

Fertigation Uniformity under Sprinkler Irrigation: Evaluation and Analysis

Zerihun D^{1*}, Sanchez CA², Subramanian J¹, Badaruddin M¹ and Bronson KF³

¹Maricopa Agricultural Center, University of Arizona, Maricopa, USA

²Departments of Soil, Water and Environmental Science and Maricopa Agricultural Center University of Arizona, Maricopa, USA

³USDA-ARS Arid-Land Agricultural Research Center, Maricopa, USA

Abstract

In modern farming systems, fertigation is widely practiced as a cost effective and convenient method for applying soluble fertilizers to crops. Along with efficiency and adequacy, uniformity is an important fertigation performance evaluation criterion. Fertigation uniformity is defined here as a composite parameter consisting of irrigation and fertilizer application uniformity indicators. The field and computational procedures for sprinkler irrigation uniformity evaluation have been the subject of various studies. The objective of the study reported in this paper, however, is the development of an analytical framework for the evaluation and analyses of test-plot scale fertilizer application uniformity under solid-set sprinkler irrigation systems. Irrigation uniformity indices are adapted for use in fertilizer application uniformity evaluation. Fertilizer application rate, given as a function of irrigation depth and fertilizer concentration, is identified as the appropriate variable to express fertilizer application uniformity indices. Pertinent mathematical properties of the uniformity indices along with their practical fertigation management implications are outlined. Carefully designed hypothetical fertigation scenarios were analyzed to examine the significance of the interactive effects, of the local spatial trends of depth and concentration data, on the test-plot scale uniformity of the resultant fertilizer application rate data. The results of the study show that the spatial overlap patterns between depth and concentration data sets are the main determinants of test-plot scale fertilizer application rate uniformity. The study also shows that often the uniformity levels of irrigation and fertilizer concentration data sets cannot be uniquely related to the uniformity of the resultant application rate data. However, some practically useful qualitative relationships between the uniformity of irrigation depth, solute concentration, and application rate data sets are defined. Application of the approach presented here in the evaluation and analysis of fertigation uniformity data sets, measured under sprinkler irrigated conditions, is highlighted.

Keywords: Sprinkler irrigation; Fertigation; Irrigation uniformity; Fertilizer application rate; Application rate uniformity

Notations

c_k = Concentration of nitrogen fertilizer in the k th rain gage of the test-plot [M/L^3];

d_k = Irrigation depth in the k th rain gage of the test-plot [L];

DU_{lq} = Test-plot scale low-quarter distribution uniformity [-];

k = Rain gage index;

K = The number of rain gages in a test-plot;

UCC = Test-plot scale Christiansen's uniformity coefficient [-];

x_k = Irrigation depth, nitrogen concentration, or nitrogen application rate in the k th grid square of the test-plot ($[L]$, $[M/L^3]$, or $[M/L^2]$);

x_{av} = Average depth, concentration, or application rate in a test-plot ($[L]$, $[M/L^3]$, or $[M/L^2]$);

Introduction

In modern farming systems, soluble fertilizers, such as inorganic sources of nitrogen, are commonly applied to crops through fertigation. Compared to conventional fertilizer application methods, fertigation presents a number of potential advantages. It allows a more precise matching of available soil fertilizer content with crop needs through the season [1-3]. In addition, reduced soil compaction, crop damage, and energy and labor costs are also cited as some of the benefits of fertigation [2,4,5]. The additional investment in fertilizer injection and safety equipment are some of the disadvantages of fertigation [1,6]. However, the wide spread use of fertigation with sprinkler systems suggest that the advantages of fertigation far outweigh the disadvantages. Sound system design and management aimed at

maximizing fertigation performance is a key to the realization of these benefits of fertigation. The irrigation method considered in this study is solid-set sprinkler systems. However, the analytical framework described here, for fertigation uniformity evaluation, can be readily adapted to other sprinkler irrigation systems.

An important fertigation performance indicator along with that of efficiency and adequacy is uniformity [7]. In the context of solid-set sprinkler systems, the practical significance of uniformity as a performance criterion stems from the fact that high uniformity is a requirement for the attainment of adequate and efficient fertigation [2,8]. Moreover, uniformity indices are generally considered as indirect indicators of the potential for soil water deficit, deep percolation losses, and nutrient leaching and groundwater pollution from fertigation.

Considering that fertigation is a process that applies both water and fertilizer to crops, it is evident that fertigation uniformity evaluation requires the use of a composite parameter consisting of irrigation and fertilizer application uniformity indicators. Because of its practical significance, and to a certain extent due to the relative simplicity of the required measurement and computational procedure, irrigation uniformity is the performance index that has been most commonly evaluated based on field measurements [8-11]. The factors and physical

***Corresponding author:** Zerihun D, Maricopa Agricultural Center, University of Arizona, Maricopa, USA, Tel: 520-374 6380; Fax No: 520-374-6394; E-mail: dawit@ag.arizona.edu

Received December 16, 2016; **Accepted** February 01, 2017; **Published** February 06, 2017

Citation: Zerihun D, Sanchez CA, Subramanian J, Badaruddin M, Bronson KF (2017) Fertigation Uniformity under Sprinkler Irrigation: Evaluation and Analysis. Irrigat Drainage Sys Eng 6: 177. doi: [10.4172/2168-9768.1000177](https://doi.org/10.4172/2168-9768.1000177)

Copyright: © 2017 Zerihun D, et al. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

mechanisms affecting sprinkler irrigation uniformity and the field and computational methods for evaluating it were examined by various authors [12-21]. The objective of the study reported here, however, is the development of an analytical framework for the evaluation and analyses of test-plot scale fertilizer application uniformity under solid-set sprinkler irrigation systems.

In the study presented here, irrigation uniformity equations are adapted for use in fertilizer application uniformity evaluation. Fertilizer application rate is identified as the appropriate variable for expressing fertilizer application uniformity indices. The mathematical properties of the uniformity equations along with their practical implications are described. Fertilizer application rate uniformity is shown to be a function of the interactive effects of the local spatial trends of the corresponding irrigation depth and solute concentration data sets. The study has also shown that often fertilizer application rate uniformity cannot be uniquely related to the uniformity of irrigation and solute concentration. However, some practically useful qualitative relationships between the uniformity of irrigation depth, solute concentration, and application rate data sets are defined. Application of the approach presented here in the evaluation and analysis of fertigation uniformity data sets, measured under sprinkler irrigated conditions, is highlighted.

Fertigation Uniformity, Pertinent Variables and Spatial Scale

Fertigation uniformity indicators

During fertigation, solute concentration may vary spatially through a sprinkler hydraulic network and temporally during the course of a fertigation event. Hence, fertilizer application uniformity cannot be automatically deduced from irrigation uniformity. The implication is that fertigation uniformity is a composite parameter consisting of irrigation and fertilizer application uniformity indicators. Accordingly, throughout this manuscript irrigation and fertilizer application uniformity indices are treated as two distinct, nonetheless, related and equally important aspects of sprinkler fertigation uniformity.

Variables for fertigation uniformity evaluation

Agricultural inputs for crop production, including irrigation and fertilizers, are typically expressed in terms of application rates: volume or mass of the input per unit area of cropland. Accordingly, sprinkler irrigation uniformity is often defined as a function of irrigation depths. For sprinkler applications, the equivalent variable, to irrigation depth, for expressing fertilizer application uniformity is a mass of fertilizer per unit area of field (e.g., gram per square meter), a variable commonly referred to as fertilizer application rate. The mass of fertilizer in irrigation water cannot be measured directly. Hence, fertilizer application rates need to be computed as a function of the directly measureable physical quantities of concentration and irrigation depth.

Spatial scale for uniformity evaluation

The basic field unit for solid-set sprinkler fertigation system uniformity evaluation is a test-plot. Typically, a uniformity evaluation test-plot consists of a rectangular area with dimensions equal to the sprinkler spacing along laterals and the lateral spacing. Rain gages arranged in a grid pattern, with suitably selected spacing, are used to measure sprinkler precipitation depths and fertilizer concentrations over the test-plot. The data collected as such is then used to calculate the test-plot scale fertigation (i.e., irrigation and fertilizer application rate) uniformity indices. Many of the factors affecting uniformity

(including system hydraulics, setting, and maintenance) can be spatially variable, as a result test-plot scale fertigation uniformity may not be representative of field-scale uniformity. A realistic evaluation of field-scale fertigation uniformity may, therefore, require the use of more than one test-plots suitably distributed over the field. In which case, the test-plots can be considered as sampling points of the field-wide variability of irrigation depth and fertilizer application rates. Test-plot scale uniformity indices can then be scaled-up to field-level with an appropriate procedure. A simple approach for deducing field-scale fertigation uniformity from test-plot scale measurements is described in reference [8]. In this paper, however, discussion on fertigation uniformity is limited to test-plot scale evaluations.

