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Estimating the Odds of Relapsing Event

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Abstract

Adopting the life table techniques this paper proposed and presents a statistical method for estimating probabilities and odds that a randomly selected subject or patient in a population is afflicted by spasm of a recurring or relapsing event or disease over a specified time period. Estimates of conditional probabilities that a randomly selected subject would experience and not experienced the relapsing event or illness at some time period given that the same patients has experienced and not experienced an attack by the relapsing event or illness at a previous time period are provided. Also provided are estimates of the number of patients expected to suffer or not to suffer the next attack of the relapsing illness within a specified time period no matter how spasmodic the attacks are. Similarly, provided are estimates of the number of subjects or patients expected to experience and not experience the relapsing event at a subsequent time period given that these same patients had early experienced and not experienced the same relapsing event or disease at a previous time period no matter short these time period. The proposed method is illustrated with some hypothetical data under the assumption that the relapsing event or disease has a known and given instantaneous attack rate in the population.

Keywords: Relapsing events; Odds ratio; Time period; Probability

Introduction

The dynamic model of relapse assumes that relapse can take the form of sudden and unexpected returns to the target behaviour. This concurs not only with clinical observations, but also with contemporary learning models stipulating that recently modified behaviour is inherently unstable and easily swayed by the context [1,2]. Relapse poses a fundamental barrier to the treatment of addictive behaviours by representing the modal outcome of behaviour change efforts [3-6]. Definitions of relapse are varied, ranging from a dichotomous treatment outcome to an on-going, transitional process [7-9]. In clinical settings disengagement from treatment is common, even and perhaps particularly, in the early stages of illness. In recognition of the associated risks, improving medication adherence and relapse prevention have been emphasised as key components of the management of illness. The client's appraisal of relapses also serves as a pivotal intervention point in that these reactions can determine whether a relapse escalates or desists [10,11].

A public health worker may sometimes wish to know the probability and odds that a certain relapsing event like illness such as cancer of a certain site, cardiovascular disease, mental illness, fever, bad habit, etc., may occur and to estimate the number of persons likely to experience such an episodic event. This would better enable the formulation of necessary management and interventionist policies and programs. We will in this paper adopt the life table techniques to develop a method of estimating these probabilities.

Methodology

In life table parlance the probability that a randomly selected individual survives up to age x is $P(x)$ and the probability that the individual survives between ages x and z, that is alive in the age interval (x,z) is $P(x,z)=\frac{P(z)}{P(x)}$ In the sequel but without loss of generality, we will re-designate ages x and z as times x and z.

Now let T be a random variable representing the length of time that elapses before a randomly selected individual or patient in a certain environment experiences another attack of a recurring or relapsing event such as illness. That is T represents the length of time the randomly selected patient lives or survives before the next attack of the illness. Then the probability that T assumes the value x, that is the probability that the patient survives up to time x before the next attack by the relapsing affliction is:

$$P(T \le x) = P(x) \tag{1}$$

The probability that the patient does not experience the next attack of the relapsing illness in the time interval (x,z) is:

$$P(x<T<z)=P(x,z)=\frac{P(z)}{P(x)} \tag{2}$$

Hence the probability that the patient experiences the next attack of the relapsing illness in the time interval (x,z) is:

$$q(x,z)=1-\frac{P(z)}{P(x)}=\frac{P(x)-P(z)}{P(x)} \tag{3}$$

Therefore the odds that a randomly selected patient experiences the next attack of the relapsing illness in the time interval (x,z) is:

$$\eta_a(x,z)=\frac{q(x,z)}{P(x,z)}=\frac{P(x)-P(z)}{P(z)} \tag{4}$$

Similarly the odds of next attack in the time interval (v,w) is:

$$\eta_a(v,w)=\frac{q(v,w)}{P(v,w)}=\frac{P(v)-P(w)}{P(w)} \tag{4a}$$

Hence the resulting odds ratio is:

$$\omega=\frac{\eta_a(v,w)}{\eta_a(x,z)}=\frac{(P(v)-P(w))P(z)}{(P(x)-P(z))P(w)} \tag{4b}$$

