

Effects of Mixed Convection and Navier Slip on a Chemically Reactive Heat and Mass Transfer MHD Fluid Flow Over a Permeable Surface with Convective Boundary Conditions

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Abstract

The work investigates the effects of mixed convection and Navier slip parameters on a chemically reactive heat and mass MHD fluid flow over a permeable surface with convective boundary conditions. By transforming the system of coupled partial differential equations governing the flow into a system of coupled ordinary differential equations using similarity transformation, the resulting ordinary differential equations are solved using the Fourth order Runge – Kutta method with shooting technique. The effects of mixed convection and Navier parameters on the velocity, temperature and concentration profiles are quantitatively discussed graphically.

Keywords: Mixed convection; Navier slip; MHD fluid flow; Convective boundary conditions

Nomenclature

MHD: Magnetohydrodynamics;
(x, y): Coordinate axes or variables;
(u, v): Velocity components along the x - and y -axes;
 ψ : Dimensionless stream functions;
 g : Acceleration due to gravity;
 β_0 : Magnetic Field Strength;
 T : Uniform Surface Temperature;
 T_∞ : Farstream Temperature or temperature at the boundary;
 T_f : Temperature at the plate surface;
 u_∞ : Free Stream Velocity;
 C : Concentration;
 C_∞ : Far stream concentration or concentration at the boundary;
 C_f : Concentration at the plate surface;
 C_p : Specific heat capacity at constant pressure;
 γ : Coefficient of Kinematic viscosity;
 σ : Electrical Conductivity;
 K_p : Permeability of the medium;
 K : Thermal conductivity;
 D_m : Mass diffusivity;
 K : Reaction Rate constant;
 q_r : Radiative heat flux;
 S_c : Schmidt Number;
 P_r : Prandtl number;
 M : Magnetic parameter;
 Gr : Thermal Grash of number;

B_i : Biot number;
 β_c : Solutal expansion coefficient;
 β_t : Thermal expansion coefficient;
 λ : Internal heat generation parameter;
 Ra : Radiation parameter;
 Da : Darcy parameter;
 Br : Brinkman number;
 Gc : Solutal Grashof number;
 Fw : Suction or Injection parameter ($S > 0$ for suction and $S < 0$ for injection);
 B_s : Convective–diffusion parameter;
 δ : Slip parameter;
(θ, ϕ): Dimensionless temperature and concentration;
 Q : Heat release;
 hm : Mass transfer;
 hf : Heat transfer

Introduction

The concept of boundary layer was first introduced by L. Prandtl in 1904 and since then it has been applied to several fluid flow problems.

A boundary layer is formed whenever there is a relative motion

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concentration equations (1) to (4) together with the boundary conditions (5) to (9), we obtain

$$\begin{aligned} f''' + \frac{1}{2}ff'' + Gr_x\theta - Gc_x\phi + (M_x + Da_x)(1 - f') &= 0 \\ (1 + \frac{4}{3}Bs_xRa)\theta'' + \frac{1}{2}Pr f\theta' + Pr\lambda_x\theta + PrEc(f'')^2 + M_xEcPr(1 - f')^2 &= 0 \\ \phi'' + \frac{1}{2}Sc_f\phi' - Sc\beta_x\phi &= 0 \end{aligned} \quad (10)$$

The corresponding boundary conditions are

$$\begin{aligned} f'(0) &= \delta_x f''(0); f(0) = F_{wx}; -\theta'(0) = Bi_x(1 - \theta(0)); -\phi'(0) = Bs_x(1 - \phi(0)); \\ f'(\infty) &= 1; \theta(\infty) = 0; \phi(\infty) = 0 \end{aligned} \quad (11)$$

where the prime symbol represents the derivative with respect to η ,

$Sc = \frac{\gamma}{D_m}$ is the Schmidt number, $Pr = \frac{\gamma}{\alpha}$ is the Prandtl number, $M_x = \frac{\sigma\beta_0^2x}{\rho u_e}$ is the magnetic parameter, $F_{wx} = \frac{\pm 2v_0x^{\frac{1}{2}}}{\sqrt{\gamma u_e}}$ is the injection/suction parameter, $Gr_x = \frac{xg\beta_x(T_f - T_\infty)}{u_e^2}$ is the thermal Grashof number, $Gc_x = \frac{xg\beta_c(C_f - C_\infty)}{u_e^2}$ is the solutal Grashof number, $\beta_x = \frac{k_fx}{u_e}$ is the reaction rate parameter, $\lambda_x = \frac{xQ}{u_e\rho C_p}$ is the internal heat generation parameter, $R_x = \frac{4\sigma^*T_\infty^3}{k\rho C_p\gamma}$ is the radiation parameter, $Da_x = \frac{x\gamma}{u_e k_p}$ is the Brinkman number, $Bs_x = \frac{D}{h_m}\sqrt{\frac{u_e}{\gamma x}}$ is the convective-diffusion parameter, $Bi_x = \frac{h_f}{k}\sqrt{\frac{\gamma x}{u_e}}$ is the Biot number, $\delta_x = L\sqrt{\frac{u_e}{\gamma x}}$.