Fertigation Uniformity Equations, Properties and Practical Implications

Uniformity can be considered as a measure of the spatial variability inherent in a data set. The practical significance of uniformity as a fertigation performance index is less intuitive than those of the application efficiency and adequacy indices. Nonetheless, in sprinkler applications high uniformity is a necessary condition for adequate and efficient fertigation.

Various indices have been proposed for use as irrigation uniformity metrics [19-23]. Some indices are developed specifically for applications in only certain irrigation methods, e.g., emission uniformity [23] and design uniformity coefficient [22] for trickle irrigation systems. In principle, the uniformity index [20], which uses the coefficient of variation of the irrigation depth data to measure variability, has a broader scope of applicability. Nonetheless, it is not commonly used in practice. Christiansen's coefficient of uniformity [21] and the low-quarter distribution uniformity [19] are currently the most widely used irrigation uniformity indices.

Often variability (uniformity) of a data set is expressed with reference to the average. In this paper two standard indices that are designed to measure different aspects of data variability, with respect to the mean value, are used to quantify fertigation (i.e., irrigation and fertilizer application rate) uniformity: Christiansen's uniformity coefficient, UCC [-], and the low-quarter distribution uniformity, DULq [-]. Although these indices are customarily used to evaluate irrigation uniformity [9-12], there is no limitation as regards their application to quantifying the spatial variability of any agricultural input applied with irrigation water.

Fertigation uniformity evaluation test-plots are generally rectangular and are further divided into elemental areas of the same shape and dimension (typically squares because of simplicity and symmetry). Each of the elemental areas are associated with a rain gage. Note that the ratio of the catchment area of the rain gage to the elemental area should be sufficiently large for the measured precipitation depth and concentration to be considered a representative average for the elemental area [15,16]. Forms of the UCC and DULq equations applicable to the conditions described above are presented in the following section.

Christiansen's uniformity coefficient

The equation for test-plot scale Christiansen's uniformity coefficient, UCC [-], of a farm input applied with irrigation water can be given as:

$$UCC = 1.0 - \frac{\sum_{k=1}^K |x_k - x_{av}|}{K x_{av}} \quad (1)$$

where k is rain gage index, K is the number of rain gages in a test-plot, x_k is application rate of a farm input (irrigation or fertilizer) computed based on measurements in the k th rain gage ($[L]$ or $[M/L^2]$), and x_{av} is the arithmetic average application rate for the test-plot ($[L]$ or $[M/L^2]$). Note that in order to maintain consistency with the definition used for fertilizer application rate, we chose here the phrase irrigation application rate in reference to the volume of irrigation per unit field area (irrigation depth). Observe that this is different from the customary usage of the phrase in the irrigation literature, where irrigation application rate refers to irrigation depth applied per unit of time.

Low-quarter distribution uniformity

The equation for test-plot scale low-quarter distribution uniformity, DU_{lq} [-], of a farm input applied with irrigation water is given as:

$$DU_{lq} = \frac{x_{lq}}{x_{av}} \quad (2)$$

where x_{lq} is the arithmetic mean of the lowest quarter of the application rates within the test-plot ($[L]$ or $[M/L^2]$). As will be noted in subsequent discussion Equations 1 and 2 are also used to compute nitrogen concentration uniformity, in which case the variables x_k , x_{lq} , and x_{av} in Equations 1 and 2 will have the dimensions of M/L^3 .

It can be noted from Equation 2 that distribution uniformity is a measure of the significance of extreme negative deviations from the average application rate. Different forms of distribution uniformity (e.g., distribution uniformity based on the minimum or lower-half of the application rate data) are commonly used, each assigning different levels of stringency to the definition of what constitutes extreme negative deviations from the average. However, the low-quarter distribution uniformity, DU_{lq} , is used here, because it has been widely applied in irrigation uniformity evaluations. The Christiansen's coefficient of uniformity, UCC , on the other hand, can be viewed as an index designed to measure the test-plot scale data variability from the average. Although in this study UCC and DU_{lq} are, the indices, used to evaluate fertigation uniformity, it is important to note that the use of any suitably selected uniformity indices along with, or in place of, these indices is equally valid.

When Equations 1 and 2 are used to quantify fertilizer application uniformity, the variable x_k represents fertilizer application rate, which can be computed as the product of fertilizer concentration in irrigation water, c_k $[M/L^3]$, and irrigation depth, d_k $[L]$:

$$x_k = c_k d_k \quad (3)$$

Properties of the fertigation uniformity equations

Although more general forms of the uniformity equations that are not limited by the shape of the test-plot or the shape and dimensions of the elemental areas constituting the test-plot can be formulated [8], Equations 1 and 2 are the most commonly used forms. The following is a list of the properties of Equations 1-3 and their practical computational implications.

(1) Considering test-plot scale irrigation depth or fertilizer application rate data, UCC and DU_{lq} indices remain unaffected if each element of the data set is multiplied by a constant.

The implication is that the volume of precipitation collected in rain gages, instead of depth, can be used directly to compute irrigation uniformity. Note that this is especially convenient if rain gages graduated in volumetric units are used in fertigation uniformity evaluation.

Likewise, the mass of fertilizer in the rain gages, instead of fertilizer application rates, can be used to calculate application rate uniformity, if the spatial distribution of fertilizer is expressed as such. This property also implies that the uniformity of a data set remains unchanged if the data is normalized with a suitably selected characteristic variable.

(2) If the fertilizer concentration over a test-plot is constant, the fertilizer application rate uniformity will be equal to irrigation uniformity. Observe that this is a corollary to the property stated above.

In such a scenario, the problem of fertigation uniformity evaluation reduces to that of irrigation uniformity evaluation. In practice this scenario can be approximated in a sprinkler system in which the effect of solute transport processes on the spatial distribution of fertilizer concentration is limited and fertilizer concentration at the system inlet is nearly constant throughout the duration of irrigation.

(3) Test-plot scale UCC and DU_{lq} are independent of the spatial distribution of the application rate data points within a test-plot.

This implies that two test-plots with the same number of data points, but different spatial distributions of application rate data, can have the same UCC and DU_{lq} provided the data sets can be shown to be equivalent after having been sorted separately in ascending/descending order. In other words, the uniformity indices associated with a given irrigation depth or fertilizer application rate data set remain unchanged under any possible spatial permutation of the data. Although the computation of irrigation uniformity or fertilizer application uniformity is independent of the spatial distribution of the data points, it should be noted that the computation of fertilizer application rates from depth and concentration data sets, Equation 3, requires a proper accounting of the spatial distribution of the data points within the test-plot.

(4) Test-plot scale fertilizer application rate uniformity is an aggregate index of the interactive effects of the local spatial trends in the irrigation depth and fertilizer concentration data sets.

This property of the uniformity indices is less intuitive than those described above, but it is key to understanding and defining the factors that affect fertilizer application uniformity and has potential fertigation system design and management implications. Hence, in the subsequent section a combination of simplified hypothetical examples and intuitive mathematical reasoning will be used to show its validity.

The Relationships between Irrigation Depth, Fertilizer Concentration and Application Rate Data Sets

Considering that fertilizer application rate is a multiplicative function of irrigation depth and fertilizer concentration, Equation 3, it can be reasoned that the interactive effects of the spatial trends and scale of variability, inherent in the irrigation depth and the concentration data sets, are the main determinants of the uniformity of the resultant application rate data. In other words, depending on the local monotonic property and scale of variability of the depth data in relation to that of the solute concentration data, the three-dimensional response surface representing the resultant application rate data can get vertically stretched (becomes relatively more variable and less uniform) or it can become more compact (less variable and more uniform) compared to the depth and/or the concentration data sets. The spatial trends of measured irrigation depth and fertilizer concentration data sets and related overlap patterns can show significant local variability over a test-plot. Hence analyses of their interactive effects on the variability of the resultant application rate data need to be based on piece-wise

(local) spatial behaviors of the (depth and concentration) data sets. Important inferences that stem from the preceding observations are:

(1) In parts of a test-plot where the local spatial trends in the irrigation depth data have the same monotonicity as that of the concentration data, the local spatial variability of the resultant application rate data tends to be larger than the variability inherent in both the depth and concentration data sets.

(2) In any given section of a test-plot the relative contributions, of the depth and concentration data sets, to the local variability of the resultant application rate data are proportional to the scale of variability inherent in the depth and concentration data sets.

(3) In parts of a test-plot where the spatial trends in the depth and concentration data sets have opposite monotonicity, the local spatial variability of the resultant application rate data tends to be smaller than that of the depth and/or concentration data set(s).

Note that the term monotonicity is used here, in relation to the spatial trends of depth and concentration data sets, to refer to the mathematical property of the data sets as increasing or decreasing functions of distance. If, for instance, both data sets are locally increasing or locally decreasing functions of distance in some part of the test-plot, then they are described as having same monotonicity there. On the other hand, if depth data is a locally increasing function of distance, whereas the concentration data is a decreasing function of distance or vice-versa, then the functions are considered to be of opposite monotonicity in that part of the test-plot. Note that in subsequent discussion the monotonic properties of depth and concentration data sets are alternatively referred to as spatial trends of depth and/or concentration data set or spatial overlap patterns between depth and concentration data sets. Furthermore, the term local function behavior should imply that a function exhibits a given mathematical property of interest (e.g., monotonicity) in some subset of its domain (which is the test-plot in the current application). Likewise, global function behavior implies that a property of interest spans the entire test-plot.

