Kenfack et al., J Nanosci Curr Res 2016, 1:1
DOI: 10.4172/2572-0813.1000104

Journal of Nanosciences: Current Research

Research Article Open Access

Decoherence of Driven Coupled Harmonic Oscillator

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Abstract

We consider a pair of linearly coupled harmonic oscillators to explore the decoherence phenomenon induces by interaction of a quantum system with a classical environment using Feynman path's integral method. We determine the DCHO propagator afterwards thermodynamics parameters associated to the system. We show numerically that one can reduce decoherence of a system by coupling this one to a driven harmonic oscillator or by carrying this system to the resonance.

Keywords: Decoherence; Propagator; Coupled oscillator

Introduction

The study of the phenomenon of decoherence has attracted increasing attention in recent years [1]. In fact, it has been recognized that decoherence is of fundamental importance in the comprehension of the nature of the boundary between the quantum and classical worlds. The nature of this boundary has been meticulously examined from both the theoretical and experimental points of view [2,3]. Decoherence is closely related to one of the greatest mysteries of physics which up to now remains unanswered, that is, the quantum measurement problem. This problem was clearly formulated by Erwin Schrodinger in 1935 in his experiment of thought known as "Schrodinger's cat paradox" [4-6] pointing out the interaction problem between quantum systems where quantum effects dominate and classical systems which operate in the classical limit. Two linearly coupled oscillators provide an ideal test model for exploring this aspect of the quantum-classical limit interaction [7]. In this work, we use the Feynman path integral formalism which enables an evaluation of the sum over all possible paths in a consistent way. This formulation which appears to be appropriate for a correct description of the reality is exact, equivalent to the Hilbert space formalism and leads to the same results for the Schrödinger equation. It is an alternative formulation of the quantum mechanics, which introduces only functions with numerical values and does completely without operators. This paper is organized as follows: In section 2, we present the model and determine the coupled harmonic oscillator (CHO) propagator from which we deduce the driven coupled harmonic oscillator(DCHO) propagator. In section 3, we derive the thermodynamic parameters associated to the system which is entropy. In section 4, we present the numerical results and then, end with the conclusion in section 5.

The DCHO propagator

The DCHO is a system constituted of two harmonic oscillators coupled by a spring and whose extremities are subjected to external driving forces represented by $f_1(t)$ and $f_2(t)$. Here, we assume that the two oscillators have equal mass m and coupling constant k.

The CHO propagator

The determination of this propagator is of great interest because it will allow us later by comparison to establish the explicit form of the DCHO propagator. The simplest version of a pair of coupled quantum-mechanical simple harmonic oscillators with linear coupling consists of two identical oscillators, with equal masses, spring constants, and frequencies, plus a connecting spring with its own spring constant. Considering the system described above, the Hamiltonian reads:

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + m\omega^2(x_1^2 - x_1x_2 + x_2^2) - f_1(t)x_1 - f_2(t)x_2 \qquad (1)$$

where x_1, x_2, p_1, p_2 are respectively the displacements and momenta of each oscillator, and ω their oscillation frequency.

From the formula $L = \sum_{i=1}^{2} p_i\dot{x}_i - H$, the Lagrangian of the system reads

$$L = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 - m\omega^2(x_1^2 - x_1x_2 + x_2^2) + f_1(t)x_1 + f_2(t)x_2 \qquad (2)$$

The differentials equations that govern the evolution of the system are given by

$$\ddot{x}_{c1} + \omega^2(2x_{c1} - x_{c2}) = \frac{f_1(t)}{m}$$
$$\ddot{x}_{c2} + \omega^2(2x_{c2} - x_{c1}) = \frac{f_1(t)}{m} \qquad (3)$$

When setting $f_1(t)=f_2(t)=0$, the system of Equation(3) becomes

$$\ddot{x}_{c1} + \omega^2(2x_{c1} - x_{c2}) = 0$$
$$\ddot{x}_{c2} + \omega^2(2x_{c2} - x_{c1}) = 0 \qquad (4)$$

The Lagrangian is transform to

$$L[x_1, x_2, \dot{x}_1, \dot{x}_2] = \frac{m}{2}(\dot{x}_1^2 + \dot{x}_2^2) - m\omega^2(x_1^2 - x_1x_2 + x_2^2) \qquad (5)$$

Then, the action is

$$S = \int_0^t d\tau\left\{\frac{m}{2}(\dot{x}_1^2 + \dot{x}_2^2) - m\omega^2(x_1^2 - x_1x_2 + x_2^2)\right\} \qquad (6)$$

From the definitions [9,10], the propagator of this system over a finite time interval will read [11]

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Received: June 06, 2016; **Accepted:** September 23, 2016; **Published:** November 04, 2016

Citation: Kenfack SC, Nguimeya GP, Talla PK, Fotue AJ, Fobasso MFC, et al. (2016) Decoherence of Driven Coupled Harmonic Oscillator. J Nanosci Curr Res 1: 104. doi: 10.4172/2572-0813.1000104

$$K(x_1,x_2,t;x_1',x_2',0)=\int_{(x_1',x_2',0)}^{(x_1,x_2,t)}Dx(t)\exp\left\{\frac{i}{\hbar}S(x_1,x_2,x_1',x_2',t)\right\} \quad (7)$$

by making the following change of variables,

$$y_1 = x_1 - x_{c1},$$
$$y_2 = x_2 - x_{c2}, \quad (8)$$

the action splits into two terms

$$S=\int_0^t d\tau\left\{\frac{m}{2}(\dot{x}_{c1}^2+\dot{x}_{c2}^2)-m\omega^2\left(x_{c1}^2-x_{c1}x_{c2}+x_{c2}^2\right)\right\}+$$
$$\int_0^t d\tau\left\{\frac{m}{2}(\dot{y}_1^2+\dot{y}_2^2-2\omega^2(y_1^2-y_1y_2+y_2^2))\right\} \quad (9)$$

that is,

$$S(x_1,x_2,x_1',x_2',t)=S_c(x_1,x_2,x_1',x_2',t)+\int_0^t d\tau$$
$$\left\{\frac{m}{2}(\dot{y}_1^2+\dot{y}_2^2-2\omega^2(y_1^2-y_1y_2+y_2^2))\right\} \quad (10)$$

Where $S_c=\int_0^t d\tau\left\{\frac{m}{2}(\dot{x}_{c1}^2+\dot{x}_{c2}^2)-m\omega^2\left(x_{c1}^2-x_{c1}x_{c2}+x_{c2}^2\right)\right\}$ (11)

is the classical action and y_i the deviation between $x_i(t)$ and theirs classical path x_{ci} which is given by $y_i = x_i - x_{ci}, (i = 1,2)$.