It is assumed that equations (10) with boundary conditions (11) have similarity solutions when the parameters M_x , F_{wx} , Gr_x , Gc_x , β_x , λ_x , Da_x , Bs_x , Bi_x and δ_x are defined as constants [19].

Results and Discussion

Numerical calculations have been carried out for different values of the thermo physical parameters controlling the fluid dynamics in the flow region using Runge-Kutta method with shooting technique

implemented by maple 17 (Table 1) illustrates the values of the Skin-friction coefficient $f''(0)$, for various values of the embedded flow parameters. Table 2 illustrate the values of the plate surface temperature $\theta(0)$ together with the Nusselt number $-\theta'(0)$ for various values of the embedded flow parameters. Table 3 shows the values of the rate of mass transfer $\phi(0)$ and Sherwood number $-\phi'(0)$ for various values of embedded flow parameters. Table 4 shows the comparison of Emmanuel et al. with the present work for prandtl number $Pr=0.72$ and the work is in perfect agreement [20].

Figures 1-6 explains the effects of emerging flow parameters on the velocity profiles. Generally, the fluid velocity is lower at the plate surface and increases gradually to its free stream values satisfying the boundary condition. In Figure 1, as Hartmann number increases, so does the retarding force and hence the velocity profile decreases. Figure 2 shows a decrease in the fluid velocity within the boundary layer due to porosity of the medium thereby decreasing the momentum

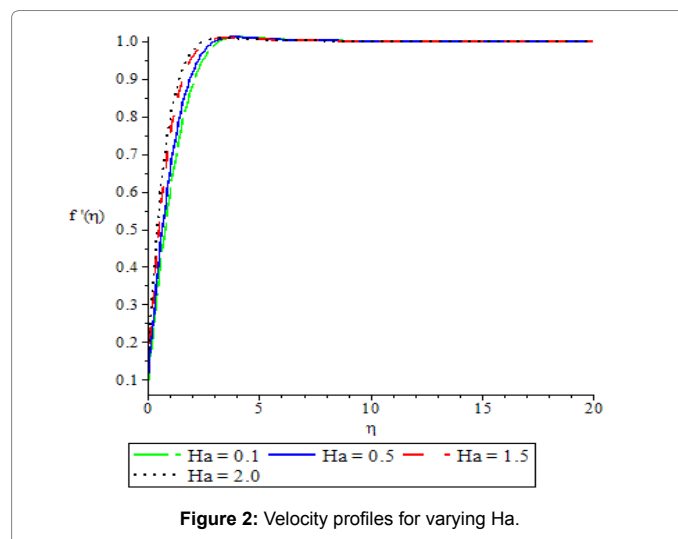


Figure 2: Velocity profiles for varying Ha.

Ha	Da	Gr	Gc	F _w	δ	f''(0)
0.3	0.1	0.1	0.1	0.1	0.1	0.00857
0.4	0.1	0.1	0.1	0.1	0.1	0.01778
0.5	0.1	0.1	0.1	0.1	0.1	0.026
0.1	0.5	0.1	0.1	0.1	0.1	0.87245
0.1	1	0.1	0.1	0.1	0.1	1.06771
0.1	2	0.1	0.1	0.1	0.1	1.35479
0.1	0.1	0.7	0.1	0.1	0.1	0.00185
0.1	0.1	0.75	0.1	0.1	0.1	0.01771
0.1	0.1	0.8	0.1	0.1	0.1	0.03663
0.1	0.1	0.1	0.2	0.1	0.1	2.29323
0.1	0.1	0.1	0.3	0.1	0.1	2.7965
0.1	0.1	0.1	0.4	0.1	0.1	3.47131
0.1	0.1	0.1	0.1	-0.5	0.1	0.55816
0.1	0.1	0.1	0.1	-0.2	0.1	0.60042
0.1	0.1	0.1	0.1	-0.1	0.1	0.61897
0.1	0.1	0.1	0.1	0.5	0.1	0.76119
0.1	0.1	0.1	0.1	0.7	0.1	0.81654
0.1	0.1	0.1	0.1	3	0.1	0.90409
0.1	0.1	0.1	0.1	0.1	2	0.31935
0.1	0.1	0.1	0.1	0.1	3	0.24789
0.1	0.1	0.1	0.1	0.1	5	0.17059

Table 1: Computations showing the values of coefficients of skin-friction with Ha, Da, Gr, Gc, F_w, δ as prescribed parameters.