Furthermore, it is important to note here that the references to scale differences in the spatial variability of depth, concentration, and application rate data sets consider only comparisons between dimensionless depth, concentration, and application rate data sets.

Evidently, actual (field) uniformity evaluations cannot be designed to produce irrigation and concentration (and hence application rate) data sets, each with, a particular spatial pattern. Thus measured fertigation data sets are not well suited for exploring the effects of specific spatial overlap patterns, of depth and concentration, on fertilizer application rate data. Furthermore, measured fertigation data typically show complex local interactions between depth and concentration and hence they are not readily amenable to simple graphical analyses. Therefore the validity of the inferences summarized above is demonstrated here with four pairs of simplified hypothetical examples presented subsequently.

Each hypothetical scenario is designed to examine the comparative significance of the effects, of test-plot scale uniformities and local spatial overlap patterns of depth and concentration, on the resultant application rate uniformity from a different perspective. The first example (section 4.1) presents a relatively simple scenario in which the irrigation and fertilizer concentration data sets have clearly discernible spatial trend that span the entire test-plot. The second example (section 4.2) is a slightly more nuanced case in which the spatial variability patterns of irrigation and concentration data sets are dominated by

local trends. However, the same local overlap patterns, between depth and concentration data sets, are repeated over the entire test-plot. The third example (section 4.3) is similar to the second, but the spatial variability of the depth and concentration data sets are of significantly different scales. Thus it is primarily designed to explore the interactive effects, of scale of variability and overlap patterns between depth and concentration, on the uniformity of the resultant application rate data. The fourth example (section 4.4) presents a relatively more realistic fertigation scenario in which the variability of both the depth and concentration data sets are dominated by local spatial trends, but also the local spatial overlap patterns are not the same over entire the test plot. Note that in many of the hypothetical fertigation scenarios considered the variability in the irrigation and/or concentration data sets is deliberately exaggerated, the objective is to make their interactive effects on the resultant application rate data readily discernible.

Scenario I: Data sets with clearly discernible dominant spatial trend that spans the entire test-plot

Consider an example in which the spatial distribution of irrigation depth and fertilizer concentration each can be approximated by a plane surface. Such an example can be considered as a simplified representation of a measured data set that has clearly discernible global spatial trend (a trend spanning the entire test-plot) with minor perturbations about a best-fit plane. Furthermore, assume that both the depth and the concentration surfaces are level in a direction parallel to one of the horizontal axes (say along the laterals) and have constant slopes along the mainline. The implication is that each data set (irrigation or concentration) is uniform in a direction parallel to the laterals and is variable only along the mainline. It can then be shown that for such a data the test-plot scale uniformity indices have the same value as those computed for a data set on any transect parallel to the mainline. Evidently, for such a data set the spatial pattern along a given transect (parallel to the mainline) will simply be a replication of the pattern along any one of the other transects. This is advantageous, because with this simplification each of the surfaces representing irrigation depth, fertilizer concentration, and application rate can be reduced to a curve with no concomitant loss of pertinent information. In which case, the application rate curve can be superimposed on a graph depicting the corresponding depth and concentration curves. In order to remove scale effects, arising from dimensions and units of measure, and allow a direct comparison between depth, concentration, and application rate, the respective data sets are nondimensionalized. In Figure 1, each of the data sets were normalized by their maximum values, hence they vary in the range 0.0 to 1.0.

Accordingly, a hypothetical example consisting of irrigation depth and fertilizer concentration data sets along a transect, through a test-plot, in a direction parallel to the mainline is depicted in Figure 1a. Both data sets have negative slopes, hence have the same monotonicity through the test-plot. Note that the equations used to generate the data sets presented in Figure 1a are summarized in Table 1. The irrigation depth data has a relatively narrower range of variation (0.27 to 1.0) compared to the fertilizer concentration data set (0.17 to 1.0). The uniformity of the irrigation depth data set is $UCC=0.688$ and $DU_{iq}=0.543$ and that of the solute concentration data set is $UCC=0.610$ and $DU_{iq}=0.429$ (Table 2).

As can be noted from Figure 1a the resultant application rate data, computed with Equation 3, is a decreasing function of distance (hence has the same monotonicity as the depth and concentration data sets), although it is curvilinear. The application rate data set has a wider range of variation (0.05 to 1.0), and hence appreciably lower

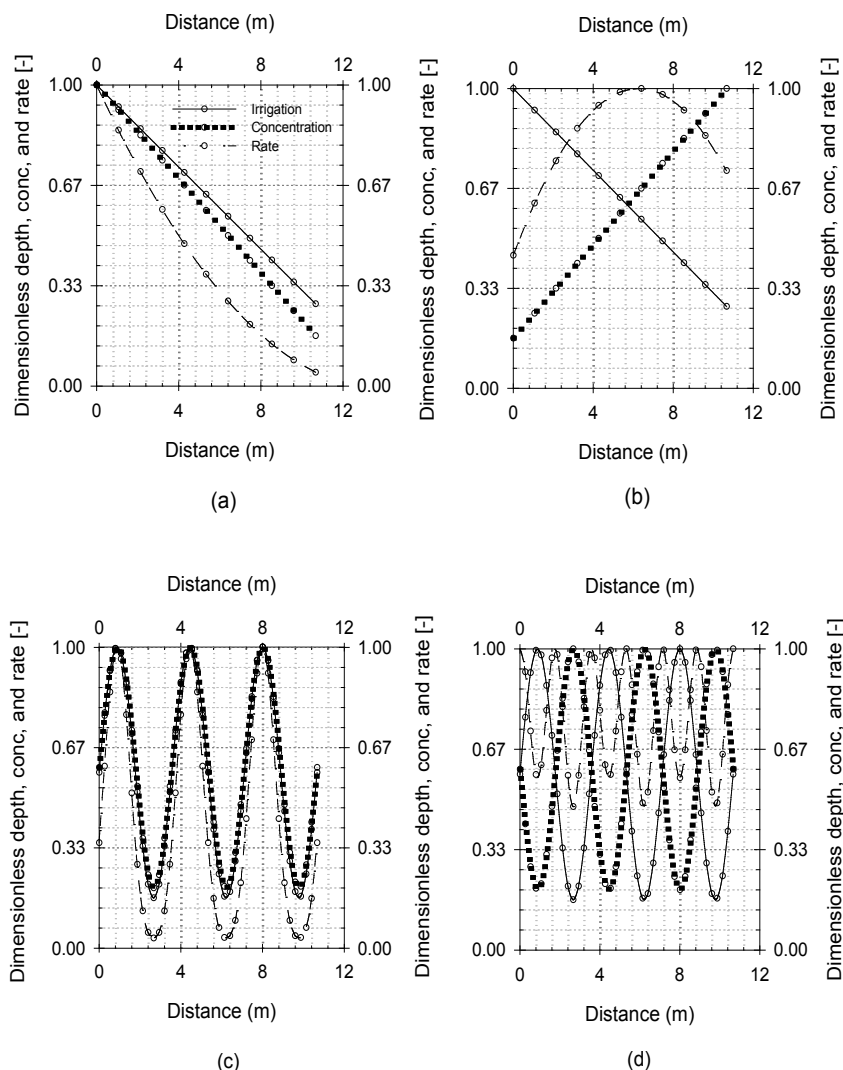


Figure 1: The relationship between spatial trends in irrigation depth, fertilizer concentration, and the spatial variability of the resultant fertilizer application rate. Scenarios where dominant spatial trend spanning the test-plot exists and depth and concentration have: (a) same monotonicity, (b) opposite monotonicity; and Scenarios where local spatial trends dominate and depth and concentration have: (c) same monotonicity, (d) opposite monotonicity.

Data set	Unit	Figures ^(a)	Equations ^(b)	Maximum values
Irrigation depth	mm	1a and 1b	$-1.874x + 27.5$	27.5
		1c, 1d, and 2b	$17.5 + 12.5 \sin(1.77x)$	30.0
		2a, 2c, and 2d	$17.5 + 3.0 \sin(1.77x)$	20.5
Solute concentration	mg/L	1a	$-14.058x + 180.0$	180.0
		1b	$14.058x + 30.0$	180.0
		1c and 2a	$105.0 + 70.0 \sin(1.77x)$	175.0
		1d	$105.0 + 17.0 \sin(1.77x + \pi)$	175.0
		2b	$105.0 + 17.0 \sin(1.77x + \pi)$	122.0
		2c	$105.0 + 15.0 \sin\left(1.77x + \frac{\pi}{2}\right)$	120.0
		2d	$105.0 + 15.0 \sin\left(1.77x + \frac{2}{3}\pi\right)$	120.0

^(a)The spatial spacing between data points in Figures 1a and 1b is 1.067m and in Figures 1c and 1d and 2a-2d is 0.267m, the length of the test-plot along the mainline is 10.67m.

