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Generalized fuzzy Lie subalgebras

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Abstract

We introduce a new kind of a fuzzy Lie subalgebra of a Lie algebra called, an (α, β)-fuzzy Lie subalgebra and investigate some of its properties. We also present characterization theorems of implication-based fuzzy Lie subalgebras.

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1 Introduction and Preliminaries

In 1965, Zadeh [15] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty. Rosenfeld [10] introduced notion of fuzzy subgroup of a group in 1971. Since then, many scholars have studied the theories of fuzzy subgroups of a group. On the other hand, the concept of quasi-coincidence of a fuzzy point in a fuzzy subset was introduced by Pu and Liu [9]. Using the concept of *belongingness* and *quasicoincidence*, Bhakat and Das [3] defined a new fuzzy subgroup called, an (α, β)-fuzzy subgroup. This concept is studied further in [4]. Yehia [11] introduced notion of a fuzzy Lie subalgebra and studied some of its properties. In this paper we introduce a new kind of a fuzzy Lie subalgebra of a Lie algebra which is generalization Yehia's fuzzy Lie subalgebra and investigate some of its properties. We also present characterization theorems of implication-based fuzzy Lie subalgebras.

Let μ be a *fuzzy set* on L, i.e., a map $\mu : L \to [0, 1]$. A fuzzy set μ in a set L of the form

$$\mu(y) = \begin{cases} t \in (0,1], & \text{if y=x ,} \\ 0, & \text{y} \neq \text{x,} \end{cases}$$

is said to be a *fuzzy point* with support x and value t and is denoted by x_t. A fuzzy point x_t is said to "*belonging to*" a fuzzy set μ, written as $x_t \in \mu$ if $\mu(x) \geq t$. A fuzzy point x_t is said to be "*quasicoincident with*" a fuzzy set μ, denoted by $x_t q\mu$ if $\mu(x) + t > 1$.

(i) "$x_t \in \mu$" or "$x_t q\mu$" will be denoted by $x_t \in \vee q\mu$.

(ii) "$x_t \overline{\in} \mu$ and $x_t \overline{\in \vee q} \mu$" mean that "$x_t \in \mu$ and $x_t \in \vee q\mu$" do not hold, respectively.

2 (α, β)- fuzzy Lie subalgebra

Definition 1. A fuzzy set μ in L is called an (α, β)-*fuzzy Lie subalgebra* of L if it satisfies the following conditions:

(1) $x_s \alpha\mu, y_t \alpha\mu \Rightarrow (x + y)_{\min(s,t)} \beta\mu$,

(2) $x_s \alpha\mu \Rightarrow (mx)_s \beta\mu$,

(3) $x_s \alpha\mu, y_t \alpha\mu \Rightarrow [x, y]_{\min(s,t)} \beta\mu$

for all x, $y \in L$, $m \in F$, s, $t \in (0, 1]$.

From (2), it follows that:

(4) $x_s \alpha \mu \Rightarrow (-x)_s \beta \mu$.
(5) $x_s \alpha \mu \Rightarrow (0)_s \beta \mu$.

The proofs of the following propositions are obvious.

Proposition 1. Every $(\in, \in)$-fuzzy Lie subalgebra is an $(\in, \in \vee q)$-fuzzy Lie subalgebra.

Proposition 2. Every $(\in \vee q, \in \vee q)$-fuzzy Lie subalgebra is an $(\in, \in \vee q)$-fuzzy Lie subalgebra.

Converse of Propositions 1 and 2 may not be true as seen in the following example.