If the attacks of the relapsing illness are frequent with the result

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that the most recent attack at time z followed within a short period of time by another attack then we may replace z with $x+\Delta x$, where Δx is a short time interval. In this case we have from Equation 2 that the probability that a randomly selected patient does not have the next attack of relapsing illness in the time interval $(x, x+\Delta x)$ is:

$$P(x, x+\Delta x) = \frac{P(x+\Delta x)}{P(x)} \quad (5)$$

And the probability that the patient experiences the illness in this time interval is from equation 3 as:

$$q(x, x+\Delta x) = \frac{P(x) - P(x+\Delta x)}{P(x)} \quad (6)$$

The corresponding odds that the patient experiences the next attack of the illness in the time interval $(x, x+\Delta x)$ is:

$$\eta_a = \frac{q(x, x+\Delta x)}{P(x, x+\Delta x)} = \frac{P(x) - P(x+\Delta x)}{P(x+\Delta x)} \quad (7)$$

If the illness is such that a patient who has just suffered the most recent attack at time x is likely to have suffered the same attack in the recent past and is also likely to experience the same attack soon, then we may replace z with $x+\Delta x$ and z with $(x-\Delta x, x+\Delta x)$. Then the probability of not experiencing and of experiencing the next attack of the relapsing illness in the time interval $(x-\Delta x, x+\Delta x)$ are obtained from equations 2 and 3 respectively as:

$$P(x-\Delta x, x+\Delta x) = \frac{P(x+\Delta x)}{P(x-\Delta x)} \quad (8)$$

and

$$q(x-\Delta x, x+\Delta x) = \frac{P(x-\Delta x) - P(x+\Delta x)}{P(x-\Delta x)} \quad (9)$$

Where, Δx is a short time interval.

Under these circumstances the odds of experiencing the next attack of the relapsing illness in the time interval $(x-\Delta x, x+\Delta x)$ is:

$$\eta_a = \frac{q(x-\Delta x, x+\Delta x)}{P(x-\Delta x, x+\Delta x)} = \frac{P(x-\Delta x) - P(x+\Delta x)}{P(x+\Delta x)} \quad (10)$$

Now suppose that in the interval the number of patients with the relapsing illness in the community is $n(0)$. Then the number of these patients who are not expected to experience the next attack of the relapsing illness before time x is:

$$n(x) = n(0)P(x) \quad (11)$$

The number of patients who are not likely to suffer the next attack in the time interval (x,z) is:

$$n(z) = \frac{n(x)P(z)}{P(x)} \quad (12)$$

Hence the number of patients expected to experience the next attack of the relapsing illness in the time interval (x,z) is:

$$d(x,z) = n(x)\left(1 - \frac{P(z)}{P(x)}\right) = n(x) - n(z) \quad (13)$$

If the relapsing illness is highly spasmodic such that its attack on the patient are very frequent then x and z may be replaced by $x-\Delta x$ and $x+\Delta x$ respectively in equations 2 and 3 for Δx approaching 0 to obtain the number of patients expected to experience the next episode of the attack by the relapsing illness during a specified interval of time no matter how small Δx is:

Now supposing that on the basis of some a-priori knowledge, it is believed that the probability that a randomly selected patient does not suffer the next bout of attack by a certain relapsing event such as illness up to time x is:

$$P(x) = 1 - \mu x \quad (14)$$

For $0 \le x \le \frac{1}{\mu}$, where $\mu (0 \le \mu \le 1)$ is the instantaneous attack rate of the illness.

Now using Equation (14) in Equation (2) we have that the probability that a randomly selected patient does not experience the next attack of the relapsing illness in the time interval (x,z) is:

$$P(x,z) = \frac{1 - \mu.z}{1 - \mu.x} \quad (15)$$

And the probability that the randomly selected patient suffers the next attack of the illness in this time interval is, (Equation (3))

$$q(x,z) = \frac{\mu(z-x)}{1 - \mu.x} \quad (16)$$

The resulting odds of the patient experiencing the next attack in the time interval is, from Equation (4)

$$\eta_a = \frac{\mu(z-x)}{1 - \mu.z} \quad (17)$$

The corresponding odds ratio based on our model of Equation (14) is:

$$\omega = \frac{\eta_a(v,w)}{\eta_a(x,z)} = \frac{(w-v)(1-\mu.z)}{(z-x)(1-\mu.w)} \quad (18)$$