The propagator is thus transformed as follows:

$$K(x_1,x_2,t;x_1',x_2',0)=F(t)\exp\left\{\frac{i}{\hbar}S_c\right\} \quad (12)$$

Where

$$F(t)=\int_0^0 Dx(t)\exp\left\{\frac{im}{2\hbar}\int_0^t d\tau\left\{\frac{m}{2}(\dot{y}_1^2+\dot{y}_2^2-2\omega^2(y_1^2-y_1y_2+y_2^2))\right\}\right\} \quad (13)$$

is a smooth function in the limit $\hbar \to 0$.

By setting

$$q_1=\frac{1}{\sqrt{2}}(y_1-y_2)$$
$$q_2=\frac{1}{\sqrt{2}}(y_1+y_2) \quad (14)$$

the normalization factor $F(t)$ of the system becomes

$$F(t)=J\int_0^0 Dq(t)\exp\left\{\frac{im}{2\hbar}\int_0^t d\tau\left\{(\dot{q}_1^2-3\omega^2q_1^2)+(\dot{q}_2^2-\omega^2q_2^2)\right\}\right\} \quad (15)$$

This factor can be written as the product of the normalization factor of each oscillator as

Follows

$$F(t)=F_1(t)F_2(t) \quad (16)$$

With

$$F_1(t)=\int_0^0 Dq_1(t)\exp\left\{\frac{im}{2\hbar}\int_0^t d\tau(\dot{q}_1^2-3\omega^2q_1^2)\right\} \quad (17)$$

And

$$F_2(t)=\int_0^0 Dq_2(t)\exp\left\{\frac{im}{2\hbar}\int_0^t d\tau(\dot{q}_2^2-3\omega^2q_2^2)\right\} \quad (18)$$

when expending q_1 and q_2 in Fourier series on eigens modes [9,10,12] one gets from Fresnel's integral (113)

$$F_1(t)=\left(\frac{\sqrt{3}m\omega}{2\pi i\hbar\sin\sqrt{3}\omega T}\right)^{\frac{1}{2}} \quad (19)$$

And

$$F_2(t)=\left(\frac{m\omega}{2\pi i\hbar\sin\omega T}\right)^{\frac{1}{2}} \quad (20)$$

Thus, it yields that [12]

$$F(t)=\left(\frac{m\omega}{2\pi i\hbar}\right)\left(\frac{\sqrt{3}}{\sin\omega T\sin\sqrt{3}\omega T}\right)^{\frac{1}{2}} \quad (21)$$

Therefore, the propagator takes the form

$$K(x_1,x_2,t;x_1',x_2',0)=\left(\frac{m\omega}{2\pi i\hbar}\right)\left(\frac{\sqrt{3}}{\sin\omega T\sin\sqrt{3}\omega T}\right)^{\frac{1}{2}}\exp\left\{\frac{i}{\hbar}S_C(0,t)\right\} \quad (22)$$

We now focus on the classical action. Performing the integration of Equation(11) and taking into account Equation(4), one finds [12]

$$S_C=\frac{m}{2}\left[(\dot{x}_{c1}(t)x_{c1}(t)+\dot{x}_{c2}(t)x_{c2}(t)\ -(\dot{x}_{c1}(0)x_{c1}(0)+\dot{x}_{c2}(0)x_{c2}(0)\ \right] \quad (23)$$

To obtain the exact form of this action, we have to solve the system of Equation(4). By setting

$$u = x_1 + x_2$$
$$v = x_1 - x_2 \quad (24)$$

This system becomes

$$\ddot{u}+\omega^2u=0$$
$$\ddot{v}+\omega^2v=0 \quad (25)$$

of which the solutions are

$$u(t)=A_1\sin\omega t+B_1\cos\omega t$$
$$v(t)=C_2\sin\sqrt{3}\omega t+D_2\cos\sqrt{3}\omega t \quad (26)$$

When returning to initials variables, one gets

$$x_1=x_1(t)=A\sin\omega t+B\cos\omega t+C\sin\sqrt{3}\omega t+D\cos\sqrt{3}\omega t$$
$$x_2=x_2(t)=A\sin\omega t+B\cos\omega t-C\sin\sqrt{3}\omega t-D\cos\sqrt{3}\omega t \quad (27)$$

thus, we have

$$\dot{x}_1=A\omega\cos\omega t-B\omega\sin\omega t+C\sqrt{3}\omega\cos\sqrt{3}\omega t-D\sqrt{3}\omega\sin\sqrt{3}\omega t$$
$$\dot{x}_2=A\omega\cos\omega t-B\omega\sin\omega t-C\sqrt{3}\omega\cos\sqrt{3}\omega t+D\sqrt{3}\omega\sin\sqrt{3}\omega t \quad (28)$$

Therefore

$$x_1'=x_1(0)=B+D$$
$$x_2'=x_2(0)=B-D \quad (29)$$

and

$$\dot{x}_1'=\dot{x}_1(0)=\omega(A+C\sqrt{3})$$
$$\dot{x}_2'=\dot{x}_2(0)=\omega(A-C\sqrt{3}) \quad (30)$$

From the system of Equation(30) we obtain

$$B=\frac{1}{2}(x_1'+x_2')$$
$$D=\frac{1}{2}(x_1'-x_2') \quad (31)$$

and by performing the computations x_1+x_2 and x_1-x_2, we get respectively

$$A=\frac{1}{2\sin\omega t}\left[(x_1+x_2)-(x_1'+x_2')\cos\omega t\right]$$

$$C=\frac{1}{2\sin\sqrt{3}\omega t}\left[(x_1-x_2)-(x_1'-x_2')\cos\sqrt{3}\omega t\right] \tag{32}$$

By using Equations. (27) - (33) one finds from Equation(23) that [12]

$$\begin{aligned}S_C=\frac{m\omega}{4}\Big\{&(x_1^2+x_2^2+x_1'^2+x_2'^2)\left[\cot\omega t+\sqrt{3}\cot\sqrt{3}\omega t\right]+\\&2(x_1x_2+x_1'x_2')\left[\cot\omega t-\sqrt{3}\cot\sqrt{3}\omega t\right]-2(x_1x_1'+x_2x_2')\\&\left[\frac{1}{\sin\omega t}+\frac{\sqrt{3}}{\sin\sqrt{3}\omega t}\right]+2(x_1x_2'+x_1'x_2)\left[-\frac{1}{\sin\omega t}+\frac{\sqrt{3}}{\sin\sqrt{3}\omega t}\right]\Big\}\end{aligned} \tag{33}$$

substituting this result into Equation(22), we obtain the CHO propagator which is [12]

$$\begin{aligned}K(x_1,x_2,t;x_1',x_2',0)=&\left(\frac{m\omega}{2\pi i\hbar}\right)\left(\frac{\sqrt{3}}{\sin\omega T\sin\sqrt{3}\omega T}\right)^{\frac{1}{2}}\\\exp\Big\{&\frac{im\omega}{4\hbar}\Big\{(x_1^2+x_2^2+x_1'^2+x_2'^2)\left[\cot\omega t+\sqrt{3}\cot\sqrt{3}\omega t\right]+\\&2(x_1x_2+x_1'x_2')\left[\cot\omega t-\sqrt{3}\cot\sqrt{3}\omega t\right]-2(x_1x_1'+x_2x_2')\\&\left[\frac{1}{\sin\omega t}+\frac{\sqrt{3}}{\sin\sqrt{3}\omega t}\right]+2(x_1x_2'+x_1'x_2)\left[-\frac{1}{\sin\omega t}+\frac{\sqrt{3}}{\sin\sqrt{3}\omega t}\right]\Big\}\Big\}\end{aligned} \tag{34}$$