^(b)The equations are in dimensional form, they were normalized by dividing them with their respective maximum values and x=distance in meter.

Table 1: Functions used to define irrigation, fertilizer concentration, and application rate curves in Figures 1 and 2.

Scenario	Uniformity indices	Spatial trends of irrigation depth data in relation to fertilizer concentration data					
		Same monotonicity		Rate	Opposite monotonicity		Rate
		Irrigation	Concentration		Irrigation	Concentration	
I	UCC	0.688	0.610	0.384	-	-	-
	DU_{iq}	0.543	0.429	0.209	-	-	-
	UCC	-	-	-	0.688	0.610	0.832
	DU_{iq}	-	-	-	0.543	0.429	0.733
II	UCC	0.557	0.587	0.291	-	-	-
	DU_{iq}	0.358	0.401	0.120	-	-	-
	UCC	-	-	-	0.557	0.587	0.797
	DU_{iq}	-	-	-	0.358	0.401	0.703
III	UCC	0.894	0.587	0.504	-	-	-
	DU_{iq}	0.846	0.401	0.322	-	-	-
	UCC	-	-	-	0.557	0.900	0.633
	DU_{iq}	-	-	-	0.358	0.855	0.434
		Variable monotonicity		Rate	Variable monotonicity		Rate
		Irrigation	Concentration		Irrigation	Concentration	
IV	UCC	0.894	0.908	0.858	-	-	-
	DU_{iq}	0.846	0.869	0.804	-	-	-
	UCC	-	-	-	0.894	0.908	0.897
	DU_{iq}	-	-	-	0.846	0.869	0.860

Table 2: A summary of computed uniformity indices for irrigation depth, fertilizer concentration, and application rate for the hypothetical examples presented in Figures 1 and 2.

uniformity ($UCC=0.384$ and $DU_{iq}=0.209$), compared to both the depth and concentration data sets (Table 2). Note that this result is consistent with the inference stated above in regard to the spatial variability of an application rate data derived from irrigation depth and fertilizer concentration data sets that are of same monotonicity.

Figure 1b presents an example in which both the depth and concentration data sets have a global spatial trend, but in contrast to the preceding example they have opposite monotonicity. The irrigation depth data in Figure 1b is exactly the same as that presented in Figure 1a. The concentration curve, on the other hand, is obtained from that presented in Figure 1a as follows: (i) the slope is set equal to -1.0 multiplied by the slope of the concentration curve in Figure 1a and (ii) the intercept on the vertical axis is set to the lower limit of the data set (Table 1). As a result, the fertilizer concentration data is an increasing function of distance and has opposite monotonicity with respect to the irrigation depth data (Figure 1b). However, it can be noted that its uniformity remains unchanged as in Figure 1a (Table 2).

In contrast to the preceding example (Figure 1a), the resultant fertilizer application rate curve is not a monotonic increasing function of distance, instead it is increasing in the range 0.0 m to about 6.5 m and is decreasing thereafter (Figure 1b). As a result, the application rate data vary in a much narrower range (0.44 to 1.0) compared to the depth and concentration data sets. The computed fertilizer application rate data also has a considerably higher uniformity ($UCC=0.852$ and $DU_{iq}=0.733$) than the corresponding depth and concentration data sets (Table 2). Note that these results are consistent with the inferences stated above in regard to the spatial uniformity of an application rate data derived from irrigation depth and concentration data sets that are of opposite monotonicity.

A more subtle and interesting point, that follows directly from the above general deductions and is revealed by this example, is that depending on the spatial overlap patterns between depth and concentration substantially different application rate uniformity levels (fertigation scenarios) can be obtained from depth and concentration data sets of given uniformity. This suggests that it is the spatial overlap patterns between depth and concentration data sets, and not their

uniformity levels, that determine the uniformity of the resultant application rate data. A more specific observation of interest here is that a combination of depth and concentration data sets, both with high spatial variability (low uniformity), does not necessarily lead to a low application rate uniformity.

Note that in the current as well as in each of the subsequent examples, different fertigation scenarios are derived from a given pair of depth and concentration data sets through spatial rearrangement of the data points. As can be noted from the properties of the uniformity indices, such a spatial reordering of the data points would leave the uniformity of the irrigation depth and fertilizer concentration data sets unchanged. However, it leads to different spatial overlap patterns between the depth and concentration data sets and hence results in different fertilizer application uniformity levels. It needs to be emphasized that the goal here is only to produce sharply contrasting fertigation scenarios in which the comparative significance of the effects, of test-plot scale uniformities and local spatial overlap patterns of depth and concentration, on application rate uniformity are clearly discernible.

Scenario II: Data sets with dominant local spatial trends in which depth and concentration have either same or opposite monotonicity throughout the test-plot

In this section a second pair of simplified hypothetical examples is presented. These examples are designed to show that the general inferences stated above apply to conditions in which the spatial variability in the irrigation depth and fertilizer concentration data sets are dominated by local trends. Here we consider irrigation depth and fertilizer concentration data sets that follow sinusoidal patterns of variation with distance along the mainline and have no gradient in a direction perpendicular to the mainline. It can then be shown that the uniformity indices computed for a data set along any transect through the test-plot in a direction parallel to the mainline is equal to the test-plot scale uniformity indices. The three-dimensional response surfaces of depth, concentration, and application rate each can then be reduced to their equivalent two-dimensional counter parts along a transect, all of which can be presented on a single graph. As discussed above in order to remove scale effects and allow a direct comparison between

depth, concentration, and application rate data, the respective data sets are presented in dimensionless graphs.

Accordingly, Figure 1c depicts a scenario in which both irrigation depth and fertilizer concentration are sinusoidal functions of distance (Table 1). The depth and concentration data sets vary in the range 0.17 to 1.0 and 0.2 to 1.0, respectively. It can be noted that in any given segment of the transect, these data sets exhibit comparable scale of variability and are of the same monotonicity. The resultant application rate data acquires the local monotonic properties of the corresponding depth and concentration data sets, but with a larger variability than both data sets (Figure 1c). The range of variation of the application rate data is 0.97, compared to 0.83 and 0.80 for the depth and concentration data sets, respectively. More importantly, the uniformity of fertilizer application rate ($UCC=0.291$ and $DU_{iq}=0.120$) is significantly lower than the corresponding irrigation uniformity ($UCC=0.557$ and $DU_{iq}=0.358$) and concentration uniformity ($UCC=0.587$ and $DU_{iq}=0.401$), Table 2. These results show that the inferences stated above on the spatial variability of the fertilizer application rate data is also applicable to scenarios in which local trends are dominant.

Figure 1d depicts a scenario in which the irrigation and concentration data sets have same uniformity and range of variation as those presented in Figure 1c, but they have opposite monotonicity in any given segment of the test-plot (Table 1). Mathematically this is accomplished by introducing a phase shift of π radian to the concentration curve with respect to the irrigation depth curve, while keeping the depth curve the same as in Figure 1c. Note that this is equivalent to moving the sinusoidal curve for concentration to the left through an angular distance of π radian, which (in the examples presented here) is equivalent to 1.77 m in linear distance. Observe that the concentration data set, in Figure 1d, is a mirror image of the data in Figure 1c about the horizontal line of symmetry. Hence, the transformation of the concentration data (through the introduction of a phase shift) from that given in Figure 1c to that given in Figure 1d in practice involves a simple spatial rearrangement of the same data set. From the property of the uniformity equations it can be noted that such a transformation leaves the uniformity of the data set unchanged at the level of Figure 1c.

From Figure 1d it can be noted that the resultant application rate data set computed with Equation 3 has a much lower range of variation (0.47 to 1.0) compared to the corresponding depth and concentration data sets. A practically more useful point, however, is that the computed application rate data here is significantly more uniform ($UCC=0.797$ and $DU_{iq}=0.703$) than the application rate data depicted in Figure 1c (Table 2). Consistent with the inference stated above, these results show that depth and concentration data sets with high local variability can lead to a resultant application rate data with a lower variability (higher uniformity), if they have opposite monotonicity.

Overall the examples presented in Figures 1c and 1d show that given irrigation depth and solute concentration data sets of fixed uniformity and comparable scale of variability, with dominant local spatial trends, substantially different application rate uniformity levels can be obtained depending on whether the depth and concentration data sets are of same or opposite monotonicity.

Scenario III: Data sets with dominant local spatial trends in which the variability of the application rate data is dominated by that of the irrigation or the concentration data set

A third pair of simplified hypothetical examples, summarized in

Figures 2a and 2b, are designed to highlight the fact that the relative contribution of a data set (i.e., depth or concentration) to the variability of the resultant application rate data is proportional to the scale of variability inherent in it. Accordingly, the examples presented here consider combinations of highly uniform irrigation data and highly variable fertilizer concentration data set and vice-versa.

