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Elastic Spring Constants for Running Shoes: A Mathematical Model

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Abstract

Background: Running shoe compliance and track surface stiffness can reduce peak vertical foot forces. It is therefore of interest to measure directly the force-deflection curve for running shoes in the heel and forefoot areas. This study compares these measurements with similar work on track and field surfaces, and derives a mathematical stress-strain model useful over the entire force range.

Methods: Six different running shoes from 4 popular brands are measured to determine vertical spring stiffness. The heel and ball areas are tested with 3.8 and 5.1 cm (1.5 and 2.0 in.) diameter heels in the force range of 0 to 0.13 kN and 2.0 to 2.6 kN (0 to 30 lbf. and 450 to 600 lbf.). The results show a factor of 2 difference from one shoe to the next, holding test area, heel diameter and force range constant. Load increments are applied on a time scale of 0.1 seconds, comparable to typical foot contact times during running.

Results: The measured spring constants are essentially independent of plunger area, a useful simplification. For a given shoe, the ball area can be three times less compliant than the heel.

Conclusion: Heel spring constants at the high force levels fall in the range from 290 kN/m to 600 kN/m (20,000 to 42,000 lbf/ft.), and thus approximate optimal track stiffness. In terms of theory, an exponential function derives from the observation that the data fall along a straight line on semi-log co-ordinates. This mathematical model enables calculation of running shoe compressive response and spring constant at physiological force levels.

Keywords: Running shoe compliance; Track stiffness; Compressive stress-strain; Orthopaedics; Force-deflection curve; Area independence

Introduction

Compliance in the track or shoe is an important design parameter that can reduce foot force significantly, as reported in many technical reports [1-11]. The effects of compliance on vertical foot force, i.e. the vertical component of the ground reaction force, are reported by Aerts and de Clercq [1] . Running shoe and track compliance is reviewed by Frederick [12], as it influences locomotion at various speeds.

The rationale and motivation for this study is force reduction. The forces on the foot during running and jogging may contribute to soft tissue injuries of the calcaneous [3], Achilles tendon [4,13] anterior compartment (shin splints) [14], and knee ligaments and cartilage [11,15]. Excessive vertical force is of particular interest to amputees [16] in terms of prosthetic stability and comfort [17]. Shoe compliance measurements can vary according to the type of compression machine, indentor, and force range [2,12,18,19]. Using optimal compliance values for tracks and running shoes, peak vertical forces can be reduced without sacrificing speed, so one objective of this report is to compare vertical spring constants of typical running shoes with those of optimal running tracks [7,8,20]. Laboratory measurements of shoe dynamics in the heel and forefoot areas of the sole enable comparison with track stiffness measurements. This report investigates whether compliance results for a flat indentor are independent of plunger area, an important experimental and theoretical simplification. Lastly, a force-deflection equation is derived, facilitating calculation of shoe compliance over the entire physiological force range, applicable to the dynamics of walking, jogging, and running. In general, running shoes are the softest during walking, least compliant during sprinting periods. Serpell et al. [21] review physiological stiffness calculations of the leg and knee. Bo [22] discusses the difficult task of computing deflection of an elastic half space.

Nomenclature

A_o=plunger area [cm^2 (sq. in.)]

ε=strain [dh/h_0 (%)]

F=force=$A_o \times \sigma(\varepsilon)$ [lbs or N],

σ=stress=A [exp($\alpha \times \varepsilon$)-1], [N/cm^2 (lbf./in^2)]

A=exponential material constant [N/cm^2 (lbf./in^2)]

α=exponential stiffening constant

h_o=sole thickness [cm (inches)]

dh=vertical deflection [cm (mils.)]

Materials and Methods

A representative sample of 6 training shoes from 4 different manufacturers was measured to determine their spring constants. They were subjected to forces between 0 to 0.13 kN and 2.0 to 2.6 kN (0-30 and 450-600 lbf.) in the heel and ball (forefoot) regions of the shoe. The important tests were in the 2.0 to 2.6 kN range (450-600 lbf.), since this is the force level during most of the foot contact time during running. As measured, the force-deflection curve is non-linear; this is mathematically modeled here using basic stress-strain constitutive equations. One purpose of this report is to explore the feasibility of these experimental procedures together with basic equations, so several representative running shoes (N=6) are selected as typical samples, in order to demonstrate the utility of these techniques.

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Sole thickness varies from the heel to ball region of the running shoe, so it was necessary to make separate measurements in each area. Two different size artificial heels were used, circular disks, with diameters D=3.8 cm and D=5.1 cm (1.5" and 2.0"), Figure 1, applied to the heel and forefoot of the shoe, to measure if an area independent index could be determined. The shoe uppers and soles were not cut in any way in order to perform these measurements, remaining intact along with the original liner inserts. The bottom of the circular aluminum heel, flat in profile, was teflon coated with a 1.6 mm (1/16") thick sheet to reduce friction during the compression tests, and to not mar the shoe. As predicted by theory, an area independent index is experimentally possible, as shown in Figure 2, because the measured effective spring constants are independent of the heel diameter. An area independent index is basically a mechanical property independent of the indentor area, an important generalization.