Deducing of the DCHO propagator: Setting $f_1(t) = f_2(t) = 0$, the DCHO is reducing to CHO. Since the Lagrangian is quadratic, then we can assume the propagator for the DCHO to have the form [13]

$$\begin{aligned}&K(x_1,x_2,t;x_1',x_2',0)=\exp\\&\left(a(t)x_1^2+b(t)x_1x_2+c(t)x_2^2+d(t)x_1+g(t)x_2+h(t)\right)\end{aligned} \tag{35}$$

Where $a(t),b(t),c(t),d(t),g(t),h(t)$ are the functions of time which have to be determined.

Taking $P_1=-i\hbar\frac{\partial}{\partial x_1}$ and $P_2=-i\hbar\frac{\partial}{\partial x_2}$, the Hamiltonian of Equation(1) takes the form

$$H=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2}+m\omega^2(x_1^2-x_1x_2+x_2)-f_1(t)x_1-f_2(t)x_2 \tag{36}$$

The Hamiltonian of Equation(37) satisfies the Schrödinger equation [14]

$$i\hbar\frac{\partial K}{\partial t}=HK \tag{37}$$

that is,

$$\begin{aligned}\frac{\partial K}{\partial t}=&-\frac{i\hbar}{2m}\frac{\partial^2K}{\partial x_1^2}-\frac{i\hbar}{2m}\frac{\partial^2K}{\partial x_2^2}+\frac{m\omega^2}{i\hbar}x_1^2K-\\&\frac{m\omega^2}{i\hbar}x_1x_2K+\frac{m\omega^2}{i\hbar}x_2K-\frac{1}{i\hbar}f_1(t)x_1K-\frac{1}{i\hbar}f_2(t)x_2K\end{aligned} \tag{38}$$

From Equation(37), when using Equation (35), we get

$$\begin{cases}\frac{\partial a(t)}{\partial t}=\frac{i\hbar}{2m}\left[4a^2(t)+b^2(t)\right]+\frac{m\omega^2}{i\hbar}\\\frac{\partial c(t)}{\partial t}=\frac{i\hbar}{2m}\left[4c^2(t)+b^2(t)\right]+\frac{m\omega^2}{i\hbar}\\\frac{\partial b(t)}{\partial t}=\frac{2i\hbar}{m}\left[a(t)b(t)+b(t)c(t)\right]-\frac{m\omega^2}{i\hbar}\\\frac{\partial d(t)}{\partial t}=\frac{i\hbar}{m}\left[2a(t)d(t)+b(t)g(t)\right]-\frac{1}{i\hbar}f_1(t)\\\frac{\partial g(t)}{\partial t}=\frac{i\hbar}{m}\left[2c(t)g(t)+b(t)d(t)\right]-\frac{1}{i\hbar}f_2(t)\\\frac{\partial h(t)}{\partial t}=\frac{i\hbar}{2m}\left[d^2(t)+g^2(t)+2a(t)+2c(t)\right]\end{cases} \tag{39}$$

From the two first relations of above system, we get, taking this relation into account

$$\frac{\partial b(t)}{\partial t}=\frac{4i\hbar}{m}b(t)c(t)-\frac{m\omega^2}{i\hbar} \tag{40}$$

When setting

$$\aleph=c+\frac{b}{2}$$

$$\Im=c-\frac{b}{2} \tag{41}$$

we have from Equation(41) the following system

$$\dot{\aleph}=\frac{2i\hbar}{m}\aleph^2+\frac{m\omega^2}{2i\hbar}$$

$$\dot{\Im}=\frac{2i\hbar}{m}\Im^2+\frac{3m\omega^2}{2i\hbar} \tag{42}$$

When solving each equation of the system of Equation(43), one obtains

$$\aleph=\frac{in\ \omega}{2\hbar}\cot(\omega t+\theta_1)$$

$$\Im=\frac{i\sqrt{3}m\omega}{2\hbar}\cot(\sqrt{3}\omega t+\theta_1) \tag{43}$$

From Equation(43) and using Equation (41), we get

$$a(t)=c(t)=\frac{in\ \omega}{4\hbar}\left[\cot(\omega t+\theta_1)+\sqrt{3}\cot(\sqrt{3}\omega t+\theta_2)\right]$$

$$b(t)=\frac{in\ \omega}{2\hbar}\left[\cot(\omega t+\theta_1)-\sqrt{3}\cot(\sqrt{3}\omega t+\theta_2)\right] \tag{44}$$

One can see that Equation (44) does not include the excitation forces $f_1(t)$ and $f_2(t)$. Thus setting $f_1(t) = f_2(t) = 0$ the functions a(t) and b(t) can be equalized with the coefficients of x_1^2 and x_2^2 of equation (34). The comparison between these two equations shows that θ_1 and θ_2 must be equal to zero. And so

$$a(t)=c(t)=\frac{in\ \omega}{4\hbar}\left[\cot(\omega t)+\sqrt{3}\cot(\sqrt{3}\omega t)\right]$$

$$b(t)=\frac{in\ \omega}{2\hbar}\left[\cot(\omega t)-\sqrt{3}\cot(\sqrt{3}\omega t)\right] \tag{45}$$

By replacing $a(t) = c(t)$ into the forth equation of system in Equation (39) and making the following change

$$d=\frac{1}{2}(\gamma+\chi)$$

$$g=\frac{1}{2}(\gamma-\chi) \tag{46}$$

we obtain

$$\dot{\gamma} = -\omega\frac{\cos\omega t}{\sin\omega t}\gamma + \left(\frac{i}{\hbar}\right)(f_1(t) + f_2(t)$$
$$\dot{\chi} = -\omega\sqrt{3}\frac{\cos\sqrt{3}\omega t}{\sin\sqrt{3}\omega t}\gamma + \left(\frac{i}{\hbar}\right)(f_1(t) + f_2(t) \tag{47}$$