Figure 2a shows an irrigation data set that vary in a relatively narrow range of 0.71 to 1.0 and a concentration data set that spans a much wider range of variation of 0.2 to 1.0. The pertinent sinusoidal functions are given in Table 1. As can be noted from Table 2, with a UCC and DU_{iq} of 0.894 and 0.846, respectively, the irrigation data set can be considered as highly uniform. Comparatively, the uniformity of the concentration data set is significantly lower ($UCC=0.587$ and $DU_{iq}=0.401$). The range of variation of the resultant application rate data set is 0.15 to 1.0 and its UCC and DU_{iq} are 0.504 and 0.322, respectively. The range of variation of the resultant application rate data is wider, and its uniformity is lower, than the depth and concentration data sets. This is consistent with the fact that the depth and concentration data sets are of same monotonicity (Figure 2a). In addition, it can be noted that the application rate data closely tracks the highly variable (non-uniform) concentration data set and its uniformity indices are much closer to those of the concentration data set than the irrigation data. This shows that the highly non-uniform concentration data has a dominant effect on the variability of the resultant application rate data. In contrast, the contribution of the highly uniform irrigation data to the variability of the application rate data is marginal. Note that this result is in agreement with the inferences stated above in regard to the significance of scale of variability and monotonicity of the data sets (i.e., depth and concentration) in terms of their effect on the scale and pattern of variability of the resultant application rate data.

The preceding example considers a scenario in which the depth and concentration data sets are of the same monotonicity. However, it can be readily shown that the essential result holds even when the depth and concentration data sets are of opposite monotonicity. The only difference is that with such a case the application rate data would have a slightly higher uniformity than the concentration data set.

Figure 2b depicts a very uniform concentration data set ($UCC=0.900$ and $DU_{iq}=0.855$) and a highly variable irrigation data ($UCC=0.557$ and $DU_{iq}=0.358$) with opposite monotonicity (Table 2). The concentration data set vary in a narrow range of 0.72 to 1.0 compared to the irrigation data, which vary between 0.17 and 1.0. The range of variation of the resultant fertilizer application rate data is 0.23 to 1.0 and its UCC and DU_{iq} are 0.633 and 0.434, respectively. The range of variation of the resultant application rate data and its uniformity indices fall between those of the depth and the concentration data, which is consistent with the fact that the depth and concentration data sets are of opposite monotonicity (Figure 2b). In addition, the resultant application rate curve closely tracks the highly variable irrigation depth data and has a range of variation and uniformity that is comparable to the irrigation data. These results show that the highly non-uniform irrigation data set has a dominant effect on the scale and pattern of variability of the resultant application rate data. By contrast, the contribution of the highly uniform concentration data to the variability of the application rate data is marginal. These observations are consistent with the inferences stated above in regard to the significance of scale of variability and monotonicity of the data sets (i.e., depth and concentration) in terms of their effect on the scale and pattern of variability of the resultant application rate data.

An important practical implication of the results presented in

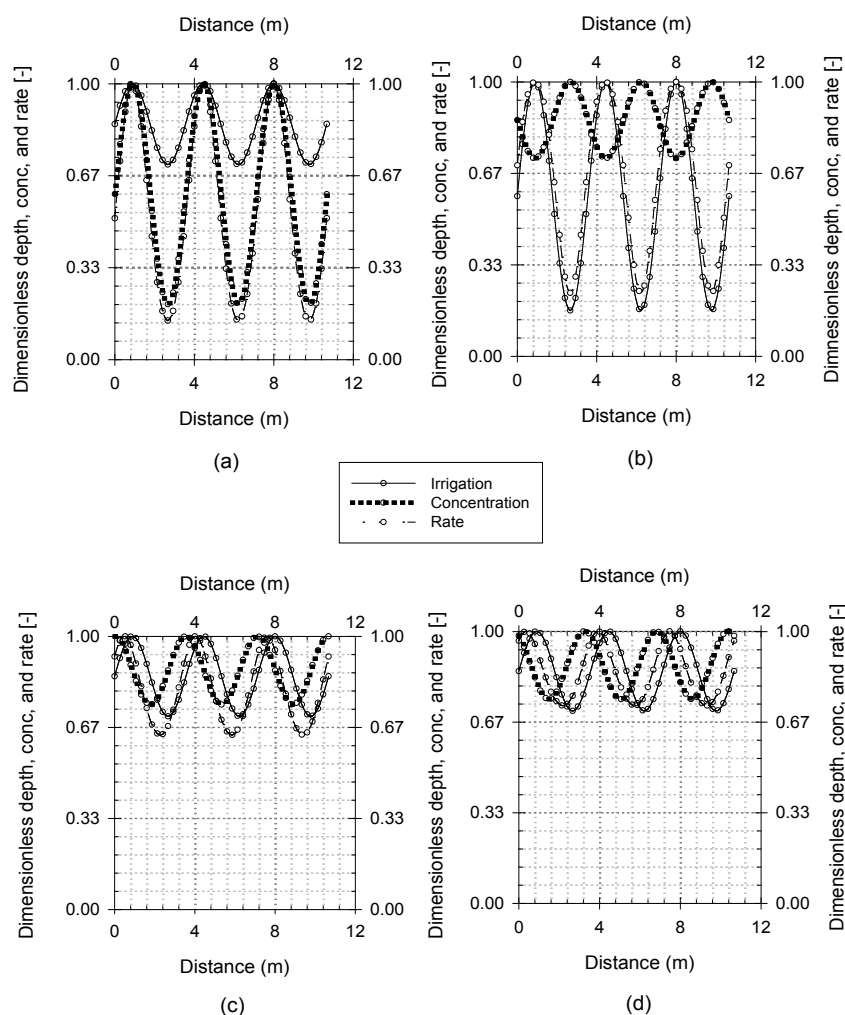


Figure 2: The relationship between spatial trends in irrigation depth, fertilizer concentration, and the spatial variability of the resultant fertilizer application rate. Scenarios where local spatial trends dominate data variability and depth and concentration have: (a) same monotonicity with a combination of high irrigation and low concentration uniformity, (b) opposite monotonicity with a combination of low irrigation and high concentration uniformity, (c) variable spatial overlap patterns over the test-plot with a combination of high irrigation and concentration uniformity, and (d) same as in Figure 2c, but depth and concentration data sets have relatively shorter segments with same monotonicity and longer segments with opposite monotonicity.

Figures 2a and 2b is that a fertigation scenario with high irrigation uniformity and low fertilizer concentration uniformity or vice-versa would likely lead to a low fertilizer application rate uniformity.

Scenario IV: Data sets with dominant local spatial trends in which depth and concentration have same monotonicity in some parts of the test-plot and opposite monotonicity in others

A fourth pair of examples, presented in Figures 2c and 2d, shows a simplified version of a more realistic scenario consisting of variable local spatial overlap patterns. Here we consider irrigation depth and concentration data sets (both following sinusoidal patterns with distance) that have the same monotonicity in some segments of the test-plot and opposite monotonicity in other parts of the test-plot. The goal is to show how the local spatial trends of the resultant application rate curve change as it transitions from a segment of the test-plot where the depth and concentration curves have the same monotonicity to those segments where depth and concentration have opposite monotonicity. In addition, these examples are designed to confirm the fact that it

is the aggregate contribution of these local effects that determine the overall test-plot scale uniformity of the application rate data.

Figure 2c depicts a dimensionless graph of irrigation depth and concentration data sets that follow a sinusoidal pattern superimposed on the resultant application rate curve. The irrigation depth data vary over a slightly wider interval of 0.70 to 1.0 compared to that of the concentration data, which vary in the range 0.75 to 1.0. As can be noted from Table 2, the corresponding irrigation uniformity ($UCC=0.894$ and $DU_{iq}=0.846$) and concentration uniformity (UCC of 0.908 and DU_{iq} of 0.869) can be described as very high. In order to form a mix of test-plot segments, where depth and concentration data sets have same monotonicity in some and opposite in others, a phase shift of 0.5π radian is introduced to the concentration data set with respect to the depth data (Table 1).

Figure 2c shows that the resultant application rate curve reaches its peaks, 1.0, and its lowest points, 0.64, in the intervals where the irrigation depth and concentration data sets have opposite monotonicity. On the other hand, in the segments where depth and concentration data sets

are of the same monotonicity, the application rate data set not only has the same monotonicity as the depth and concentration data sets, but also has steeper slopes (higher variability) than the curves of both data sets. The implication is that in segments of the test-plot where depth and concentration data sets have opposite monotonicity, the overall effect of the interaction of the local spatial trends is to limit the variability in the resultant application rate data. Conversely, in those segments of the test-plot where depth and concentration data sets have same monotonicity, the overall effect of the interaction of the local spatial trends is to enhance the variability in the resultant application rate data. Note that these results are consistent with the inferences stated above in regard to the effects of the local overlap patterns, of the depth and concentration data sets, on the variability of the resultant fertilizer application rates.