Example. Let V be a vector space over a field F such that $dim(V) = 5$. Let $V = \{e_1, e_2, \ldots, e_5\}$ be a basis of a vector space over a field F with Lie brackets as follows:

$$\begin{aligned}
&[e_1, e_2] = e_3, \quad [e_1, e_3] = e_5, \quad [e_1, e_4] = e_5, \quad [e_1, e_5] = 0 \\
&[e_2, e_3] = e_5, \quad [e_2, e_4] = 0, \quad [e_2, e_5] = 0, \quad [e_3, e_4] = 0 \\
&[e_3, e_5] = 0, \quad [e_4, e_5] = 0, \quad [e_i, e_j] = -[e_j, e_i]
\end{aligned}$$

and $[e_i, e_j] = 0$ for all $i = j$. Then V is a Lie algebra over F. We define a fuzzy set $\mu : V \to [0, 1]$ by

$$\mu(x) := \begin{cases} 0.5 & \text{if } \ x = 0, \\ 0.6 & \text{if } \ x \in \{e_3, e_5\}, \\ 1.0 & \text{if } \ x \in \{e_1, e_2, e_4\}. \end{cases}$$

By routine computations, it is easy to see that μ is an $(\in, \in \vee q)$-fuzzy Lie subalgebra of L. But, it is easy to see that μ is not $(\in, \in)$- and $(\in \vee q, \in \vee q)$-fuzzy Lie subalgebra of L.

For a fuzzy set μ in L, we define support of μ by $L_0 = \{x \in L : \mu(x) > 0\}$. We now establish a series of Lemmas.

Lemma 1. If μ is a nonzero $(\in, \in)$-fuzzy Lie subalgebra of L, then the set L_0 is a fuzzy Lie subalgebra of L.

Proof. Let $x, y \in L_0$. Then $\mu(x) > 0$ and $\mu(y) > 0$.
(1) If $\mu(x + y) = 0$. Then we can see that $x_{\mu(x)} \in \mu$ and $y_{\mu(y)} \in \mu$, but $(x + y)_{\min(\mu(x), \mu(y))} \overline{\in} \mu$ since $\mu(x + y) = 0 < \min(\mu(x), \mu(y))$. This is clearly a contradiction, and hence $\mu(x + y) > 0$, which shows that $x + y \in L_0$.
(2) If $\mu(mx) = 0$. Then we can see that $x_{\mu(x)} \in \mu$, but $(mx)_{\mu(x)} \overline{\in} \mu$ since $\mu(mx) = 0 < \mu(x)$. This is clearly a contradiction, and hence $\mu(mx) > 0$, which shows that $mx \in L_0$.
(3) If $\mu([x, y]) = 0$. Then we can see that $x_{\mu(x)} \in \mu$ and $y_{\mu(y)} \in \mu$, but $([x, y])_{\min(\mu(x), \mu(y))} \overline{\in} \mu$ since $\mu([x, y]) = 0 < \min(\mu(x), \mu(y))$, a contradiction, and hence $\mu([x, y]) > 0$, which shows that $[x, y] \in L_0$. Consequently L_0 is a Lie subalgebra of L. □

Lemma 2. If μ is a nonzero $(\in, q)$-fuzzy Lie subalgebra of L, then the set L_0 is a fuzzy Lie subalgebra of L.

Proof. Let $x, y \in L_0$. Then $\mu(x) > 0$ and $\mu(y) > 0$.
(1) Suppose that $\mu(x + y) = 0$, then

$$\mu(x + y) + \min(\mu(x), \mu(y)) = \min(\mu(x), \mu(y)) \leq 1$$

Hence $(x + y)_{\min(\mu(x), \mu(y))} \overline{q} \mu$, which is a contradiction since $x_{\mu(x)} \in \mu$ and $y_{\mu(y)} \in \mu$. Thus $\mu(x + y) > 0$, so $x + y \in L_0$.

(2) Suppose that $\mu(mx) = 0$, then

$$\mu(mx) + \mu(x) = \mu(x) \leq 1$$

Hence $mx_{\mu(x)}\overline{q}\mu$, which is a contradiction since $x_{\mu(x)} \in \mu$. Thus $\mu(mx) > 0$, so $mx \in L_0$.