Rd	Br	Ha	Pr	λ	Bi	F_w	$\theta(0)$	$-\theta'(0)$
0.5	0.1	0.1	0.72	0.2	0.1	0.1	0.46704	0.0533
1	0.1	0.1	0.72	0.2	0.1	0.1	0.47016	0.05298
1.5	0.1	0.1	0.72	0.2	0.1	0.1	0.47634	0.05237
0.1	1.5	0.1	0.72	0.2	0.1	0.1	2.36778	0.13678
0.1	1.7	0.1	0.72	0.2	0.1	0.1	2.69636	0.16964
0.1	2	0.1	0.72	0.2	0.1	0.1	3.22784	0.22278
0.1	0.1	8	0.72	0.2	0.1	0.1	0.6329	0.03671
0.1	0.1	9	0.72	0.2	0.1	0.1	0.64146	0.03585
0.1	0.1	10	0.72	0.2	0.1	0.1	0.64911	0.03509
0.1	0.1	0.1	1.2	0.2	0.1	0.1	0.43898	0.0561
0.1	0.1	0.1	1.7	0.2	0.1	0.1	0.42118	0.05788
0.1	0.1	0.1	2	0.2	0.1	0.1	0.41469	0.05853
0.1	0.1	0.1	0.1	0.05	0.1	0.1	0.31646	0.06835
0.1	0.1	0.1	0.1	0.07	0.1	0.1	0.33	0.067
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.35335	0.06467
0.1	0.1	0.1	0.1	0.2	0.2	0.1	0.6089	0.07822
0.1	0.1	0.1	0.1	0.2	0.3	0.1	0.68795	0.09362
0.1	0.1	0.1	0.1	0.2	1	0.1	0.87004	0.12996
0.1	0.1	0.1	0.1	0.2	0.1	-0.5	0.87786	0.01221
0.1	0.1	0.1	0.1	0.2	0.1	-0.2	0.62587	0.03741
0.1	0.1	0.1	0.1	0.2	0.1	-0.1	0.56692	0.04331
0.1	0.1	0.1	0.1	0.2	0.1	0.5	0.35461	0.06454
0.1	0.1	0.1	0.1	0.2	0.1	0.7	0.31468	0.06853
0.1	0.1	0.1	0.1	0.2	0.1	1	0.26956	0.07304

Table 2: Computations showing values of the plate temperature $\theta(0)$ and Nusselt number $-\theta'(0)$ with Rd, Br, Ha, Pr, λ , Bi, F_w , $\theta(0)$, $-\theta'(0)$ as prescribed parameters.

Sc	β	F_w	Bs	$\phi(0)$	$-\phi'(0)$
0.24	0.1	0.1	0.1	0.32577	0.06742
0.64	0.1	0.1	0.1	0.24267	0.07573
0.78	0.1	0.1	0.1	0.22684	0.07732
0.1	0.01	0.1	0.1	0.43841	0.05616
0.1	0.05	0.1	0.1	0.41818	0.05818
0.1	0.09	0.1	0.1	0.40041	0.05996
0.1	0.1	-0.5	0.1	0.37328	0.06267
0.1	0.1	-0.2	0.1	0.36112	0.06389
0.1	0.1	-0.1	0.1	0.35699	0.0643
0.1	0.1	0.5	0.1	0.33277	0.06672
0.1	0.1	0.7	0.1	0.32512	0.06749
0.1	0.1	1	0.1	0.31413	0.06859
0.1	0.1	0.1	0.3	0.66294	0.10112
0.1	0.1	0.1	0.5	0.76617	0.11692
0.1	0.1	0.1	1	0.86756	0.13244