Figure 2d depicts the same irrigation depth data as that presented in Figure 2c. However, in order to show the effect of different local overlap patterns on the variability of the resultant application rate data, here the concentration data is shifted by an angular distance of 0.67π radian instead of the 0.5π radian used in Figure 2c (Table 1). Note that this leaves the uniformity of the concentration data unchanged as in Figure 2c. Evidently, the observations noted in the preceding example (Figure 2c), with respect to the local behavior of the application rate data as affected by the overlap patterns of depth and concentration data sets, hold here as well. However, compared to that of Figure 2c, with the current example the intervals over which the depth and concentration curves have opposite monotonicity are slightly longer and the segments over which they exhibit same monotonicity are slightly shorter. The combined effect of which is to reduce the range of variation of the resultant application rate data to between 0.73 and 1.0 compared to that of 0.64 and 1.0 in the preceding example (Figure 2c). The uniformity of the resultant fertilizer application rate data ($UCC=0.897$ and $DU_{iq}=0.860$) as well is higher than that computed for the preceding example ($UCC=0.858$ and $DU_{iq}=0.804$), Figure 2c and Table 2.

The results presented in Figures 2c and 2d confirm, the preceding observation, that the uniformity indices of the depth and concentration data sets do not determine the specific values of the uniformity indices of the resultant application rate data set. Instead it is the interactive effects of the local spatial trends and scales of variability, inherent in the depth and concentration data sets, that are the main determinants of the test-plot scale uniformity of the resultant application rate data set. In other words, test-plot scale application rate uniformity is an aggregate index of the interplay of these local spatial effects over the test-plot.

Although the spatial overlap patterns of the depth and concentration data sets considered in Figures 2c and 2d are different, for both examples the uniformity of the resultant application rate data sets can be described as high. This is related to the very high uniformity of the corresponding depth and concentration data sets and will be discussed in the next section.

Fertigation scenarios that lead to acceptably high fertilizer application rate uniformity

Considering the irrigation depth and fertilizer concentration functions presented in Figures 2c and 2d, it can be noted that a fairly large number of overlap patterns, and hence different resultant application rate functions with specific uniformity levels, can be derived by simply varying the phase shifts between the irrigation and concentration data sets. Evidently, the minimum fertilizer application rate uniformity, that

can be obtained from these depth and concentration functions, should correspond to an overlap pattern in which the functions are completely in phase, (i.e., they are of exactly the same monotonicity) throughout the test-plot. Accordingly, it can be shown that the minimum resultant fertilizer application rate uniformity indices are $UCC=0.807$ and $DU_{iq}=0.729$. Considering, for instance, a fertilizer application rate uniformity acceptability threshold of $UCC=0.75$ and $DU_{iq}=0.7$ (a criteria that closely parallels the irrigation uniformity thresholds recommended for field crops [11]), this minimum uniformity level can be described as acceptably high. The implication here is that, regardless of the overlap patterns, the uniformity of any application rate data set derived from the depth and concentration data sets, given in Figures 2c and 2d, will remain within the range considered acceptably high.

For comparison let us now consider two additional groups of data sets presented in the preceding sections. Considering depth and concentration data sets presented in Figures 1c and 1d, it can be readily observed that the corresponding minimum fertilizer application rate uniformity is well below the uniformity acceptability threshold given above ($UCC=0.291$ and $DU_{iq}=0.120$). Similarly, for data sets depicted in Figures 2a and 2b, the minimum application rate uniformity is significantly lower than the threshold ($UCC=0.504$ and $DU_{iq}=0.322$). Note that the overlap patterns, between the depth and concentration data sets, that correspond to the respective minimum application rate uniformity levels are the same for all these data sets. Thus, the difference between these data sets lies in the variability (uniformity) of their respective depth and concentration data sets. Considering the data sets presented in Figures 2c and 2d, the uniformity indices for both depth and concentration are very high, exceeding the indicated uniformity acceptability thresholds by a minimum of 15.0%. By comparison, the uniformity indices of the depth and concentration data sets depicted in Figures 1c-1d are both well below the uniformity thresholds (Table 2). For the data sets presented in Figures 2a-2b, however, irrigation uniformity is well above the threshold, but the uniformity of fertilizer concentration is below the threshold by a significant margin and as such it has a dominant effect on the uniformity of the resultant application rate uniformity. A useful observation that stems from the preceding discussion is that high uniformity, of both irrigation depth and fertilizer concentration, is a requisite condition for attaining acceptably high fertilizer application rate uniformity.

Summary of significant results

Evidently the analyses presented in the preceding sections are based on simplified hypothetical examples in which functions of the same form are used to define the spatial variations of both irrigation and concentration. Furthermore, the functions considered have uniform, or locally variable yet repetitive, spatial trends and overlap patterns spanning the test-plot. In addition, the hypothetical data sets considered here generally have higher spatial resolution than typical measured data. Nonetheless, the basic mathematical relationship that determines the effects of the interactions between irrigation and concentration data sets on the spatial patterns of the resultant application rate data is the same for both hypothetical and measured fertigation data. The implication is that results of the preceding analyses, which are based on simplified hypothetical scenarios, have relevance to measured fertigation data sets. Accordingly, interesting inferences with potential applications in the evaluation, design, and management of fertigation systems can be deduced on the relationships between irrigation, fertilizer concentration, and application rate:

- (1) The interactive effects of the local spatial trends and scales of

variability, of the irrigation depth and fertilizer concentration data sets, are the main determinants of the uniformity of the resultant application rate data. Thus, test-plot scale application rate uniformity can be considered as an aggregate index of these local effects over the area of the test-plot.

An important practical implication of this observation is that irrigation or fertilizer concentration uniformity alone may not often be adequate to even qualitatively characterize fertilizer application rate uniformity.

(2) The local spatial trends of a fertilizer application rate data are functions of the respective spatial trends of the depth and concentration data sets:

(2a) If irrigation depth and fertilizer concentration data sets have the same local monotonicity in a given section of the test-plot, the resultant application rate data as well will have the same local spatial trend as the depth and concentration data sets;

(2b) If irrigation depth and fertilizer concentration data sets have opposite local monotonicity in a given section of the test-plot and they are of significantly different scale, the resultant application rate data will have the same local spatial trend as either the depth or the concentration data set, whichever has the larger scale of variability; and

(2c) If irrigation depth and fertilizer concentration have opposite local monotonicity in a given section of the test-plot and they are of comparable scale, the local spatial trend of the resultant application rate data may have a larger frequency of spatial variability elements than the underlying depth and concentration data sets and the scale of variability will be less than those of the corresponding depth and concentration data sets (Figure 1d);

(3) The test-plot scale uniformity of a fertilizer application rate data set cannot be predicted based on the uniformity levels of the corresponding irrigation and fertilizer concentration data sets. Note that this inference excludes the unique scenario whereby either depth or concentration or both have perfect uniformity, in which case the local spatial overlap patterns have no effect on the uniformity of the resultant application rate;

(4) The results presented in the preceding sections show that under a special set of conditions a qualitative characterization of the uniformity of the resultant application rate data can be made based on the uniformity of the corresponding depth and concentration data sets, these include:

(4a) Given a fertilizer uniformity acceptability threshold, high uniformity, of both irrigation depth and fertilizer concentration, is a requisite condition for attaining acceptably high fertilizer application rate uniformity.

(4b) A combination of low irrigation and low concentration uniformity may not necessarily lead to low application rate uniformity; and

(4c) A combination of low irrigation uniformity and high concentration uniformity or vice-versa will likely lead to low application rate uniformity.

Applications in Fertigation Uniformity Evaluation

In this section field data sets collected through test-plot scale measurements are presented. The aim is to highlight the practical applications of the results, presented in the preceding sections, in the evaluation and analysis of the uniformity of fertigation data sets.

A concise description of the approach and materials used to measure test-plot scale irrigation depths and concentrations is presented. Computation of the resultant application rate data and the uniformity indices for irrigation, concentration, and application rate with Equations 1-3 is discussed. Finally, the inferences deduced in section 4 are used to explain and analyze the observed relationships between local spatial trends, and test-plot scale uniformities, of measured depth, concentration, and resultant application rate data sets.