(3) Suppose that $\mu([x, y]) = 0$, then

$$\mu([x, y]) + \min(\mu(x), \mu(y)) = \min(\mu(x), \mu(y)) \leq 1$$

Hence $[x, y]_{\min(\mu(x),\mu(y))}\overline{q}\mu$, which is a contradiction since $x_{\mu(x)} \in \mu$ and $y_{\mu(y)} \in \mu$. Thus $\mu([x, y]) > 0$, so $[x, y] \in L_0$. Hence L_0 is a fuzzy Lie subalgebra of L. □

Lemma 3. If μ is a nonzero $(q, \in)$-fuzzy Lie subalgebra of L, then the set L_0 is a fuzzy Lie subalgebra of L.

Proof. Let $x, y \in L_0$. Then $\mu(x) > 0$ and $\mu(y) > 0$. Thus $\mu(x)+1 > 1$ and $\mu(y)+1 > 1$, which imply that $x_1 q\mu$ and $y_1 q\mu$.

(1) If $\mu(x + y) = 0$, then $\mu(x + y) < 1 = \min(1, 1)$. Therefore, $(x + y)_{\min(1,1)}\overline{\in}\mu$, which is a contradiction. It follows that $\mu(x + y) > 0$ so that $x + y \in L_0$.

(2) If $\mu(mx) = 0$, then $\mu(mx) < 1 = 1$. Therefore, $mx_1\overline{\in}\mu$, a contradiction. It follows that $\mu(mx) > 0$ so that $mx \in L_0$.

(3) If $\mu([x, y]) = 0$, then $\mu([x, y]) < 1 = \min(1, 1)$. Therefore, $[x, y]_{\min(1,1)}\overline{\in}\mu$, which is a contradiction. It follows that $\mu([x, y]) > 0$ so that $[x, y] \in L_0$. □

Lemma 4. If μ is a nonzero (q, q)-fuzzy Lie subalgebra of L, then the set L_0 is a fuzzy Lie subalgebra of L.

Proof. Let $x, y \in L_0$. Then $\mu(x) > 0$ and $\mu(y) > 0$. Thus $\mu(x) + 1 > 1$ and $\mu(y) + 1 > 1$. This implies that $x_1 q\mu$ and $y_1 q\mu$.

(1) If $\mu(x + y) = 0$, then $\mu(x + y) + \min(1, 1) = 0 + 1 = 1$, and so $(x + y)_{\min(1,1)}\overline{q}\mu$. This is impossible, and hence $\mu(x + y) > 0$, i. e., $x + y \in L_0$.

(2) If $\mu(mx) = 0$, then $\mu(mx)+1 = 0+1 = 1$, and so $(mx)_1\overline{q}\mu$. This is impossible, and hence $\mu(mx) > 0$, i. e., $mx \in L_0$.

(3) If $\mu([x, y]) = 0$, then $\mu([x, y]) + \min(1, 1) = 0 + 1 = 1$, and so $[x, y]_{\min(1,1)}\overline{q}\mu$. This is impossible, and hence $\mu([x, y]) > 0$, i. e., $[x, y] \in L_0$. □

By using similar method as given in the above Lemmas, we can also prove the following Lemma.

Lemma 5. If μ is a nonzero $(\in, \in \vee q)$-, $(\in, \in \wedge q)$-,$(\in \vee q, q)$-, $(\in \vee q, \in)$-, $(\in \vee q, \in \wedge q)$-, $(q, \in \wedge q)$-, $(q, \in \vee q)$-, or $(\in \vee q, \in \vee q)$-fuzzy Lie subalgebra of L. Then the set L_0 is a fuzzy Lie subalgebra of L.

In summarizing the above lemmas, we obtain the following theorem.

Theorem 1. If μ is a nonzero (α, β)-fuzzy Lie subalgebra of L, then the set L_0 is a fuzzy Lie subalgebra of L.

Theorem 2. Let $L_0 \subset L_1 \subset \cdots \subset L_n = L$ be a strictly increasing chain of an $(\in, \in)$-fuzzy Lie subalgebras of a Lie algebra L, then there exists $(\in, \in)$-fuzzy Lie subalgebra μ of L whose level subalgebras are precisely the members of the chain with $\mu_{0.5} = L_0$.