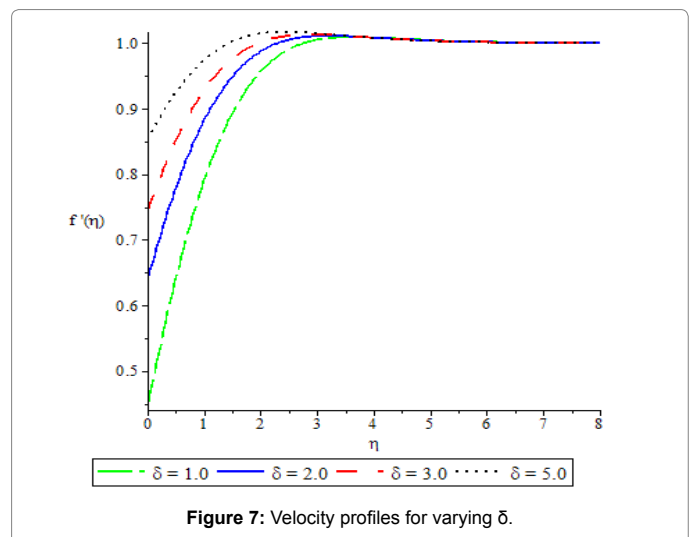
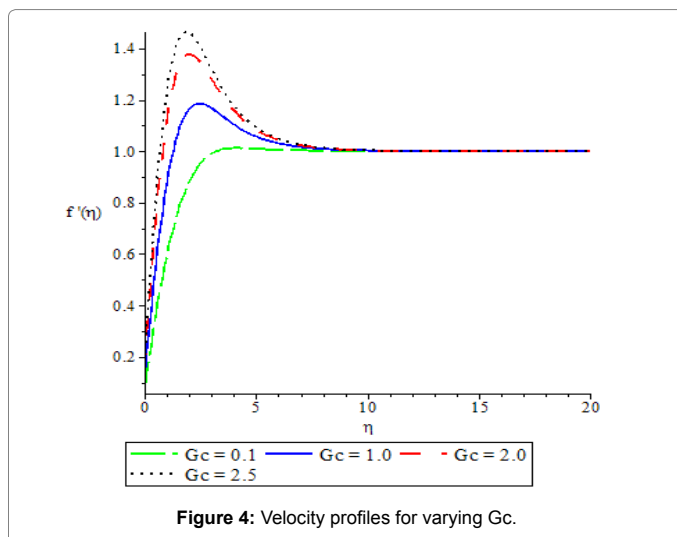
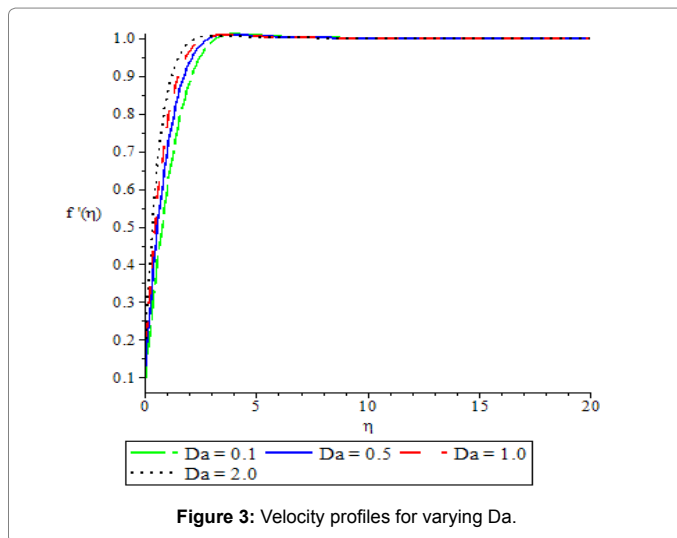
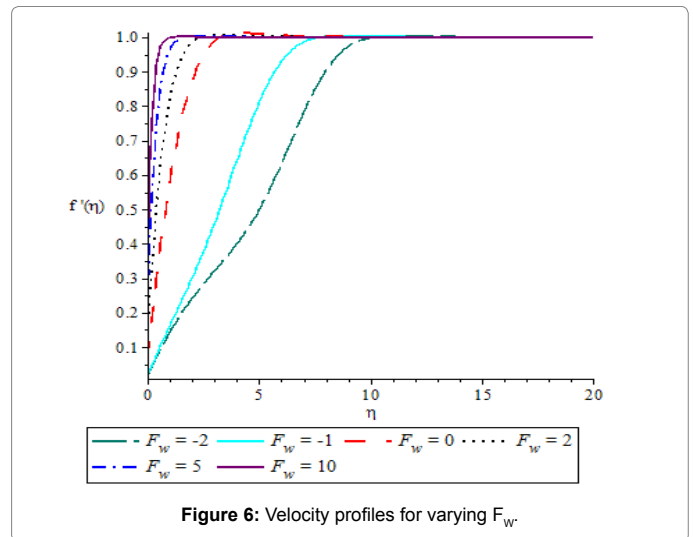
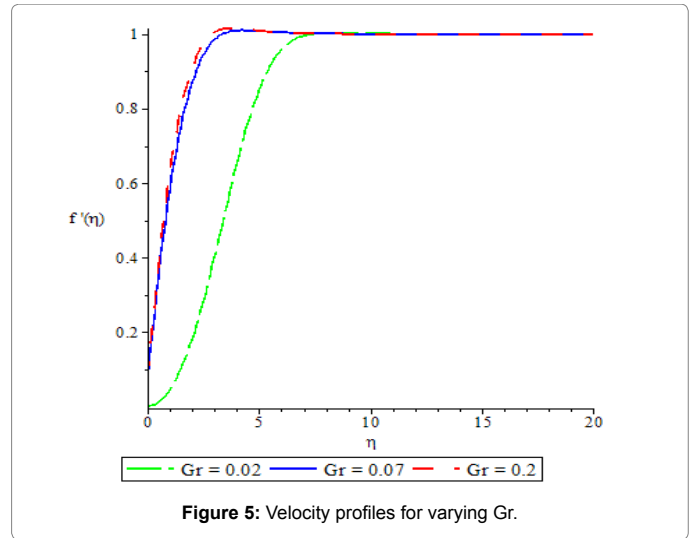
Table 3: Computations showing values of Rate of Mass Transfer $\phi(0)$ and Sherwood number $-\phi'(0)$ with Sc, β , F_w , Bs as prescribed parameters.

