_1)=(0, 0)$ we have

$F(x_0, y_0)=F(y_0, x_0)=\{0\}$, $(x_1, y_1)=(0, 0) \in \{(0, 0)\}=F(x_0, y_0) \times F(y_0, x_0)$,

$\alpha((x_0, y_0), (x_1, y_1))=\alpha((1, 1), (0, 0))=1$,

$\alpha((y_0, x_0), (y_1, x_1))=\alpha((1, 1), (0, 0))=1$.

Now let $(x, y) \in X^2$, $(u, v) \in F(x, y) \times F(y, x)$ and $\alpha((x, y), (u, v)) \ge 1$. Then $x \ge y$, $u \ge v$. Hence $F(v, u)=\{0\}$. Now if $(w, z) \in F(u, v) \times F(v, u)$, then $w \ge 0=z$. Therefore $\alpha((u, v), (w, z)) \ge 1$.This shows that F is modified α-admissible. It is easy to see that F is continuous. Hence by theorem 2.2, F has a coupled fixed point in X^2. For example $(0,0)$ is a coupled fixed point of F.

Example 2.6: Let X be the space of real numbers with the usual metric $d(x, y)=|x-y|$ and define $F: X^2 \to CB(X)$ by $F(x, y)=[0, 2|x-y|]$ for all $x, y \in X$. Also define $\alpha: X^2 \times X^2 \to [0, \infty)$ by

$$\alpha((x,y),(u,v)) = \begin{cases} \frac{1}{8} & (x,y,u,v) \ne (0,0,0,0) \\ 1 & x=y=u=v=0 \end{cases}$$

Now put $\psi(t) = \frac{t}{2}$. We see that

$$\alpha((x,y),(u,v))H(F(x,y),F(u,v)) \le \frac{1}{2}\psi(d(x,u)+d(y,v)).$$

Also for $(x_0, y_0)=(x_1, y_1)=(0, 0)$ we have

$F(x_0, y_0)=F(y_0, x_0)=\{0\}$, $(x_1, y_1)=(0, 0) \in \{(0, 0)\}=F(x_0, y_0) \times F(y_0, x_0)$, $\alpha((x_0, y_0), (x_1, y_1))=\alpha((0, 0), (0, 0))=1$,

$\alpha((y_0, x_0), (y_1, x_1))=\alpha((0, 0), (0, 0))=1$.

It is easy to see that F is modified α-admissible. Obviously F is continuous. Hence by theorem 2.2, F has a coupled fixed point in X^2. For example (0,0) is a coupled fixed point of F. Note that coupled fixed point of F is not unique for example (2, 1) is a coupled fixed point of F too. In fact for any $x, y \ge 0$ that $x \ge 2y$ or $y \ge 2x$,(x, y) is a coupled fixed point of F.

Corollary 2.3: Let (X, d) be a complete metric space and $\preceq$ be an order on X^2. Suppose $\psi \in \Psi$ a strictly increasing map and $F: X^2 \to CB(X)$ a multifunction such that

$$H(F(x,y),F(u,v)) \le \frac{1}{2}\psi(d(x,u)+d(y,v)),$$

for all comparable elements $(x, y), (u, v) \in X^2$. Assume that the following assertions hold.

(i) If $(x, y) \in X^2$,$(u, v) \in F(x, y) \times F(y, x)$ and $(x, y), (u, v)$ are comparable, then $(u, v), (w, z)$ are comparable, for all $(w, z) \in F(u, v) \times F(v, u)$.

(ii) There exist $(x_0, y_0) \in X^2$ and $(x_1, y_1) \in F(x_0, y_0) \times F(y_0, x_0)$ such that (x_0, y_0), $(x1, y_1)$ are comparable and (y_0, x_0), (y_1, x_1) are comparable.

(iii) Either F is continuous or for any two sequences $\{x_n\}$ and $\{y_n\}$ in X, that $x_n \to x$, $y_n \to y$ and (x_n, y_n), (x_{n+1}, y_{n+1}) are comparable for all n, then (x_n, y_n), (x, y) are comparable for all n.