Proof. Let $\{t_i : t_i \in (0, 0.5], i = 1, 2, \cdots, n\}$ be such that $t_1 > t_2 > t_3 > \cdots > t_n$. Let $\mu : L \to [0, 1]$ defined by

$$\mu(x) = \begin{cases} t, & \text{if } x = 0, \\ n, & \text{if } x = 0, x \in L_0, \\ t_1, & \text{if } x \in L_1 \setminus L_0, \\ t_2, & \text{if } x \in L_2 \setminus L_1, \\ \vdots & \\ t_n, & \text{if } x \in L_n \setminus L_{n-1}. \end{cases}$$

Let x, $y \in L$. If $x + y \in L_0$, then

$$\mu(x + y) \geq 0.5 \geq \min(\mu(x), \mu(y), 0.5)$$

On the other hand, If $x + y \notin L_0$, then there exists i, $1 \leq i \leq n$ such that $x + y \in L_i \setminus L_{i-1}$ so that $\mu(x + y) = t_i$. Now there exists $j(\geq i)$ such that $x \in L_j$ or $y \in L_j$. If x, $y \in L_k(k < i)$, then L_k is a Lie subalgebra of L, $x + y \in L_k$ which contradicts $x + y \notin L_{i-1}$. Thus

$$\mu(x + y) \geq t_i \geq t_j \geq \min(\mu(x), \mu(y), 0.5)$$

The verification is analogous (2-3) and we omit the details. Hence μ is $(\in, \in)$-fuzzy Lie subalgebra of L. It follows from the contradiction of μ that $\mu_{0.5} = L_0$, $\mu_{t_i} = L_i$ for $i = 1, 2, \ldots, n$. □

3 Implication-based fuzzy Lie subalgebras

Fuzzy logic is an extension of set theoretic multivalued logic in which the truth values are linguistic variables or terms of the linguistic variable truth. Some operators, for example $\vee$; $\wedge$; $\neg$; $\to$ in fuzzy logic are also defined by using truth tables and the extension principle can be applied to derive definitions of the operators. In fuzzy logic, the truth value of fuzzy proposition p is denoted by $[p]$. For a universe of discourse U, we display the fuzzy logical and corresponding set-theoretical notations used in this paper.

(1) $[x \in \mu] = \mu(x)$,
(2) $[p \wedge q] = \min([p], [q])$,
(3) $[p \to q] = \min(1, 1 - [p] + [q])$,
(4) $[\forall x p(x)] = \inf_{x \in U}[p(x)]$,
(5) $\models p$ if and only if $[p] = 1$ for all valuations.

The truth valuation rules given in (4) are those in the Lukasiewicz system of continuous-valued logic. Of course, various implication operators have been defined. We show only a selection of them as follows:

(A) Gaines-Rescher implication operator (I_{GR}):

$$I_{GR}(x, y) := \begin{cases} 1 & \text{if } x \leq y, \\ 0 & \text{otherwise.} \end{cases}$$

(B) Gödel implication operator (I_G):

$$I_G(x, y) := \begin{cases} 1 & \text{if } x \leq y, \\ y & \text{otherwise.} \end{cases}$$

(C) The contraposition of Gödel implication operator ($\overline{I}_G$):

$$\overline{I}_G(x,y) := \begin{cases} 1 & \text{if } x \le y, \\ 1-x & \text{otherwise.} \end{cases}$$

Ying [13] introduced the concept of fuzzifying topology. We can expand this concept to Lie algebras, and we define a fuzzifying Lie subalgebra as follows:

Definition 2. A fuzzy set μ in L is called a *fuzzfying Lie subalgebra* of L if it satisfies the following conditions:

(a) for any $x,\ y \in L$,

$$\models \min([x \in \mu],[y \in \mu]) \to [x+y \in u]$$

(b) for any $x \in L$ and $m \in F$,

$$\models [x \in \mu] \to [mx \in u]$$

(c) for any $x,\ y \in L$,

$$\models \min([x \in \mu],[y \in \mu]) \to [[x,y] \in u]$$

Obviously, Definition 3 is equivalent to the Definition 2.4 [1]. Hence a fuzzifying Lie subalgebra is an ordinary fuzzy Lie subalgebra.