Measurements of test-plot scale irrigation depth and fertilizer concentration

As part of a field-scale sprinkler fertigation study a series of fertigation uniformity evaluations were conducted in the Yuma Valley Irrigation Districts of Southwest Arizona in the winter seasons of 2013 and 2014 [8]. Six of the test-plot scale data sets, collected in these evaluations, are presented here. Figure 3 depicts the layout of a uniformity evaluation test-plot used in this study. It covers a rectangular area, circumscribed by four adjacent sprinklers, and measuring 9.14 m × 10.67 m (30.0 ft × 35.0 ft). The test-plot is discretized into 42 grid squares, each measuring 1.524 m × 1.524 m (5 ft × 5 ft). A rain gage is placed in each of the grid squares constituting the test-plot. The rain gages used in these field evaluations were obtained from the Irrigation Training & Research Center, California Polytechnic State University, San Luis Obispo, CA. They have a catchment area of 104.84 cm² and are graduated in 5.0 mL increments up to 100.0 mL volume. For measurements ranging between 100.0 mL and 200.0 mL they are graduated in 25.0 mL increments. The maximum measurable depth with these rain gages is about 19.1 mm with an estimated precision ranging between 0.1 mm and 0.5 mm (computed based on assumed volumetric reading errors ranging between ± 1.0 mL to ± 5.0 mL).

Nitrogen fertilizer was applied in the form of ammonium nitrate. The duration of irrigation in each of the field evaluations was three hours. The duration of nitrogen fertilizer application vary from a third of the irrigation application time to the entire irrigation application time. Further details related to the fertigation studies are presented in Zerihun and Sanchez.

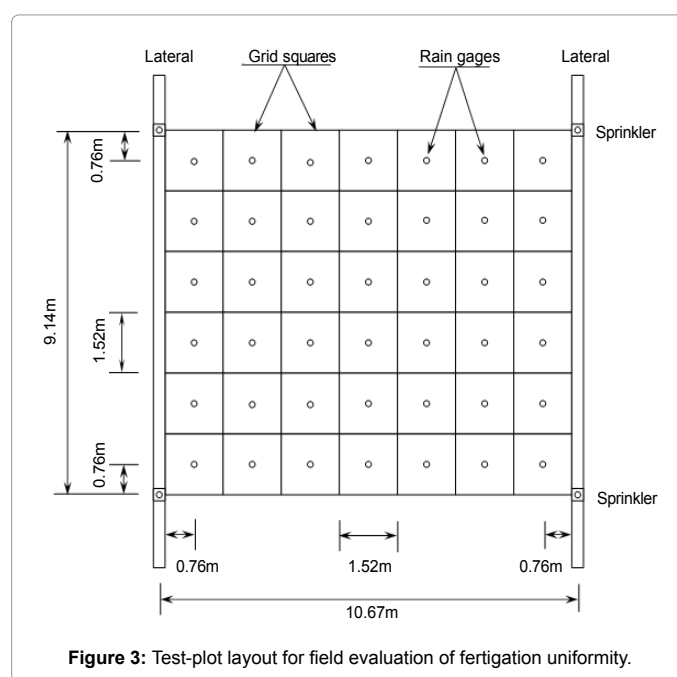


Figure 3: Test-plot layout for field evaluation of fertigation uniformity.

Precipitation depths, collected in the rain gages, were recorded immediately following a fertigation event and are used subsequently to compute test-plot scale irrigation uniformity. Water samples were then collected in appropriately labeled vials from each of the rain gages, which were sealed and frozen shortly after sampling, in order to preserve the integrity of the sample constituents (i.e., mineral nitrogen forms) until laboratory analysis. The total nitrogen concentration here consists of the sum total of elemental nitrogen concentrations present in solution in the form of ammonium- and nitrate-nitrogen. Total ammonium- and nitrate-nitrogen were determined colorimetrically using an Astoria Pacific A2. Ammonium was determined using the indophenol blue method and nitrate was determined using Griess-Ilosvay method after reduction with copperized cadmium [24].

Computation of nitrogen application rates and uniformities

Measured total nitrogen concentrations and irrigation depths were used to compute the resultant application rates with Equation 3. The irrigation depths, nitrogen concentrations, and nitrogen application rates are then used to compute the respective test-plot scale uniformity indices with Equations 1 and 2.

Comparison of the uniformity indices of measured irrigation and nitrogen concentration data sets and the resultant application rate data

Figure 4 presents a comparison of the test-plot scale UCC and DU_{lq} of six field measured irrigation and nitrogen concentration data sets and the corresponding application rate data. For each of the data sets fertilizer application rate uniformity more closely tracks the lower of the irrigation and concentration uniformity levels and often times fall below it (Figures 4a and 4b). Evidently, these results can be explained based on observations made in section 4.3, which states that the data set (i.e., either depth or concentration) with the larger scale of variability (lower uniformity) has a dominant effect on the variability (uniformity) of the resultant application rate data. In addition, the application rate uniformity levels (i.e., the UCC and DU_{lq} values) for data sets 3 and 4 are lower than the concentration uniformity levels by appreciable margins. As can be noted from the discussion in section 4.2, the likely explanation for this is that in a significant fraction of the test-plot area the depth and concentration data for data sets 3 and 4 may have same monotonicity.

Irrigation, nitrogen concentration, and application rate contours

Test-plot scale dimensionless contours of the irrigation, concentration, and application rate data sets for one of the fertigation events, summarized in Figure 4, are presented in this section. The contours are generated with Surfer (Golden Software Inc., Golden, Colorado) using the gridding option of Kriging. The goal is to examine the effects of local spatial trends and overlap patterns, of the measured irrigation and concentration data sets, on the spatial variability of the resultant nitrogen application rate data.

Test-plot scale normalized contours of irrigation depth, nitrogen concentration, and nitrogen application rates for data sets 1 (Figure 4) are shown in Figures 5a-5c. Note that the spatial variability patterns of the nitrogen application rate surface more closely approximate the rather less uniform nitrogen concentration surface than the irrigation depth surface (Figures 5a-5c).

Considering the lower left-hand corner of the test-plot, it can be observed that both irrigation and nitrogen concentration decrease

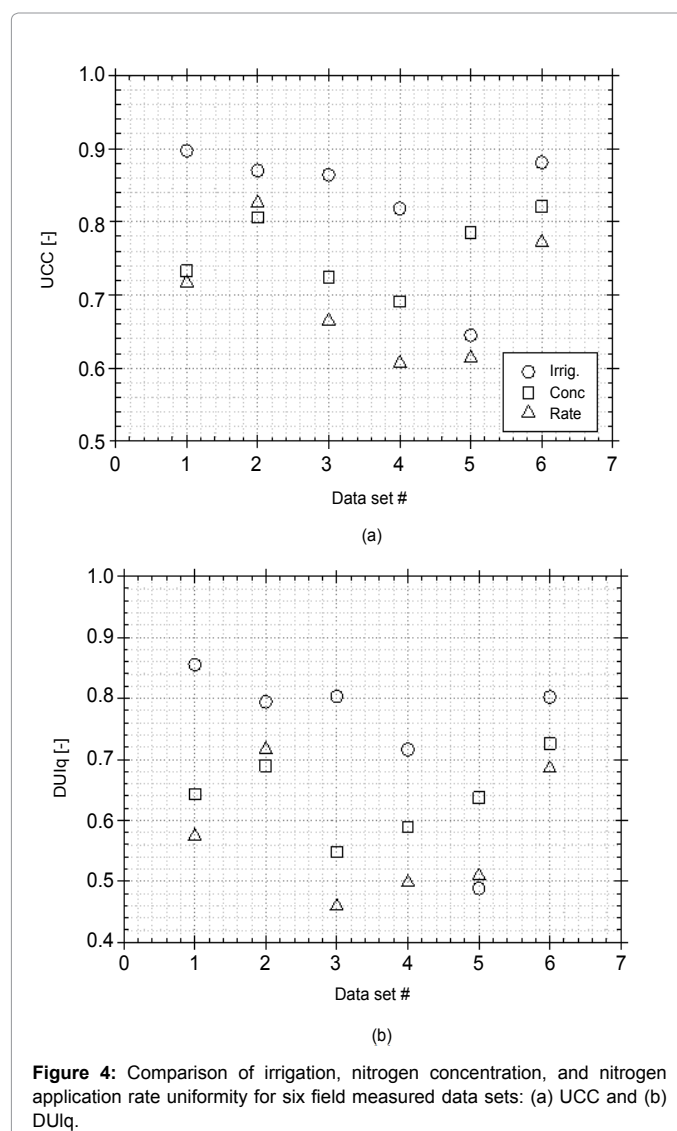


Figure 4: Comparison of irrigation, nitrogen concentration, and nitrogen application rate uniformity for six field measured data sets: (a) UCC and (b) DU_{lq} .

as one moves toward that corner of the test-plot along the direction of the respective arrows (Figures 5a and 5b). Note that the resultant application rate data (Figure 5c) has a similar local spatial trend as the irrigation and nitrogen concentration data sets. Furthermore, a close look at an area of the test-plot that is slightly above the middle section and adjacent to the left edge shows that both the irrigation depth and concentration contours have similar local spatial trends, decreasing in the direction indicated by the arrows (Figures 5a and 5b). Observe that the resultant application rate surface (Figure 5c) as well follows the same general spatial trend as these data sets, but it more closely approximates the relatively more variable concentration surface than that of the irrigation surface. Considering the area slightly above the middle section of the test-plot and adjacent to the right edge, it can be noted that the irrigation and concentration data sets have opposite local spatial trends along the directions indicated by the respective arrows (Figures 5a and 5b). The concentration data set shows a significantly larger local variability than the irrigation data set, as a result the local variability pattern of the application rate data closely approximates that of the concentration data.