Solving these two differentials equations gives

$$\gamma = \frac{1}{\sin\omega t}\left\{\int_0^t d\tau\left(\frac{i}{\hbar}\right)\left[f_1(\tau) + f_2(\tau)\right]\sin\omega\tau + \Lambda_1\right\}$$

$$\chi = \frac{1}{\sin\sqrt{3}\omega t}\left\{\int_0^t d\tau\left(\frac{i}{\hbar}\right)\left[f_1(\tau) - f_2(\tau)\right]\sin\omega\tau + \Lambda_2\right\} \tag{48}$$

Where Λ_1 and Λ_2 are the constants to be determine. When taking into account Equation(48), then, Equation(46) gives [12]

$$\begin{aligned}
d(t) &= \left(\frac{i}{2\hbar\sin\omega t}\right)\int_0^t d\tau\left[f_1(\tau)+f_2(\tau)\right]\sin\omega\tau + \frac{\Lambda_1}{2\sin\omega t} + \\
&\left(\frac{i}{2\hbar\sin\sqrt{3}\omega t}\right)\int_0^t d\tau\left[f_1(\tau)-f_2(\tau)\right]\sin\sqrt{3}\omega\tau + \frac{\Lambda_2}{2\sin\sqrt{3}\omega t} \\
g(t) &= \left(\frac{i}{2\hbar\sin\omega t}\right)\int_0^t d\tau\left[f_1(\tau)+f_2(\tau)\right]\sin\omega\tau + \frac{\Lambda_1}{2\sin\omega t} - \\
&\left(\frac{i}{2\hbar\sin\sqrt{3}\omega t}\right)\int_0^t d\tau\left[f_1(\tau)-f_2(\tau)\right]\sin\sqrt{3}\omega\tau - \frac{\Lambda_2}{2\sin\sqrt{3}\omega t}
\end{aligned} \tag{49}$$

When combining Equations (45) and (49), we find [12]

$$\begin{aligned}
h(t) &= -\frac{i\hbar}{4m\omega}\left[\Lambda_1^2\cot\omega t + \frac{\Lambda_2^2}{\sqrt{3}}\cot\sqrt{3}\omega t\right] - \frac{\Lambda_1}{2m\omega\sin\omega t}\int_0^t d\tau\left[f_1(\tau)+f_2(\tau)\right]\sin\omega(t-\tau) - \\
&-\frac{\Lambda_2}{2\sqrt{3}m\omega\sin\sqrt{3}\omega t}\int_0^t d\tau\left[f_1(\tau)-f_2(\tau)\right]\sin\sqrt{3}\omega(t-\tau) + \\
&+\frac{1}{4i\hbar m\omega\sin\omega t}\int_0^t d\tau\int_0^\tau d\nu\left[f_1(\tau)+f_2(\tau)\right]\left[f_1(\nu)+f_2(\nu)\right]\sin\omega(t-\tau)\sin\omega\nu + \\
&+\frac{1}{4\sqrt{3}i\hbar m\omega\sin\sqrt{3}\omega t}\int_0^t d\tau\int_0^\tau d\nu\left[f_1(\tau)-f_2(\tau)\right]\left[f_1(\nu)-f_2(\nu)\right]\sin\sqrt{3}\omega(t-\tau)\sin\sqrt{3}\omega\nu - \\
&-\ln\left(\sin\omega t.\sin\sqrt{3}\omega t\right)^{\frac{1}{2}} + \vartheta
\end{aligned} \tag{50}$$

Where ϑ is a constant to be determined.

To determine the constants Λ_1, Λ_2 and ϑ, we make a comparison between Equations (34) and (35) for $f_1(t) = f_2(t) = 0$. From this comparison, it comes that

$$\begin{aligned}
\Lambda_1 &= -\frac{im\omega}{\hbar}\left(x_1' + x_2'\right) \\
\Lambda_2 &= -\frac{im\omega\sqrt{3}}{\hbar}\left(x_1' - x_2'\right) \\
\vartheta &= \ln\frac{3^{1/4}m\omega}{2\pi i\hbar}
\end{aligned} \tag{51}$$

When inserting Equations (45) and (49)-(51) into Equation(35), gathering the similar terms and replacing $\exp\left[\ln\left(\sin\omega t.\sin\sqrt{3}\omega t\right)^{-1/2} + \delta\right]$ by $\left(\frac{m\omega}{2\pi i\hbar}\right)\left(\frac{\sqrt{3}}{\sin\omega t\sin\sqrt{3}\omega t}\right)^{\frac{1}{2}}$ we work out the DCHO propagator which reads [12]

$$\begin{aligned}
K(x_1,x_2,t;x_1',x_2',0)=&\left(\frac{m\omega}{2\pi i\hbar}\right)\left(\frac{\sqrt{3}}{\sin\omega t\sin\sqrt{3}\omega t}\right)^{\frac{1}{2}}\exp\left\{\frac{im\omega}{4\hbar}\left\{\left[(x_1^2+x_2^2+x_1'^2+x_2'^2)(\cot\omega t+\sqrt{3}\cot\sqrt{3}\omega t)\right]+\right.\right.\\
&+2(x_1x_2+x_1'x_2')(\cot\omega t-\sqrt{3}\cot\sqrt{3}\omega t)-2(x_1x_1'+x_2x_2')\left(\frac{1}{\sin\omega t}+\frac{\sqrt{3}}{\sin\sqrt{3}\omega t}\right)+2(x_1x_2'+x_1'x_2)\left(-\frac{1}{\sin\omega t}+\frac{\sqrt{3}}{\sin\sqrt{3}\omega t}\right)+\\
&+\frac{2x_1}{m\omega}\left[\left(\frac{1}{\sin\omega t}\right)\int_0^t d\tau[f_1(\tau)+f_2(\tau)]\sin\omega\tau+\left(\frac{1}{\sin\sqrt{3}\omega t}\right)\int_0^t d\tau[f_1(\tau)-f_2(\tau)]\sin\sqrt{3}\omega\tau\right]+\\
&+\frac{2x_2}{m\omega}\left[\left(\frac{1}{\sin\omega t}\right)\int_0^t d\tau[f_1(\tau)+f_2(\tau)]\sin\omega\tau-\left(\frac{1}{\sin\sqrt{3}\omega t}\right)\int_0^t d\tau[f_1(\tau)-f_2(\tau)]\sin\sqrt{3}\omega\tau\right]+\\
&+\frac{2x_1'}{m\omega}\left[\left(\frac{1}{\sin\omega t}\right)\int_0^t d\tau[f_1(\tau)+f_2(\tau)]\sin\omega(t-\tau)+\left(\frac{1}{\sin\sqrt{3}\omega t}\right)\int_0^t d\tau[f_1(\tau)-f_2(\tau)]\sin\sqrt{3}\omega(t-\tau)\right]+\\
&+\frac{2x_2'}{m\omega}\left[\left(\frac{1}{\sin\omega t}\right)\int_0^t d\tau[f_1(\tau)+f_2(\tau)]\sin\omega(t-\tau)-\left(\frac{1}{\sin\sqrt{3}\omega t}\right)\int_0^t d\tau[f_1(\tau)-f_2(\tau)]\sin\sqrt{3}\omega(t-\tau)\right]-\\
&-\left(\frac{1}{m^2\omega^2\sin\omega t}\right)\int_0^t d\tau\int_0^\tau d\nu[f_1(\tau)+f_2(\tau)][f_1(\nu)+f_2(\nu)]\sin\omega(t-\tau)\sin\omega\nu-\\
&\left.\left.-\left(\frac{1}{\sqrt{3}m^2\omega^2\sin\sqrt{3}\omega t}\right)\int_0^t d\tau\int_0^\tau d\nu[f_1(\tau)-f_2(\tau)][f_1(\nu)-f_2(\nu)]\sin\sqrt{3}\omega(t-\tau)\sin\sqrt{3}\omega\nu\right\}\right\}
\end{aligned} \quad \textbf{(52)}$$