Definition 3. Let μ be a fuzzy set of L and $t \in (0,1]$. Then μ is called a *t-implication-based Lie subalgebra* of L if it satisfies the following conditions:

(d) For any $x,\ y \in L \models_t \min([x \in \mu],[y \in \mu]) \to [x+y \in \mu]$,
(e) For any $x \in L$, $m \in F \models_t [x \in \mu] \to [mx \in \mu]$,
(f) For any $x,\ y \in L \models_t \min([x \in \mu],[y \in \mu]) \to [[x,y] \in \mu]$.

Proposition 3. Let I be an implication operator. A fuzzy set μ of L is a t-implication based fuzzy Lie subalgebra of L if and only if it satisfies the following:

(g) $I(\min(\mu(x),\mu(y)),\mu(x+y)) \ge t$,
(h) $I(\mu(x),\mu(mx)) \ge t$,
(i) $I(\min(\mu(x),\mu(y)),\mu([x,y])) \ge t$

for all $x,\ y \in L$, $m \in F$.

Proof. Straightforward. □

Definition 4. Let $\lambda_1,\ \lambda_2 \in [0,1]$ and $\lambda_1 < \lambda_2$. If μ is a fuzzy set of a Lie algebra L, then μ is called a *fuzzy Lie subalgebra with thresholds* if

(j) $\max(\mu(x+y),\lambda_1) \ge \min(\mu(x),\mu(y),\lambda_2)$,
(k) $\max(\mu(mx),\lambda_1) \ge \min(\mu(x),\lambda_2)$,
(l) $\max(\mu([x,y]),\lambda_1) \ge \min(\mu(x),\mu(y),\lambda_2)$

for all $x,\ y \in L$, $m \in F$.

We now give characterization theorems.

Theorem 3. Let μ be a fuzzy set in L. If $I = I_G$, then μ is a 0.5-implication- based fuzzy Lie subalgebra of L if and only if μ is a fuzzy Lie subalgebra with thresholds $\lambda_1 = 0$ and $\lambda_2 = 0.5$ of L.

Proof. Suppose that μ is a 0.5-implication based Lie subalgebra of L. Then

(i) $I_G(\min(\mu(x), \mu(y)), \mu(x + y)) \geq 0.5$, and hence

$$\mu(x + y) \geq \min(\mu(x), \mu(y)) \quad \text{or} \quad \min(\mu(x), \mu(y)) \geq \mu(x + y) \geq 0.5$$

It follows that

$$\mu(x + y) \geq \min(\mu(x), \mu(y), 0.5)$$

(ii) $I_G(\min(\mu(x), \mu(mx)) \geq 0.5$, and hence

$$\mu(mx) \geq \mu(x) \quad \text{or} \quad \mu(x) \geq \mu(mx) \geq 0.5$$

It follows that

$$\mu(mx) \geq \min(\mu(x), 0.5)$$

(iii) $I_G(\min(\mu(x), \mu(y)), \mu([x, y])) \geq 0.5$, and hence

$$\mu([x, y]) \geq \min(\mu(x), \mu(y)) \quad \text{or} \quad \min(\mu(x), \mu(y)) \geq \mu([x, y]) \geq 0.5$$

It follows that

$$\mu([x, y]) \geq \min(\mu(x), \mu(y), 0.5)$$

so that μ is a fuzzy Lie subalgebra with with thresholds $\lambda_1 = 0$ and $\lambda_2 = 0.5$ of L.