Pr	Sc	M	Ra	Br	β	Bi	Emmanuel et al. [9]		Present Paper	
							$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0.71	0.24	0.1	0.1	0.1	0.1	0.1	0.45184	0.06828	0.45184	0.06828
0.72	0.24	0.1	0.1	0.1	0.1	0.1	0.45184	0.06842	0.45184	0.06842
0.72	0.24	0.1	0.1	0.1	0.1	0.1	0.77079	0.06422	0.77079	0.06422
0.72	0.24	0.1	0.1	0.1	0.1	0.1	0.45184	0.06698	0.45184	0.06698
0.72	0.24	0.1	0.1	0.1	0.1	0.1	0.45184	0.04266	0.45184	0.04266

Table 4: Computations showing comparison of the work with Emmanuel et al. [9] for $n=1$ and $M=Ra=Br=\beta=Bi=0$.

boundary layer thickness. In Figure 3, the fluid velocity increases to a more distinctive peak value due to increase in the buoyancy force. Figure 4 shows that an increase in the thermal Grashof number leads to a thinning in the momentum boundary layer thickness and hence increasing the velocity of the flow. In Figure 5 increase in fluid suction $F_w > 0$ retarded the rate of transport and reduced the boundary layer thickness and an opposite phenomenon was noticed for fluid injection $F_w < 0$ while $F_w = 0$ refers to a non-porous plate. In Figure 6 it was observed that the fluid velocity at the plate surface increases with increase in the Navier slip parameter δ resulting in the lubrication and slipperiness of the surface [21].

Figure 7 shows the effects of the emerging flow parameter on the temperature profile. The maximum value of the fluid temperature is attained at the plate surface but decreases to the free stream zero value away from the plate satisfying the boundary condition. The thermal boundary layer thickness increases with an increase in Biot number B_i . This is because as B_i increases, the heat transfer rate from the hot fluid at the lower side of the plate to the cold fluid at the upper side increases. The effects of the slip parameter and Prandtl number on the temperature profile are seen in Figures 8. The temperature profile decreases with increase in slip parameter δ and Prandtl number P_r (Figures 9). This is



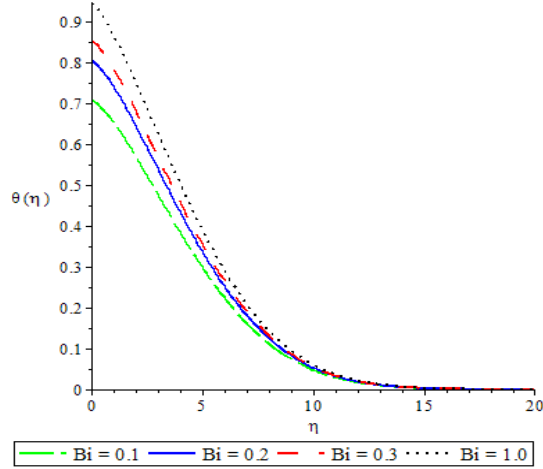


Figure 8: Temperature profiles for varying Bi.

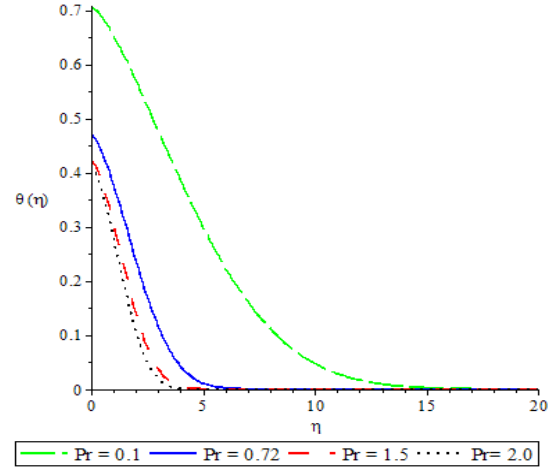


Figure 10: Temperature profiles for varying Pr.