If the time period between the most recent attack and the next attack is small, with a length of only Δx time units, then we have from Equations (5) and (6) that the probabilities that a randomly selected patient does not experience and experiences the next attack of the illness during this short time period are respectively.

$$P(x, x+\Delta x) = \frac{1 - \mu(x+\Delta x)}{1 - \mu.x} \quad (19)$$

and

$$q(x, x+\Delta x) = \frac{\mu.\Delta x}{1 - \mu.x} \quad (20)$$

In this situation the corresponding odds is from Equation (7) as:

$$\eta_a = \frac{\mu.\Delta x}{1 - \mu(x+\Delta x)} \quad (21)$$

If furthermore, bouts of successive attacks of the relapsing illness occur within short time periods, we have from Equations (8) and (9) that the probabilities that a randomly selected patient does not suffer and suffers the next attack of the illness in the time interval $(x-\Delta x, x+\Delta x)$ are respectively:

$$P(x-\Delta x, x+\Delta x) = \frac{1 - \mu(x+\Delta x)}{1 - \mu.(x-\Delta x)} \quad (22)$$

and

$$q(x-\Delta x, x+\Delta x) = \frac{2\mu.\Delta x}{1 - \mu.(x-\Delta x)} \quad (23)$$

Therefore the resulting odds of the patient experiencing the next attack of the relapsing illness under these circumstances is from Equation 10.

$$\eta_a = \frac{2\mu.\Delta x}{1-\mu(x+\Delta x)} \quad (24)$$

Following the specifications in Equation (14) we have from Equations (11) and (12) that the number of patients not expected to experience the next attack of the relapsing illness before time x and in the time interval (x,z) are respectively:

$$n(x) = n(0)(1-\mu.x) \quad (25)$$

and

$$n(z) = \frac{n(x)(1-\mu.z)}{1-\mu.x} \quad (26)$$

Therefore the number of patients to expected to experience the next attack of the relapsing illness in the time interval (x,z) is from Equation (13).

$$d(x,z) = \frac{n(x)\mu(z-x)}{1-\mu.x} \quad (27)$$

Note that the probability that a randomly selected patient who survive to the next attack of the relapsing illness or up to time x experiences the next attack before time $z(x<z)$ is:

$$P(z\mid x) = P(T<z\mid T>x) = \frac{P(x<T<z)}{P(T>z)} \quad (28)$$

or

$$q(z\mid x) = \frac{P(z)\mid P(x)}{1-P(x)} = \frac{P(z)}{P(x)}.q(x) \quad (29)$$

The probability that this randomly selected patient does not also experience the next attack of the illness before time Z is:

$$q(z) = p(T>z \mid T>x) = 1- P(T<z \mid T>x) \quad (30)$$

or

$$q(z) = 1-\frac{P(z)}{P(x)}.q(x) \quad (31)$$

Therefore the number of patients who have not suffered the next attack of the illness before the time z but experiences the attack before time z is:

$$n(z\mid x) = \frac{n(x)P(z)}{P(x)q(x)} \quad (32)$$

And the number of patients not expected to experience the attack by the relapsing illness before time z given that they have not experienced the same illness before time x is:

$$n(z) = \frac{n(x)(1-P(z))}{P(x).q(x)} = n(x)-n(z\mid x) \quad (33)$$

Using these results in Equation (29) we have that

$$q(z\mid x) = \frac{1-\mu.z}{\mu.x(1-\mu.x)} \quad (34)$$

And from Equation (31) we have that

$$P(z) = 1-\frac{1-\mu.z}{\mu.x(1-\mu.x)} \quad (35)$$

From Equation (32), we have that the number of patients who survived the next attack of the illness up to age z but are likely to experience the next attack before the time $z(x < z)$

$$n(z\mid x) = n(x)\frac{(1-\mu.z)}{\mu.x(1-\mu.x)} \quad (36)$$

From Equation (34) we have that the number of patients who survive the next attack by the relapsing illness up to time z and also survive up to time $z$is:

$$n(z) = n(x)\left(1-\frac{1-\mu.z}{\mu.x(1-\mu.x)}\right) = n(x)-n(z\mid x) \quad (37)$$

As noted above if the attacks by the relapsing illness occur in rapid successions then z and z may be replaced by x-Δx and x+Δx in the above equations to obtain the required probabilities, odds and expected number of patients under these circumstances.