Conversely, if μ is a fuzzy Lie subalgebra with with thresholds $\lambda_1 = 0$ and $\lambda_2 = 0.5$ of L. Then

(i) $\mu(x+y) = \max(\mu(x+y), 0) \geq \min(\mu(x), \mu(y), 0.5)$. If $\min(\mu(x), \mu(y), 0.5) = \min(\mu(x), \mu(y))$, then

$$I_G(\min(\mu(x), \mu(y)), \mu(x + y)) = 1 \geq 0.5$$

Otherwise,

$$I_G(\min(\mu(x), \mu(y)), \mu(x + y)) \geq 0.5$$

(ii) $\mu(mx) = \max(\mu(mx), 0) \geq \min(\mu(x), 0.5)$. If $\min(\mu(x), 0.5) = \mu(x)$, then

$$I_G(\mu(x), \mu(mx)) = 1 \geq 0.5$$

Otherwise,

$$I_G(\mu(x), \mu(mx)) \geq 0.5$$

(iii) $\mu([x, y]) = \max(\mu([x, y]), 0) \geq \min(\mu(x), \mu(y), 0.5)$. If $\min(\mu(x), \mu(y), 0.5) = \min(\mu(x), \mu(y))$, then

$$I_G(\min(\mu(x), \mu(y)), \mu([x, y])) = 1 \geq 0.5$$

Otherwise,

$$I_G(\min(\mu(x), \mu(y)), \mu([x, y])) \geq 0.5$$

Hence μ is a 0.5-implication based Lie subalgebra of L.

This completes the proof. □

Theorem 4. Let μ be a fuzzy set in L. If $I = \overline{I}_G$, then μ is a 0.5-implication- based fuzzy Lie subalgebra of L if and only if μ is a fuzzy Lie subalgebra with thresholds $\lambda_1 = 0.5$ and $\lambda_2 = 1$ of L.

Proof. Suppose that μ is a 0.5-implication based Lie subalgebra of L. Then

(i) $\overline{I}_G(\min(\mu(x), \mu(y)), \mu(x+y)) \geq 0.5$ which implies that

$$\mu(x+y) \geq \min(\mu(x), \mu(y)) \quad or \quad 1 - \min(\mu(x), \mu(y)) \geq 0.5, \quad \text{i.e} \quad \min(\mu(x), \mu(y)) \leq 0.5$$

Thus

$$\max(\mu(x+y), 0.5) \geq \min(\mu(x), \mu(y), 1)$$

(ii) $\overline{I}_G(\min(\mu(mx)\mu(mx)) \geq 0.5$ which implies that

$$\mu(mx) \geq \mu(x) \quad \text{or} \quad 1 - \mu(x) \geq 0.5, \quad \text{i.e} \quad \mu(x) \leq 0.5$$

Thus

$$\max(\mu(mx), 0.5) \geq \min(\mu(x), 1)$$

(iii) $\overline{I}_G(\min(\mu(x), \mu(y)), \mu([x, y])) \geq 0.5$ which implies that

$$\mu([x, y]) \geq \min(\mu(x), \mu(y)) \quad \text{or} \quad 1 - \min(\mu(x), \mu(y)) \geq 0.5 \quad \text{i.e} \quad \min(\mu(x), \mu(y)) \leq 0.5$$

Thus

$$\max(\mu([x, y]), 0.5) \geq \min(\mu(x), \mu(y), 1)$$

Hence μ is a fuzzy Lie subalgebra with thresholds $\lambda_1 = 0.5$ and $\lambda_2 = 1$ of L The proof of converse part is obvious.

This completes the proof. □

Theorem 5. Let μ be a fuzzy set in L. If $I = I_{GR}$, then μ is a 0.5-implication- based fuzzy Lie subalgebra of L if and only if μ is a fuzzy Lie subalgebra with thresholds $\lambda_1 = 0$ and $\lambda_2 = 1$ of L.

Proof. Obvious. □

As a consequence of the above Theorems we obtain the following

Corollary 1. (1) Let $L = L_{GR}$. Then μ is an implication-based fuzzy Lie subalgebra of L if and only if μ is a Yehia's fuzzy Lie subalgebra of L.

(2) Let $L = L_G$. Then μ is an implication-based fuzzy Lie subalgebra of L if and only if μ is an $(\in, \in \vee q)$- fuzzy Lie subalgebra of L.

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