Thermodynamics Parameters for the DCHO

To determine thermodynamic parameters associated to the system, the knowledge of the time-dependent wave function is necessary as well as the wave function at initial time.

Wave function at initial time: The considered system here can be modeled as follows (Figure 1)

The potential energy $U(x_1',x_2')$ of this system is:

$$U(x_1',x_2')=\frac{1}{2}k\left(x_1'\right)^2+\frac{1}{2}k\left(x_2'\right)^2+\frac{1}{2}k_C(x_1'-x_2')^2 \quad (53)$$

Where both k and k_c are the elastic constant and k_c characterize the coupling of particles at point $x`=0$ with each other.

The kinetic energy of this system is written as:

$$T=\frac{1}{2}m\dot{x}_1'^2+\frac{1}{2}m\dot{x}_2'^2 \quad (54)$$

When introduce the coordinate of the center of mass Y and the relative coordinate X.

$$X=\frac{1}{\sqrt{2}}(x_1'-x_2'),$$
$$Y=\frac{1}{\sqrt{2}}(x_1'+x_2'), \quad (55)$$

that is,

$$x_1'=\frac{1}{\sqrt{2}}(Y+X)$$
$$x_2'=\frac{1}{\sqrt{2}}(Y-X) \quad (56)$$

Equations (53) and (54) are transformed to

$$T=\frac{1}{2}m(\dot{Y}^2+\dot{X}^2)$$
$$U(X,Y)=\frac{1}{2}(k+2k_C)X^2+\frac{1}{2}kY^2 \quad (57)$$

Thus, the Hamiltonian reads

$$H(X,Y)=\frac{1}{2}m\dot{X}^2+\frac{1}{2}m\dot{Y}^2+\frac{1}{2}(k+2k_C)X^2+\frac{1}{2}kY^2 \quad (58)$$

From the Hamilton canonical equations, we get

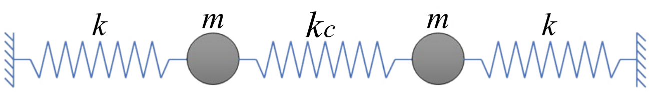

Figure 1: Diagram of the model.

$$\ddot{X} + \omega_C^2 X = 0$$
$$\ddot{Y} + \omega^2 Y = 0$$
(59)

With

$$\omega_C^2 = \frac{(k+2k_C)}{m}, \ \omega^2 = \frac{k}{m} \tag{60}$$

Then the Schrodinger equation can be written as:

$$-\frac{\hbar^2}{2m}\Delta_X\psi(X,Y) - \frac{\hbar^2}{2m}\Delta_Y\psi(X,Y) + U(X,Y) = E\psi \tag{61}$$

By separation of variables $\psi(X,Y) = \phi(X)\varphi(Y)$ while taking $E = E_1 + E_2$ this equation yields to

$$\frac{\partial^2\phi}{\partial\xi^2} + (E_{11} - \xi^2)\phi = 0$$
$$\frac{\partial^2\varphi}{\partial\mu^2} + (E_{22} - \mu^2)\varphi = 0 \tag{62}$$

With

$$\xi^2 = \frac{m\omega_C}{\hbar}X^2, \ \mu^2 = \frac{m\omega}{\hbar}Y^2, \ E_{11} = \frac{2E_1}{\hbar\omega_C}, \ E_{22} = \frac{2E_2}{\hbar\omega} \tag{63}$$

The solutions of system in Equation(62) are looking for under the forms

$$\phi(\xi) = A_{n_1}H_{n_1}(\xi)\exp\left\{-\frac{\xi^2}{2}\right\}$$
$$\varphi(\mu) = A_{n_2}H_{n_2}(\mu)\exp\left\{-\frac{\mu^2}{2}\right\} \tag{64}$$

Where

$$A_{n_1} = \left(\frac{1}{2^{n_1} n_1! x_{01}\sqrt{\pi}}\right)^{1/2}, \ A_{n_2} = \left(\frac{1}{2^{n_2} n_2! x_{02}\sqrt{\pi}}\right)^{1/2};$$

$$x_{01} = \left(\frac{\hbar}{m\omega_C}\right)^{1/2}, \ x_{02} = \left(\frac{\hbar}{m\omega}\right)^{1/2} \tag{65}$$

Then

$$\psi_{n_1 n_2}(\xi,\mu) = A_{n_1}A_{n_2}H_{n_1}(\xi)H_{n_2}(\mu)\exp\left\{-\frac{(\xi^2+\mu^2)}{2}\right\} \tag{66}$$

In the fundamental state $n_1=0$ and, $n_2=0$, what leads to:

$$\psi_0(\xi,\mu) = \left(\frac{1}{2\pi\sigma^2}\right)^{1/2}\exp\left\{-\frac{(\xi^2+\mu^2)}{2}\right\} \tag{67}$$

With

$$\sigma^2 = \frac{\hbar}{2m\sqrt{\omega\omega_C}} \tag{68}$$

When returning to the initials variables, we obtain

$$\psi_0(x_1', x_2') = \left(\frac{1}{2\pi\sigma^2}\right)^{1/2}\exp\left\{-\frac{\lambda_C}{4\hbar}(x_1'-x_2')^2 - \frac{\lambda}{4\hbar}(x_1'+x_2')^2\right\} \tag{69}$$

Where

$$\lambda = \sqrt{mk}, \lambda_C = \sqrt{\lambda^2 + 2mk_C} \tag{70}$$

Time-dependent wave function: Having the propagator and Knowing the initial wave function, the solution of the time-dependent Schrodinguer equation is found by performing the integral [10,13].

$$\psi(x_1,x_2,t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dx_1'dx_2'K(x_1,x_2,t;x_1',x_2',0)\psi(x_1',x_2',0) \tag{71}$$

with $K(x_1,x_2,t;x_1',x_2',0)$ and $\psi(x_1',x_2',0)$ given by Equations. (52) and (69) respectively.