Then F has a coupled fixed point in X^2.

Proof Define $\alpha: X^2 \times X^2 \to [0, +\infty)$ by $\alpha((x, y), (u, v))=1$ if $(x, y), (u, v)$ are com- parable and $\alpha((x,y), (u, v))=0$ otherwise and apply theorem 2.2

Corollary 2.4: Let (X, d) be a complete metric space and $\preceq$ be an order on X^2. Fix $(x^*, y^*) \in X^2$. Suppose $\psi \in \Psi$ a strictly increasing map and $F: X^2 \to CB(X)$ a multifunction such that $H(F(x,y),F(u,v)) \le \frac{1}{2}\psi(d(x,u)+d(y,v))$, *for all comparable elements* $(x, y), (u, v) \in X^2$ *with* (x, y). Assume that the following assertions hold.

(i) If $(x, y) \in X^2$,$(u, v) \in F(x, y) \times F(y, x)$ and $(x, y), (u, v)$ are comparable with (x^*, y^*), then $(u, v), (w, z)$ are comparable with (x^*, y^*), for all $(w, z) \in F(u, v) \times F(v, u)$.

(ii) There exist $(x_0, y_0) \in X^2$ and $(x_1, y_1) \in F(x_0, y_0) \times F(y_0, x_0)$ such that (x_0, y_0), (x_1, y_1) are comparable with (x^*, y^*) and (y_0, x_0), (y_1, x_1) are comparable with (x^*, y^*).

(iii) Either F is continuous or for any two sequences $\{x_n\}$ and $\{y_n\}$in X, that $x_n \to x$, $y_n \to y$ and (x_n, y_n), (x_{n+1}, y_{n+1}) are comparable with (x^*, y^*) for all n, then (x_n, y_n), (x, y) are comparable with (x^*, y^*) for all n.Then F has a coupled fixed point in X^2.

Proof Define $\alpha: X^2 \times X^2 \to [0, +\infty)$ by $\alpha((x, y), (u, v))=1$ *if* $(x, y), (u, v)$ are comparable with (x^*,y^*) and $\alpha((x, y), (u, v))=0$ otherwise and apply theorem 2.2.

Corollary 2.5: Let $(X, \preceq, d)$ be a partial ordered complete metric space. Suppose $\psi \in \Psi$ a strictly increasing map and $F: X^2 \to CB(X)$ a multifunction such that

$$H(F(x,y),F(u,v)) \le \frac{1}{2}\psi(d(x,u)+d(y,v)),$$

for all elements (x, y), (u, v) $\in X^2$ that $x \ge u$ *or* $y \ge v$.

Assume that the following assertions hold.

(i) If $(x, y) \in X^2$,$(u, v) \in F(x, y) \times F(y, x)$ and $x \ge u$ *or* $y \ge v$, then $u \ge w$ *or* $v \ge z$, for all $(w, z) \in F(u, v) \times F(v, u)$.

(ii) There exist $(x_0, y_0) \in X^2$ and $(x_1, y_1) \in F(x_0, y_0) \times F(y_0, x_0)$ such that $x_0 \ge x_1$ *or* $y_0 \ge y_1$.

(iii) Either F is continuous or for any two sequences $\{x_n\}$ and $\{y_n\}$ in X, that $x_n \to x$, $y_n \to y$ and $x_n \ge x_{n+1}$ *or* $y_n \ge y_{n+1}$ for all n, then $x_n \ge x$ *or* $y_n \ge y$ for all n.

Then F has a coupled fixed point in X^2.

Proof Define $\alpha: X^2 \times X^2 \to [0, +\infty)$ by $\alpha((x, y), (u, v))=1$ *if* $x \ge u$ *or* $y \ge v$ are comparable and $\alpha((x, y), (u, v))=0$ otherwise and apply theorem 2.2.

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