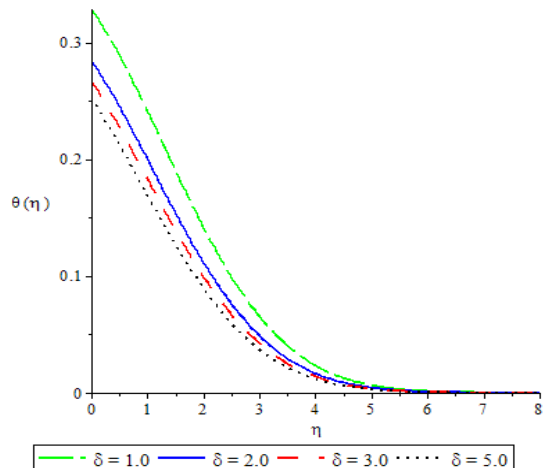


Figure 9: Temperature profiles for varying δ .

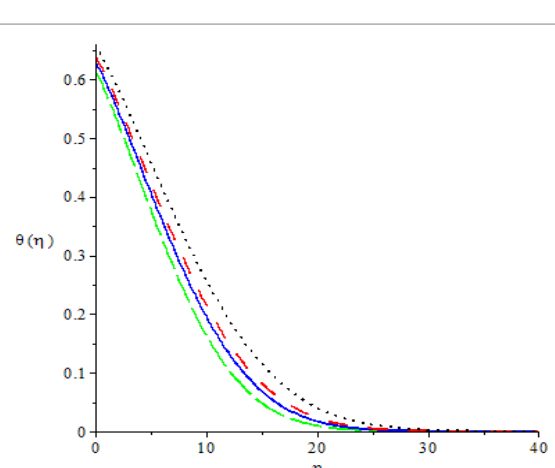


Figure 11: Temperature profiles for varying Rd.

because at higher Prandtl number, the fluid has relatively low thermal conductivity, which reduces conduction together with the thermal boundary layer thickness and thereby decreasing the temperature. Figure 10 shows the graph of temperature against the coordinate η for various values of radiation parameter. Due to absorption, the thermal boundary layer thickness increases as the radiation parameter R_a increases. Also, in Figure 11, increase in the magnetic parameter M increases the fluid temperature which in turn, increases the thermal boundary layer thickness due to ohmic heating on the flow system. In Figure 12, progressive rise in fluid temperature and thickening of the thermal boundary layer were observed phenomena as Brinkman number B_r and internal heat generation parameter λ increases. Fluid suction was found to make the thermal boundary layer thinner as shown in Figures 13.

Figure 14 exhibit the variations of concentration profile against dimensionless variable η under the influence of chemical reaction rate parameter β , Schmidt number Sc , convective-diffusion parameter B_s and fluid Suction/Injection F_w (Figures 15). In Figures 15 and 16, concentration decreases with increase in chemical reaction parameter β and fluid Injection $F_w > 0$ but increases with increase in the convective-diffusion parameter B_s (Figures 16). The solutal boundary

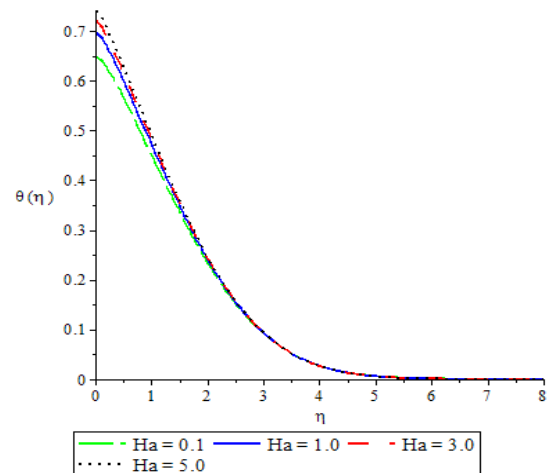


Figure 12: Temperature profiles for varying Ha.