As a reminder, x_1', x_2' and x_1, x_2 are the initial and final coordinates of the oscillators respectively. In a much more compact form easy to manipulate, the propagator in Equation(54) can be written as follows:

$$K(x_1,x_2,t;x_1',x_2',0) = \left(\frac{m\omega}{2\pi i\hbar}\right)\left(\frac{\sqrt{3}}{\sin\omega t\sin\sqrt{3}\omega t}\right)^{\frac{1}{2}}$$
$$\exp\left\{\frac{im\omega}{4\hbar}\left[(x_1^2+x_2^2)I_1 + 2x_1x_2I_2 + (x_1'^2+x_2'^2)I_1 + 2x_1'x_2'I_2\right.\right.$$
$$+2(x_2I_4 - x_1I_3)x_1' + 2(x_1I_4 - x_2I_3)x_2' +$$
$$\left.\frac{2J_3}{m\omega}x_1' + \frac{2J_4}{m\omega}x_2' + \frac{2J_1}{m\omega}x_1 + \frac{2J_2}{m\omega}x_2 - J_5 - J_6\right\} \tag{72}$$

with

$$I_1 = \cot\omega t + \sqrt{3}\cot\sqrt{3}\omega t \quad I_2 = \cot\omega t - \sqrt{3}\cot\sqrt{3}\omega t$$
$$I_3 = \frac{1}{\sin\omega t} + \frac{\sqrt{3}}{\sin\sqrt{3}\omega t} \quad I_4 = -\frac{1}{\sin\omega t} + \frac{\sqrt{3}}{\sin\sqrt{3}\omega t} \tag{73}$$

$$J_1 = \frac{1}{\sin\omega t}\int_0^t d\tau\left(f_1(\tau)+f_2(\tau)\right)\sin\omega\tau + \frac{1}{\sin\sqrt{3}\omega t}\int_0^t d\tau\left(f_1(\tau)-f_2(\tau)\right)\sin\sqrt{3}\omega\tau$$

$$J_2 = \frac{1}{\sin\omega t}\int_0^t d\tau\left(f_1(\tau)+f_2(\tau)\right)\sin\omega\tau - \frac{1}{\sin\sqrt{3}\omega t}\int_0^t d\tau\left(f_1(\tau)-f_2(\tau)\right)\sin\sqrt{3}\omega\tau$$

$$J_3 = \frac{1}{\sin\omega t}\int_0^t d\tau\left(f_1(\tau)+f_2(\tau)\right)\sin\omega(t-\tau) + \frac{1}{\sin\sqrt{3}\omega t}\int_0^t d\tau\left(f_1(\tau)-f_2(\tau)\right)\sin\sqrt{3}\omega(t-\tau)$$

$$J_4 = \frac{1}{\sin\omega t}\int_0^t d\tau\left(f_1(\tau)+f_2(\tau)\right)\sin\omega(t-\tau) - \frac{1}{\sin\sqrt{3}\omega t}\int_0^t d\tau\left(f_1(\tau)-f_2(\tau)\right)\sin\sqrt{3}\omega(t-\tau)$$

$$J_5 = \frac{1}{m^2\omega^2\sin\omega t}\int_0^t d\tau\int_0^\tau d\nu\left(f_1(\tau)+f_2(\tau)\right)\left(f_1(\nu)+f_2(\nu)\right)\sin\omega(t-\tau)\sin\omega\nu$$

$$J_6 = \frac{1}{\sqrt{3}m^2\omega^2\sin\sqrt{3}\omega t}\int_0^t d\tau\int_0^\tau d\nu\left(f_1(\tau)-f_2(\tau)\right)\left(f_1(\nu)-f_2(\nu)\right)\sin\sqrt{3}\omega(t-\tau)\sin\sqrt{3}\omega\nu \tag{74}$$

When changing I_1, I_2, I_3 and I_4 into

$$I_1 = a+b,\ I_2 = a-b,\ I_3 = a'+b',\ I_4 = -a'+b' \tag{75}$$

Where

$$a = \cot\omega t,\ b = \sqrt{3}\cot\sqrt{3}\omega t,\ a' = \frac{1}{\sin\omega t},\ b' = \frac{\sqrt{3}}{\sin\sqrt{3}\omega t} \tag{76}$$

this propagator can finally be expressed as follows:

$$\begin{aligned} K(x_1,x_2,t;x_1',x_2',0) &= \left(\frac{m\omega}{2\pi i\hbar}\right)\left(\frac{\sqrt{3}}{\sin\omega t\sin\sqrt{3}\omega t}\right)^{\frac{1}{2}} \\ &\exp\left\{\frac{im\omega}{4\hbar}\left[\cot\omega t(x_1+x_2)^2+\sqrt{3}\cot\sqrt{3}\omega t(x_1-x_2)^2+\frac{2J_1}{m\omega}x_1+\frac{2J_2}{m\omega}x_2-J_5-J_6\right]\right\}\times \\ &\times\exp\left\{\frac{im\omega}{4\hbar}\left[\cot\omega t(x_1'+x_2')^2+\sqrt{3}\cot\sqrt{3}\omega t(x_1'-x_2')^2+\left[\frac{2J_3}{m\omega}-\frac{2(x_1+x_2)}{\sin\omega t}-\frac{2\sqrt{3}(x_1-x_2)}{\sin\sqrt{3}\omega t}\right]x_1'\right.\right. \\ &\left.\left.+\left[\frac{2J_4}{m\omega}-\frac{2(x_1+x_2)}{\sin\omega t}+\frac{2\sqrt{3}(x_1-x_2)}{\sin\sqrt{3}\omega t}\right]x_2'\right]\right\} \end{aligned} \tag{77}$$

When putting Equation(77) into Equation(71) and doing some arrangement, we then get

$$\begin{aligned}\psi(x_1,x_2,t) &= \left(\frac{m\omega}{2\pi i\hbar}\right)\left(\frac{\sqrt{3}}{2\pi\sigma^2\sin\omega t\sin\sqrt{3}\omega t}\right)^{\frac{1}{2}} \\ &\exp\{i\Omega\}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}dx_1'dx_2'\exp\left\{-\alpha(x_1'+x_2')^2-\beta(x_1'-x_2')^2++\gamma x_1'+\eta x_2'\right\}\end{aligned} \tag{78}$$