The dynamic velocity component of the shoe sole, i.e. the dashpot component in the constituitive relation, was not measured directly. Typical foot contact times are the order of 0.10 to 0.15 sec during running, 0.3 to 0.8 sec during walking, jogging, accelerating, and turning [20,23-25]. The incremental loads in these experiments are applied on a time scale of 0.1 secs, so the elastic spring constants reported here are physiologically realistic, particular in the high force range.

As shown in Figure 1, steel weights are used as a source of vertical force. In order to alleviate operator fatigue, a significant problem when performing repeated tests in the 2.6 kN (600 lbf.) range, a fulcrum arrangement was devised to decrease the required amount of applied weight by a factor of three. Because of the 3:1 moment arm, an applied 880 N (200 lbf.) weight exerts a 2.64 kN (600 lbf.) vertical force on the disc plunger. This is a manually operated device, not yet automated, requiring considerable weight-lifting work for each shoe. Depending on the test, the artificial heel has an area of either 11 cm^2 or 20 cm^2 (1.77 or 3.14 sq. in.). At t=0 , for the low stress tests, a 44 N (10 lbf.) weight was added to the extended end of the U-beam around the 25 cm (10") bolt, the force is amplified by a factor of 3, and the shoe is compressed by a 132 N (30 lbf.) force in less than 0.1 seconds. The deflection dial indicates the amount of compression. The deflection gauge is accurate within +/-1.0 mil., readable within +/-5.0 mil. In the high-stress region, the last of four 220 N (50 lbf.) weights is added and the deflection recorded. The same techniques are used in the ball region of the shoe. Incremental 660 N (150 lbf.) compressive force is transferred to the shoe on a time scale of 0.1 secs. An adjustable counter-weight is used, shown at the left in Figure 1, to null out and level the weight of the moment arm before the steel weights are applied. All shoes used in these experiments were men's size 8-1/2. We recorded shoe stiffness constant k=dF/dx at the two different load ranges to characterize the non-linear material properties. An exponential constituitive relation, Fung [23] enables mathematical prediction of the stress-strain and load-deflection curves using the equation:

Eq. (1) $F=A_o \times \sigma(\varepsilon)$, where

A_o=plunger area [cm^2 (sq. in.)]

$\boldsymbol{\sigma}$=stress [N/cm^2 ($lbf./in^2$)]

$\boldsymbol{\varepsilon}$=strain [dh/h_0 (%)]

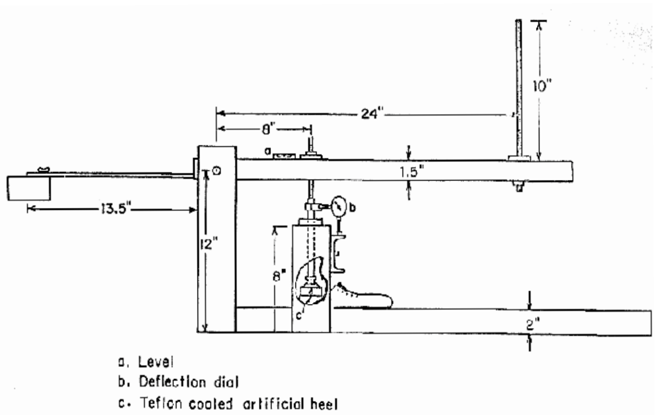

Figure 1: Scale drawing of the shoe compression machine. Steel weights are loaded on the 25 cm (10 inch) spindle at the end of the moment arm. Teflon artificial heel is ball-jointed to accommodate slope changes in the shoe. Deflection gauge is accurate within +/-1.0 mil.

Results

Figure 2 shows the results of all the force-deflection tests. For any given combination of shoe, test locale, plunger area, and force range, each test was repeated 4 times. The error bars indicate the standard deviation of these 4 repeated measurements. Overall there is a decrease in spring constant for increasing sole thickness, i.e. the shoe is more compliant as expected. Of the 6 running shoes tested, No. 5, Figure 2, has the smallest average spring constant in both the heel and ball area, and thus is the most compliant.

The heel value of k=332 kN/m (23,000 lbf./ft.) is close to the optimal value for running tracks [7,8]. The same weights, deflection gauge, and testing procedure used for the track and field experiments were also used for the shoe compressions. As shown in Figure 2, the ball area of the shoe is several times stiffer than the heel for all of the shoes tested. During running, a significant portion of the foot contact phase takes place with either both the heel and ball or just the ball of the foot in contact with the ground. Thus, the measured running shoes are several times stiffer than the optimal vertical compliance for track and field surfaces.