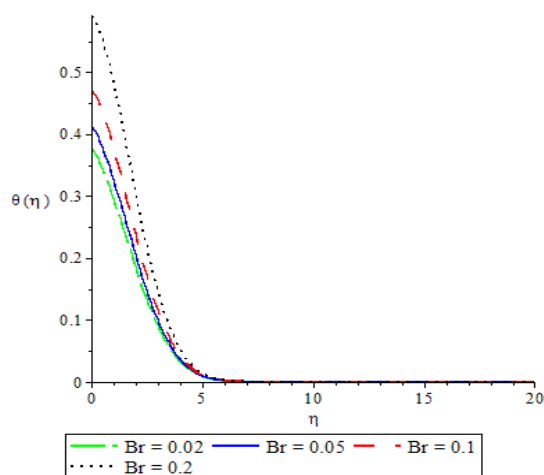


Figure 13: Temperature profiles for varying Br.

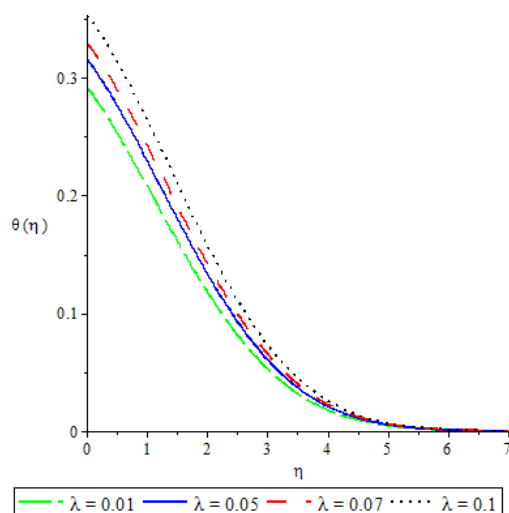


Figure 14: Temperature profiles for varying λ .

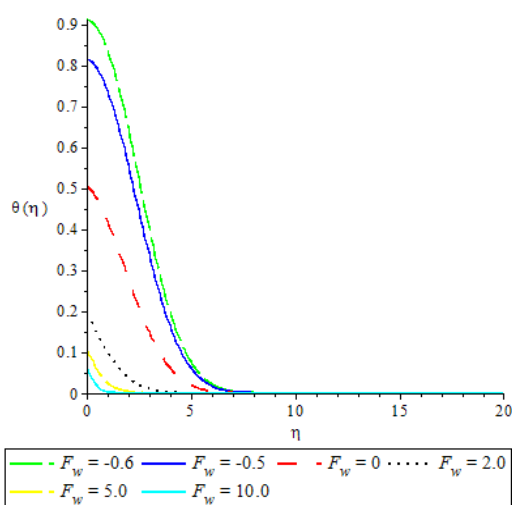


Figure 15: Temperature profiles for varying F_w .

layer thickness was also seen to decrease for appreciable values of B_s (Figure 17). The plate surface concentration was increased as a result of increase in B_s but reduced for increase in the values of Sc , β and F_w . It was observed from Figure 18 that the concentration level of the fluid drops due to increasing chemical reaction parameter, fluid injection and Schmidt number because mass diffusivity raises the concentration level steadily (Figure 19) [22].

Conclusions

From the numerical solutions and graphical representations, we can conclude that:

- The fluid velocity decreases at the plate and increases gradually to its free stream satisfying the boundary condition.

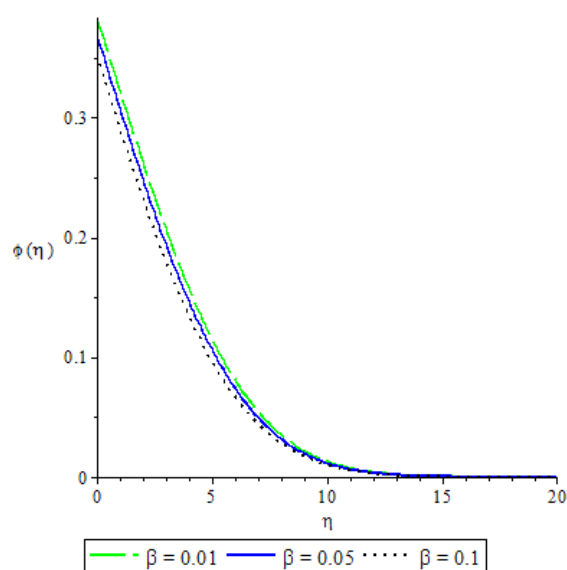


Figure 16: Concentration profiles for varying β .