With

$$\alpha = \frac{1}{4\hbar}\left(\lambda - im\omega a\right),\quad \alpha^* = \frac{1}{4\hbar}\left(\lambda + im\omega a\right)$$

$$\beta = \frac{1}{4\hbar}\left(\lambda_c - im\omega b\right),\quad \beta^* = \frac{1}{4\hbar}\left(\lambda_c + im\omega b\right)$$

$$\gamma = \frac{im\omega}{2\hbar}\left[\frac{J_3}{m\omega} - a'(x_1+x_2) - b'(x_1-x_2)\right]$$

$$\eta = \frac{im\omega}{2\hbar}\left[\frac{J_4}{m\omega} - a'(x_1+x_2) + b'(x_1-x_2)\right]$$

$$\Omega = \frac{m\omega}{4\hbar}\left[a(x_1+x_2)^2 + b(x_1-x_2)^2 + \frac{2J_1}{m\omega}x_1 + \frac{2J_2}{m\omega}x_2 - J_5 - J_6\right] \tag{79}$$

When changing x_1 and x_2 into

$$x_1' = \frac{1}{2}(X'+Y')$$

$$x_2' = \frac{1}{2}(X'-Y') \tag{80}$$

then, Equation(78) becomes

$$\psi\left(x_1,x_2,t\right)=\left(\frac{m\omega}{4\pi i\hbar}\right)\left(\frac{\sqrt{3}}{2\pi\sigma^2\sin\omega t\sin\sqrt{3}\omega t}\right)^{\frac{1}{2}}$$
$$\exp\left\{i\Omega\right\}\int_{-\infty}^{+\infty}dX'\exp\left\{-\alpha X'^2+\xi'X'\right\}\int_{-\infty}^{+\infty}dY'\exp\left\{-\beta Y'^2+\upsilon'Y'\right\} \tag{81}$$

where

$$\xi'=\frac{1}{2}(\gamma+\eta)$$

$$\upsilon'=\frac{1}{2}(\gamma-\eta) \tag{82}$$

Equation (81) is easily integrate using Gaussian integral, then, one gets the expression of the time-dependent wave function which is

$$\psi(x_1,x_2,t)=\left(\frac{m^2\omega^2\sqrt{3}}{2\pi\sigma^2(im\omega\cot\omega t-\lambda)(\lambda_c-im\omega\sqrt{3}\cot\sqrt{3}\omega t)\sin\omega t\sin\sqrt{3}\omega t}\right)^{\frac{1}{2}}\times$$
$$\times\exp\left\{\frac{im\omega}{4\hbar}\left[\cot\omega t(x_1+x_2)^2+\sqrt{3}\cot\sqrt{3}\omega t(x_1-x_2)^2+\frac{2J_1}{m\omega}x_1+\frac{2J_2}{m\omega}x_2-J_5-J_6\right]\right.$$
$$-\frac{m^2\omega^2}{16\hbar(\lambda-im\omega\cot\omega t)}\left(\frac{1}{m\omega}\left(J_3+J_4\right)-\frac{2(x_1+x_2)}{\sin\omega t}\right)^2-$$
$$\left.\frac{m^2\omega^2}{16\hbar(\lambda_c-im\omega\sqrt{3}\cot\sqrt{3}\omega t)}\left(\frac{1}{m\omega}\left(J_3-J_4\right)-\frac{2\sqrt{3}(x_1-x_2)}{\sin\sqrt{3}\omega t}\right)^2\right\} \tag{83}$$

Where we recall that

$$\lambda=\sqrt{mk}\ ,\ \lambda_C=\sqrt{\lambda^2+2mk_C}\ ,\ \omega^2=\frac{k}{m},$$
$$\omega_C^2=\frac{(k+2k_C)}{m}\ ,\ \sigma^2=\frac{\hbar}{2m\sqrt{\omega\omega_C}} \tag{84}$$

Probability density: In term of the time-dependent wave function, the probability density is define by

$$P(x_1,x_2;t)=\psi^*(x_1,x_2,t)\psi(x_1,x_2,t) \tag{85}$$

Carrying out this product using Gaussian integrals and taking into account Equations (83), we obtain after few arrangements the probability density of the system, which reads

$$P(x_1,x_2;t)=\left(\frac{m^2\omega^2\sqrt{3}}{2\pi\sigma^2\sin\omega t\sin\sqrt{3}\omega t}\right)\left(\frac{1}{(\lambda^2+m^2\omega^2\cot^2\omega t)(\lambda_C^2+3m^2\omega^2\cot^2\sqrt{3}\omega t)}\right)^{\frac{1}{2}}\times$$
$$\exp\left\{-\frac{m^2\omega^2}{8\hbar}\left(\frac{\lambda}{\lambda^2+m^2\omega^2\cot^2\omega t}\right)\left(\frac{1}{m\omega}\left(J_3+J_4\right)-\frac{2(x_1+x_2)}{\sin\omega t}\right)^2-\right.$$
$$\left.\frac{m^2\omega^2}{8\hbar}\left(\frac{\lambda_C}{\lambda_C^2+3m^2\omega^2\cot^2\sqrt{3}\omega t}\right)\left(\frac{1}{m\omega}\left(J_3-J_4\right)-\frac{2\sqrt{3}(x_1-x_2)}{\sin\sqrt{3}\omega t}\right)^2\right\} \tag{86}$$

Entropy: By definiton, the Gibbs-Shannon entropy reads

$$S=-\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}P(x_1,x_2;t)\ln P(x_1,x_2;t)dx_1dx_2 \tag{87}$$

When setting

$$A_1=\frac{m^2\omega^2\lambda}{2\hbar(\lambda^2+m^2\omega^2\cot^2\omega t)\sin^2\omega t},$$

$$A_2=\frac{3m^2\omega^2\lambda_C}{2\hbar(\lambda_C^2+3m^2\omega^2\cot^2\sqrt{3}\omega t)\sin^2\sqrt{3}\omega t}$$

Citation: Kenfack SC, Nguimeya GP, Talla PK, Fotue AJ, Fobasso MFC, et al. (2016) Decoherence of Driven Coupled Harmonic Oscillator. J Nanosci Curr Res 1: 104. doi: 10.4172/2572-0813.1000104

$$B_1 = \frac{m\omega\lambda}{2\hbar(\lambda^2 + m^2\omega^2\cot^2\omega t)\sin\omega t}(J_3 + J_4)$$

$$B_2 = \frac{\sqrt{3}m\omega\lambda_C}{2\hbar(\lambda_C^2 + 3m^2\omega^2\cot^2\sqrt{3}\omega t)\sin\sqrt{3}\omega t}(J_3 - J_4) \quad (88)$$

$$C_1 = \frac{\lambda}{8\hbar(\lambda^2 + m^2\omega^2\cot^2\omega t)}(J_3 + J_4)^2$$

$$C_2 = \frac{\lambda_C}{8\hbar(\lambda_C^2 + 3m^2\omega^2\cot^2\sqrt{3}\omega t)}(J_3 - J_4)^2$$

$$M = \left(\frac{m^2\omega^2\sqrt{3}}{2\pi\sigma^2\sin\omega t\sin\sqrt{3}\omega t}\right)\left(\frac{1}{(\lambda^2 + m^2\omega^2\cot^2\omega t)(\lambda_C^2 + 3m^2\omega^2\cot^2\sqrt{3}\omega t)}\right)^{\frac{1}{2}}$$

and proceeding to the changes

$$x_1 = \frac{1}{2}(X + Y)$$

$$x_2 = \frac{1}{2}(X - Y) \quad (89)$$

Equation(89) becomes

$$S = -\frac{1}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P(X,Y;t)\ln P(X,Y;t)\,dXdY \quad (90)$$