Following the theoretical results of Fung [26], the data are fit to an exponential stress-strain curve:

Eq. (2) $\sigma(\varepsilon)=A\,[\exp(\alpha \times \varepsilon)-1]$, where

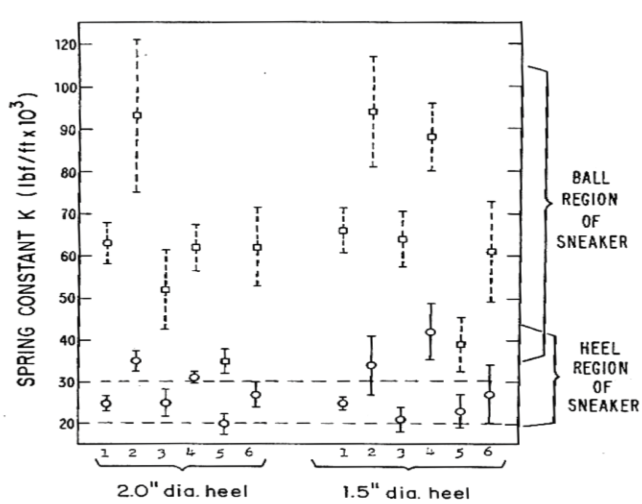

Figure 2: Vertical spring constants in the 2 kN to 2.6 kN (450-600 lbf.) range for the heel and ball regions of 6 different running shoes. Dotted lines indicate optimal stiffness for running tracks [7,8].

σ=stress [N/cm^2 (lbf./in^2)]

ε=strain [dh / h_0]

A=material constant [N/cm^2 (lbf./in^2)]

α=exponential stiffening constant

Experimental results are shown in Figures 3 and 4. Figure 3 is a force-deflection graph for the ball region (squares) and the heel region (circles) of a running shoe. Typical load levels for walking, jogging, and running are shown, indicating that the effective stiffness does vary with the applied force level. Figure 4 is a semi-log stress-strain graph for the ball region of one of the six running shoes indicating that the sole material has an exponential stress-strain constituitive law as derived by Fung [26]. The +/-rms error bars are larger in the forefoot area (Figure 2) because the deflection in this area is approximately 1/3 that of the heel area.

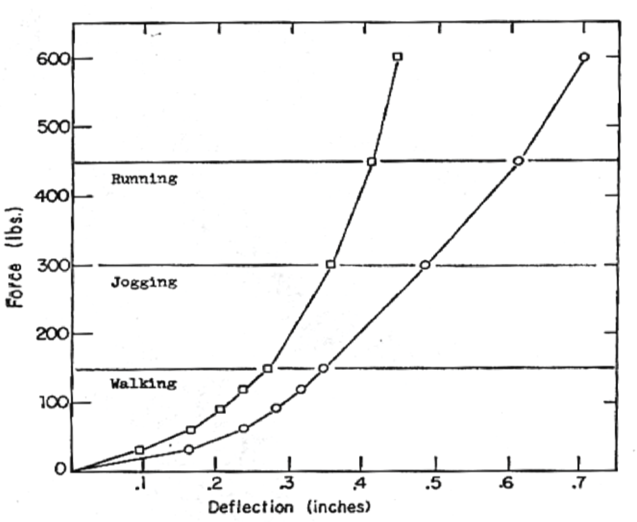

Figure 3: Force-deflection graph for the ball (forefoot) region (squares) and the heel region (circles) of a running shoe. Typical load levels for walking, jogging, and running are shown, indicating that the effective stiffness does vary with the applied force level.

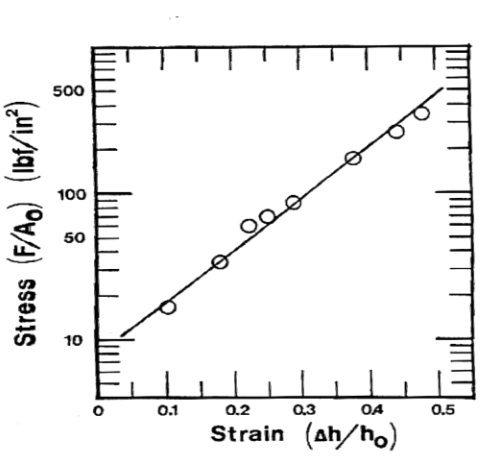

Figure 4: A semi-log stress-strain graph for the ball region of one of the six running shoes indicates that the sole material has an exponential stress-strain constituitive law as derived by [23] h_o=sole thickness, A_o=plunger area.