Evidently, the local spatial variability and overlap patterns of measured irrigation and concentration data sets are irregular and three

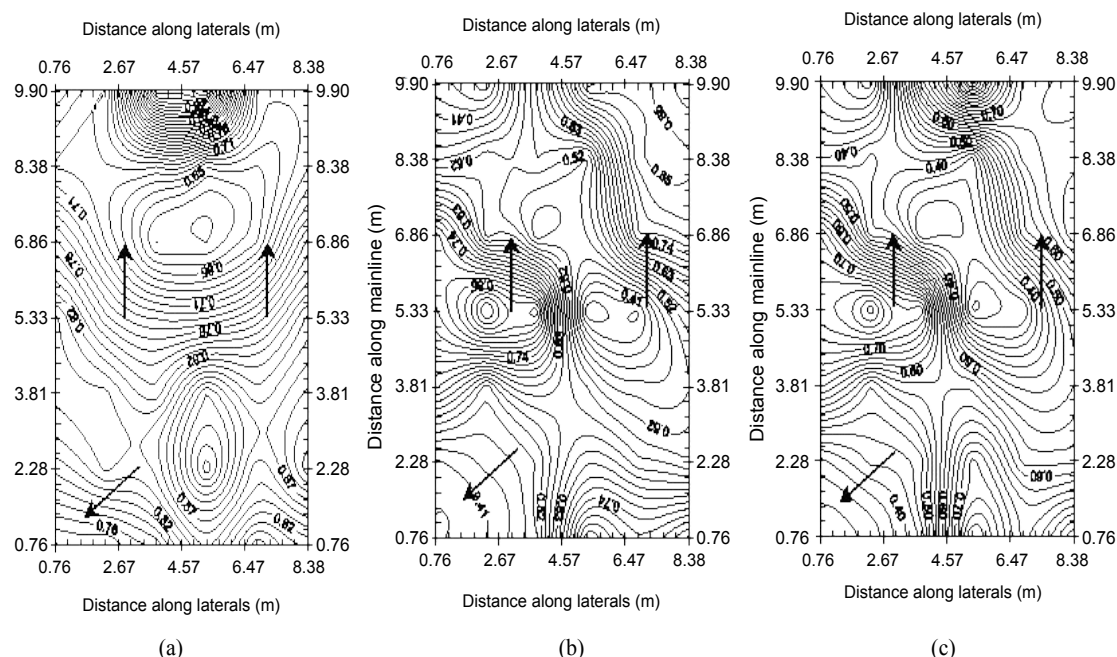


Figure 5: Normalized contours of irrigation, nitrogen concentration, and application rate for data set #1 (Figure 4): (a) irrigation depth, (b) nitrogen concentration, and (c) nitrogen application rate.

dimensional, hence too complex to relate directly to the test-plot scale uniformity of the resultant application rate data. Nonetheless, the results presented above highlight the fact that the local spatial trends and scale of variability of the application rate data (the aggregate effect of which eventually determines the application rate uniformity) are functions of the local overlap patterns and scale of variability of the irrigation and concentration data sets. Note that these observations can be explained by the inferences deduced, in sections 4.1-4.6, based on analyses of hypothetical fertigation scenarios.

Summary and Conclusions

In modern farming systems, fertigation is widely practiced as a convenient and cost effective method for applying soluble fertilizers to crops. Along with efficiency and adequacy, uniformity is an important fertigation system performance criterion. Fertigation uniformity is defined here as a composite parameter consisting of two independent but equally important indices: irrigation and fertilizer application uniformity indicators. The field and computational procedures related to irrigation uniformity evaluation have been studied extensively. Hence, the study reported here focusses on the development of an analytical framework for the evaluation and analyses of fertilizer application uniformity under sprinkler irrigated conditions.

Equations for fertigation uniformity indices are presented. Fertilizer application rate, given as a function of irrigation depth and fertilizer concentration, is identified as the appropriate variable for expressing fertilizer application uniformity indices. Pertinent mathematical properties of the uniformity equations are listed and their practical implications are described. Carefully designed hypothetical examples were analyzed to demonstrate the significance of the effect, of the local spatial overlap patterns between irrigation depth and fertilizer concentration data sets, on the uniformity of the resultant application rate data.

The results of the study show that fertilizer application rate

uniformity is an aggregate index of the interactive effects of the local spatial trends inherent in the irrigation depth and fertilizer concentration data sets. The study also demonstrated that often the uniformity of irrigation and fertilizer concentration cannot be uniquely related to the uniformity of the resultant application rate data set. However, some practically useful qualitative relationships between the uniformity of irrigation depth, solute concentration, and the resultant application rate data sets are presented. Application of the approach presented here in the evaluation and analysis fertigation uniformity data sets, measured under sprinkler irrigated conditions, is highlighted.

References

1. Wright J, Bergsrud F, Rehm G, Rosen C, Malzer G et al., (2013) Nitrogen application with irrigation water: Chemigation.
2. Burt C, O'Conner K, Ruehr T (1998) Fertigation. Irrigation Training and Research Center, California Polytechnic State University.
3. Muirhead WA, Melhuish FM, White RJG (1984) Comparison of Several Nitrogen Fertilizers Applied in Surface Irrigation Systems I. *Crop Response Fert Res* 6: 97-109.
4. Fares A, Abbas F (2009) Irrigation Systems and Nutrient Sources for Fertigation. College of Tropical Agriculture and Human Resources, University of Hawaii at Manoa.
5. van der GTW, Evans RG, Eisenhauer DE (2007) Chemigation. In *Design and operation of farm irrigation systems*; pp: 725-753.
6. Threadgill DE (1985) Chemigation via Sprinkler Irrigation: Current Status and Future Development. *App Eng Agric* 1: 16-23.
7. Zerihun D, Sanchez CA, Farrell-Poe KL, Admsen FJ, Hunsaker DJ (2003) Performance Indices for Surface N Fertigation. *J Irrig Drain Eng* 129: 173-183.
8. Zerihun D, Sanchez CA (2014) Evaluation of Sprinkler Fertigation of Vegetables. Report submitted to the Arizona Specialty Crops Council.
9. Heerman DF, Solomon KH (2007) Efficiency and Uniformity. In *Design and Operation of Farm Irrigation Systems*. Pp: 108-119.
10. Martin DL, Dennis CK, Lyle WM (2007) Design and Operation of Sprinkler Systems. In *Design and operation of farm irrigation systems*. Pp: 557-631.

11. Keller J, Bliesner R (1990) *Sprinkle and Trickle Irrigation*. Van Nostrand Reinhold. New York.
12. Burt CM, Clemmens AJ, Strelkoff TS, Solomon KH, Bliesner RD, et al., (1997) Irrigation Performance Measures: Efficiency and Uniformity. *J Irrig Drain Eng* 123: 423-442.
13. Clemmens AJ, Solomon KH (1997) Estimation of Global Irrigation Distribution Uniformity. *J Irrig Drain Eng* 123: 454-461.
14. Nderitu SM, Hills DJ (1993) Sprinkler Uniformity as Affected by Riser Characteristics. *Trans ASAE* 9: 515-521.
15. Fischer GR, Wallender WW (1988) Collector Size and Test Duration Effects on Sprinkler Water Distribution Measurement. *Trans ASAE* 31: 539-542.
16. Livingston P, Loftis JC, Duke HR (1985) A Wind Tunnel Study of Sprinkler Catch-can Performance. *Trans ASAE* 28: 1961-1965.
17. Warrick WA (1983) Interrelationships of Irrigation Uniformity Terms. *J Irrig Drain Eng* 109: 317-332.
18. Kruse EG (1978) Describing Irrigation Efficiency and Uniformity. *J. Irrig Drain Div* 104: 35-41.
19. Merriam JL, Keller J (1978) *Farm Irrigation System Evaluation: A Guide to Management*. Utah State University. Utah.
20. Wilcox JC, Swailes GE (1947) Uniformity of Water Distribution by Some Under Tree Orchards Sprinklers. *Sci Agric* 27: 565-583.
21. Christiansen JR (1942) *Irrigation by Sprinkling*. Food and Agriculture Organization of the United Nations.
22. Nakayama FS, Bucks DA, Clemmens AJ (1979) Assessing Trickle Emitter Application Uniformity. *Trans ASAE* 22: 816-821.
23. Keller J, Karmeli D (1975) Trickle Irrigation Design Parameters. *Trans ASAE* 17: 678-684.
24. Mulvaney RR (1996) Nitrogen-Inorganic Forms. In *Methods of Soil Analysis: Part 3—Chemical Methods*. Ed. D. L. Sparks Soil Sci Soc Am.