Where

$$\begin{aligned} P(X,Y;t)\ln P(X,Y;t) &== M\exp\{-C_1 - C_2\}\exp\{-A_1X^2 + B_1X\} \\ &\exp\{-A_2Y^2 + B_2Y\}\left(\ln M - A_1X^2 + B_1X - A_2Y^2 + B_2Y - (C_1 + C_2)\right) \end{aligned} \quad (91)$$

Integrating Equation(92) using Gaussian integrals yields to the entropy of the system which is

$$\begin{aligned} S = -\frac{M\pi}{8A_1A_2}\left(\frac{1}{A_1A_2}\right)^{\frac{1}{2}}\exp\left\{\frac{B_1^2}{4A_1} + \frac{B_2^2}{4A_2} - C_1 - C_2\right\} \\ \left(\ln M + \frac{1}{2}B_1^2 + \frac{1}{2}B_2^2 - A_1 - A_2 - C_1 - C_2\right) \end{aligned} \quad (92)$$

Numerical results

We notice in figure 2 that the entropy decreases with time and tends towards a limit.

Therefore, decoherence decrease (Figure 2).

This result is in accordance with that obtained by M.A. de Ponte et al.[14] for two coupled dissipative oscillators. Physically, this means that the whole system is organizes itself and evolves towards a coherent state in which information will be preserved.

From figure 3, we note that the entropy of the system oscillates by keeping a constant amplitude which means that the exchange between the system and its surrounding are quasi-perfect so that the system keeps a certain coherence. The system is slowly affected by external forces. Physically, this coherence can be interpreted as being due to the fact that information remains preserved in time. This result is in accordance with that obtained by Tabue et al. [15] for the pure states of a damped harmonic oscillator.

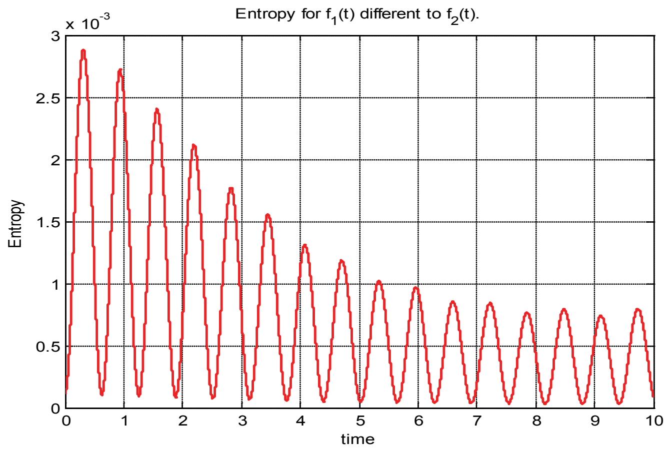

Figure 2: Entropy with parameter k = 0.001, k_c = 0.005, ωo = 5.

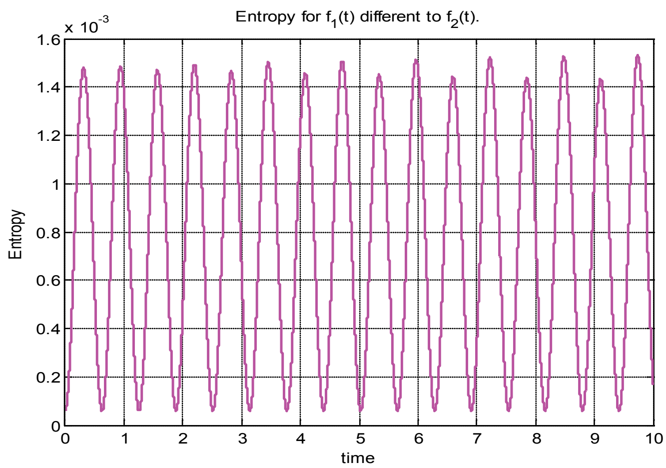

Figure 3: Entropy with parameter k = 0.001, k_c = k, ωo = 5.

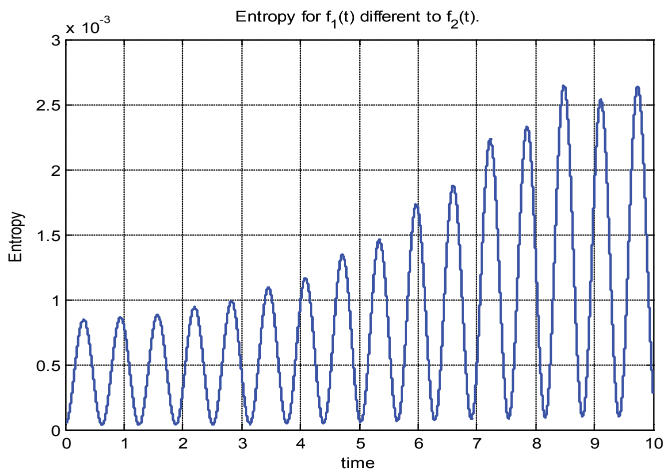

Figure 4: Entropy with parameter k = 0.001, k_c = 0, ωo = 5.

Figure 4 shows that entropy increases in time, and thus decoherence increases. This means that, the whole system collects environment which accentuates decoherence and so the system disorganizes itself.

One could expect such a result which is in accordance with that obtained by Tabue et al. [15] for a damped harmonic oscillator.

Conclusion

Using the Feynman path integral method we have determined the DCHO propagator after which, considering the particular case where the external driving forces are sinusoidal, we have determined the entropy associated to the system. We reveal numerically that, when two harmonic oscillators subjected to an external driving force are coupled, one compensates the effects of the other so that the whole system organizes itself and evolves towards an equilibrium position in which it will conserve certain coherence. That is explained by the decrease of the entropy which thereafter tends towards a limit. We also reveal that at resonance the system is in a coherent state which is explained by the conservation, with time of the oscillation amplitudes of the entropy of the system. Thus, we have shown that to reduce decoherence in a system one can just couple such a system with a driven harmonic oscillator or the system in question to resonance.

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