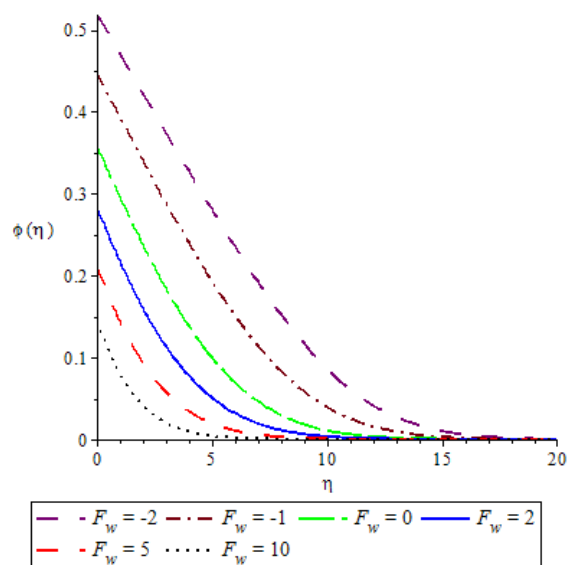


Figure 17: Concentration profiles for varying F_w .

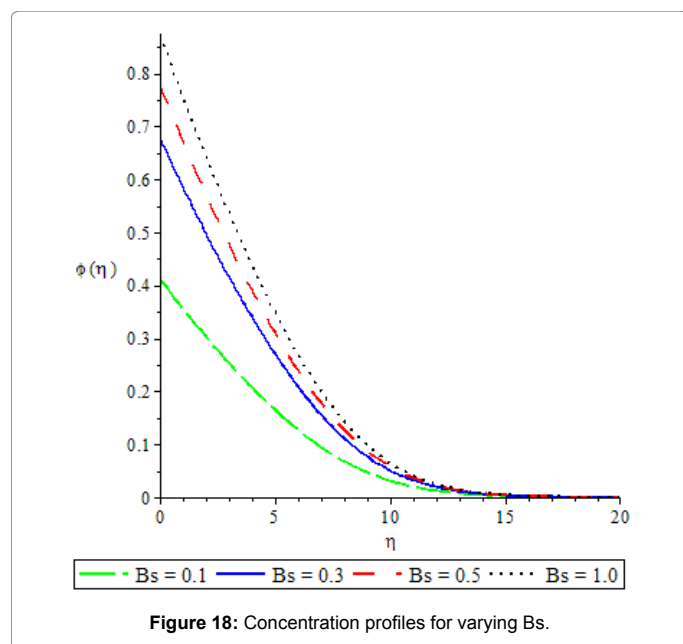


Figure 18: Concentration profiles for varying B_s .

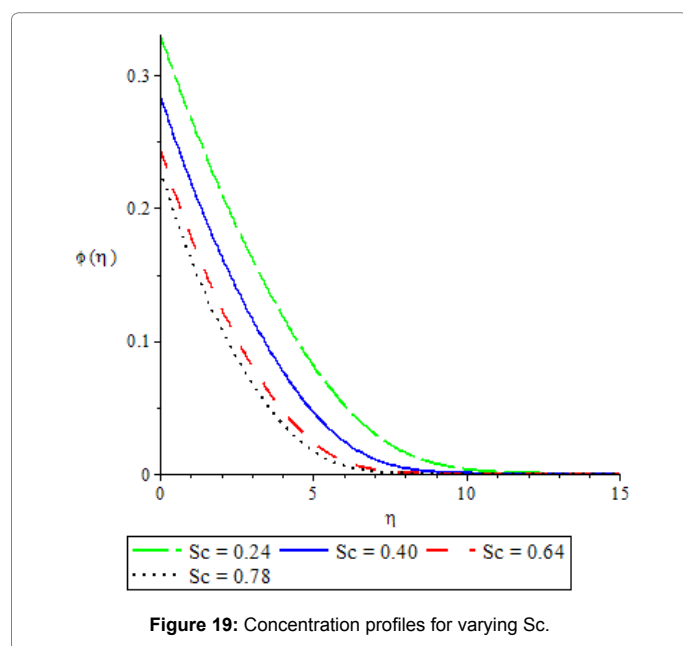


Figure 19: Concentration profiles for varying Sc .

- The fluid velocity at plate surface increases with increases in Navier slip parameter resulting in the lubrication and slipperiness of the surface.
- The maximum value of the fluid temperature is attained at the plate surface but decreases to the free stream value of zero away from the plate satisfying the boundary condition.
- The fluid temperature decreases with increase in Navier slip parameter and prandtl number due to relatively low conductivity.
- Fluid temperature and the thickening of the thermal boundary layer increases as Brinkman number Br and internal heat generation parameter λ increases.

- Thermal boundary layer thickness increases with increase in biot numbers and fluid suction makes the boundary layer thinner.
- The rate of heat transfer (from the plate to the fluid) decreases with thermal radiation and heat source parameter but increases with increase in Brinkman number, Hartmann number, Pandtl number and ohmic heating of the flow system.
- Concentration of the flow decreases with increase in chemical reaction β_x , Schmidt number Sc , suction parameter F_w and convective-diffusion parameter B_s .

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