Discussion

In terms of practical clinical applications, excessive ground reaction forces on the foot during running and jogging may contribute to soft tissue injuries of the calcaneous, Achilles tendon, anterior compartment (shin splints), and knee ligaments and cartilage [11,14,16]. Excessive vertical force is of particular interest to amputees [16], in terms of shock transmission characteristics after foot strike [17], from the ground plain sequentially on upwards through the body. Thus, optimizing the running shoe spring constant to minimize foot force is a well-defined design objective. Other important design and experimental parameters include indentor area, hysteresis, and exponential stress-strain, as discussed below. Mathematical theory is important because it enables continuous calculation of the force-deflection and stress-strain curves over the entire force range, applicable to the forces and stresses typical of walking, jogging, and running, a useful generalization. These equations (Eq.1 and Eq. 2, Fung [26]) are an important economy, in terms of computing time and effort, because the exact elastic half-space calculations can be quite involved [3,22].

This study shows that compliance results for a flat indentor are independent of plunger area, an important experimental and theoretical simplification. An exponential force-deflection equation allows calculation of shoe compliance over the entire physiological force range, applicable to the dynamics of human locomotion at varying speeds. It remains to be seen with future research if we can confirm that compliant running shoes exhibit a 2% to 3% increase in top speed, as reported by McMahon and Greene [7,8] for tuned tracks.

Area independence

The data show that there is no essential difference in measured vertical stiffness constant between the small and large plungers, although the disk area changes by a factor of 77%. This effect can be predicted on theoretical grounds [26] as a result of the exponentially stiffening stress-strain curve of the polymer materials, a result which is also true for tendons stressed in tension[13]. Thus, a possible area independent index may be the exponential stiffening constant [26], typically $3<\alpha<10$ for running shoes, Eq. 2.

Hysteresis

Viscoelastic effects, i.e. hysteresis, with different trajectories for loading and unloading, are not measured for all the shoes reported here. Preliminary measurements, not presented here, indicated there is a 10% to 15% difference in effective stiffness, resulting from the viscous dashpot effects.

Exponential stress-strain relation

Following the theoretical results of Fung [26], Eq. (2), the data are fit to an exponential stress-strain curve $\sigma(\varepsilon)=A\,[\exp(\alpha \times \varepsilon)-1]$, as shown in Figures 3 and 4, with data from one of the shoes. All six shoes demonstrate similar exponential stress-strain curves. Exponential material constants range from $25<A<45$ and $4<\alpha<6$ for the heel region. Figure 3 shows that the effective spring constant of the shoe varies considerably according to whether one is walking, jogging, or running.

Shoe design for other sports

When walking, jogging, accelerating [25], or running on turns [23,24], foot contact time is lengthened considerably from 0.15 secs. to 0.3-0.8 secs., although the forces remain at a high level. Thus, the dynamics of these longer impulse foot-strikes are quite different from the dynamics of sprinting, suggesting different design parameters for basketball, soccer, and football shoes [27].

Study limitations

Extrapolating these results to shoes of different sizes, i.e. scaling laws, is a challenging question, beyond the scope of this report, requiring further research. Sole stiffness is found to be proportional to the reciprocal of thickness h^{-1} by substituting Eq. 1 into Eq. 2 and differentiating: $K=dF/dx \sim h^{-1}$. This is a useful simplification. Since only 6 representative running shoes from various companies were used, this study is by no means exhaustive. For commercial reasons, we do not want to exclude any of the excellent manufacturers, nor do we wish to emphasize one brand over another, so the company names and models are deleted.

Summary

The vertical spring stiffness constants of six popular running shoes are reported. The measured average spring stiffness in the heel area for the 2.0 to 2.6 kN range (450-600 lbf.), loaded on a time scale of 0.1 seconds, is close to that for an optimal running track for some of the shoes reported here. Spring constant is found to be independent of plunger area, a useful theoretical and experimental simplification, which can be derived from the basic exponential stress-strain law. In the forefoot area of the shoe, the spring constant is larger than in the heel by a factor of 2 to 3, as shown in Figure 2.

From the theoretical point of view, it is shown that the stress-strain data falls along a straight line on semi-log co-ordinates, Figure 4, indicating an exponential constitutive relation [26]. Vertical stiffness constants are measured for the six shoes in both the heel and ball areas, both for the small and large plunger disks. The independence of plunger area is true in both regions. Thus, using basic equations, we can calculate the compression characteristics and vertical spring constant of a running shoe at any force level. This then is directly applicable to typical physiological forces and stresses experienced during locomotion [20]. Stiffness is found to be proportional to the reciprocal of the sole thickness h^{-1}, a useful simplification.

This report demonstrates that compliance results for a flat indentor are independent of plunger area, an important experimental and theoretical simplification. An exponential force-deflection equation enables calculation of shoe compliance over the entire physiological force range, applicable to the dynamics of walking, jogging, and running.

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