Ilustration

We here illustrate some of these results with numbers assuming that the relapsing illness is relatively spasmodic and virulent. Thus suppose a randomly selected patient is likely to experience attacks of s relapsing illness once every one month and that the instantaneous attack rate of the illness is $\mu = 0.20$ also we assume that initially $n(0) = 100$ individuals or patients in a certain community are afflicted with the relapsing illness. Then from equation 14 we have that

$P(x) = 1\text{-}0.20x$

We here estimate the probabilities and odds that a patient attacked by the illness in the third month of the year (x=3) is likely to have been attacked in the second month of the year ($x - \Delta x = 2$) and is also likely to be attacked in the fourth month of the year ($x + \Delta x = 4$) so that Δx = 1 month.

Under these conditions we have that the probability that a randomly selected patient is not attacked by the relapsing illness between the second and the fourth month of the year is from equation 8, that is P(3 – 1; 3 + 1)

$$= P(2,4) = \frac{P(3+1)}{P(3-1)} = \frac{P(4)}{P(2)}$$

Or $P(2,4)\frac{1-(0.20)(4)}{1-(0.20)(2)} = \frac{0.20}{0.6} = 0.333$

And from Equation (9) the probability that the patient experiences the relapsing illness in the time interval (2, 4) that is between the month of February and April of the year inclusively is:

$q(2,4) = 1\text{-}0.333 = 0.667$

Hence the odds that the patient experiences the next attack of the relapsing illness in the time interval (2, 4) months is from Equation (24) as:

$$= \frac{2(0.20)(1)}{1-(0.20)(4)}\frac{0.40}{0.20} = 2$$

Note that the same value of the odds is obtained when calculated directly from the definition thus

$$\eta_a = \frac{q(2,4)}{P(2,4)} = \frac{0.667}{0.333} = 2.003 = 2$$

Approximately, from Equation (36), we have that the probability that a randomly selected patient who survives the next attack of illness up to time $x = 2$ (February) experienced the next attack before time z = 4 (April) is

$$q(z\mid x) = \frac{1-(0.20)(4)}{(0.20)(2)(1-(0.20)(2))} = \frac{0.20}{(0.40)(0.60)} = \frac{0.20}{0.24} = 0.833$$

From Equation (35) we have that the probability that a randomly selected patient does not suffer the next attack of illness before both times $x = 2$ (February) and $z = 4$ (April) is:

$P(z,x) = 1\text{-}0.833 = 0.167$

The number of patients not expected to experience the next bout of the relapsing illness before time $x = 2$ but experiences it before time $z = 4$ (April) is:

$n(z \mid x) = 60(0.833)$ or 50 patients.

Hence from Equation (37) we have that the number of patients who are not expected to experience the next bout of attack of relapsing illness is:

$n(z) = 60 - 50 = 10$ patients

The number of patients not expected to experience the next attack of the relapsing illness in the time interval (2,4) is from Equation (12):

$n(4) = 60(0.333) = 19$ or 20

And from Equation (13) the number of patients expected to experience the next attack of the relapsing illness during the time interval (2,4) that is between February and April of the year inclusively is:

$d(2,4) = 60(0.667) = 40.02 = 40$ Patients

To evaluate the odds ration when $(x,z) = (1,2)$, $(v,w) = (3.4)$ and $\mu = 0.20$ we have that

$$\omega = \frac{\eta_a(3,4)}{\eta_a(1,2)} = \frac{1-(0.20)(2)}{1(0.20)(4)} - = \frac{1-0.40}{1-0,80} = \frac{0.60}{0.20} = 3.00$$

Summary and Conclusion

We have in this paper proposed and presented a statistical model for the estimation of the probabilities and odds that a randomly selected patient experiences the next attack of a recurring or relapsing illness over a space of time. Also provided are estimates of the number of patients to suffer or not to suffer the next attack of the relapsing illness within a specified time period no matter how spasmodic the attack is.

Estimates of conditional probabilities that a randomly selected subject would experience and not experience the relapsing event or illness at some time period given that the same patient has experienced or has not experienced an attack by a relapsing event or illness at a previous time period were provided as well as the estimates of the corresponding number of subjects expected to experience a relapsing event under these circumstances

The proposed model is illustrated with some hypothetical examples.

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