

## GPS-Compatible Lorentz Transformation that Satisfies the Relativity Principle

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### Abstract

In relativity theory there are two versions of time dilation: symmetric and asymmetric. In the first case, it is assumed that a moving clock always runs slower than the observer's local clock, so it is just a matter of perspective which of two clocks runs faster. By contrast, asymmetric time dilation assumes that if two clocks are running at different rates, one of them is unambiguously slower. The Lorentz transformation (LT) of Einstein's special theory of relativity (STR) predicts that only symmetric time dilation occurs in nature. However, experimental studies of the rates of atomic clocks on airplanes, as well as of the second-order Doppler effect using high-speed rotors, find that time dilation is exclusively asymmetric, in clear contradiction to the LT. In the present work, it is shown that there is another space-time transformation that also satisfies Einstein's two postulates of relativity, but one which assumes that clock rates in different rest frames are *strictly proportional* to one another. It is therefore in complete agreement with the results of the above time-dilation experiments and also with the clock-rate adjustment procedure applied to satellite clocks in the methodology of the Global Positioning System; hence the designation GPS-LT for this alternative space-time transformation. Unlike the original LT, the GPS-LT is consistent with the absolute remote simultaneity of events, and it eliminates the necessity of assuming that space and time are inextricably mixed. It also disagrees with the FitzGerald-Lorentz length-contraction prediction of STR, finding instead that isotropic length expansion always accompanies time dilation in a given rest frame. The results of the Ives-Stilwell study of the transverse Doppler effect and also those of experiments with accelerated muons are shown to be in complete agreement with the latter conclusion.

**Keywords:** Relativity Principle; Einstein's special theory; High-speed rotors; Compatible

### Introduction

The Lorentz transformation (LT) is the centerpiece of Einstein's special theory of relativity (STR [1]). It satisfies the two postulates of relativity: the relativity principle (RP) and the assumption of the constancy of the speed of light in free space (LSP). It was pointed out as early as 1898 by Lorentz [2] that there is a degree of freedom in the definition of a space-time transformation that satisfies the LSP and/or leaves Maxwell's equations invariant. He introduced a series of four equations that can be referred to as the General Lorentz transformation (GLT) in which a common normalization function appears on the right-hand side of each relation. It therefore follows that there are an infinite number of such transformations that satisfy the LSP. However, the RP puts another constraint on the definition of a fully relativistic space-time transformation. In addition, there is the obvious criterion that the equations must be in agreement with all relevant experimental data.

In the following it will be shown that, although the LT satisfies both of the relativity postulates, it fails to predict the results of a number of experiments that were carried out in the latter half of the 20<sup>th</sup> century. Evaluation of these experimental results, which were not available to Einstein at the time of his landmark paper, allows for a more precise statement of the physical requirements that must be satisfied in order to obtain a completely satisfactory theory of motion in the absence of gravitational fields. The technology of the Global Positioning System (GPS), particularly the manner in which it makes use of atomic clocks in obtaining accurate estimates of distances on the earth's surface, will prove to be a key element in the following discussion.

### Asymmetric time dilation

In Einstein's derivation of the LT [1], there is a step in which the

Lorentz normalization factor must be determined. The GLT given below contains this factor in all four of its equations, referred to as  $\epsilon$  by Lorentz [2] but as  $\phi$  in Einstein's work:

$$\Delta t' = \gamma \epsilon (\Delta t - v \Delta x / c^2) = \gamma \epsilon \eta^{-1} \Delta t \quad (1a) \quad \Delta x' = \gamma \epsilon (\Delta x - v \Delta t) \quad (1b) \quad \Delta y' = \epsilon \Delta y \quad (1c) \quad \Delta z' = \epsilon \Delta z. \quad (1d)$$

The equations are given in terms of intervals of space  $\Delta x$ ,  $\Delta y$  and  $\Delta z$  and time  $\Delta t$ , i.e.

$\Delta x = x_2 - x_1$  etc. for two events ( $c$  is the speed of light,  $v$  is the relative speed of the participating inertial systems  $S$  and  $S'$  and  $\gamma = (1 - v^2/c^2)^{-0.5}$ ). In addition, the quantity  $\eta$  is defined in Equation 1a as  $(1 - v^2/c^2)^{-1}$ . Einstein asserted without proof that the normalization factor can only depend on the relative speed  $v$ . He went on to show that under this restriction the only allowable value is  $\epsilon = \phi = 1$ , which upon substitution in the GLT equations leads directly to the LT. This choice for the normalization function ensures that the LT also is consistent with the RP, as will be discussed in more detail in the following section. One of the main conclusions of the LT is that there is a definite symmetry between the results of measurements made by two observers in relative motion. For example, two separate equations for their elapsed times follow directly from the LT:

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$$\Delta t = \gamma \Delta t' \quad (2a)$$

$$\Delta t' = \gamma \Delta t. \quad (2b)$$

These two equations indicate that each observer will find that the other's clock runs slower than his own. Similar relations are expected for all other properties such as distance and inertial mass. It should be clearly seen that this characteristic of the LT implies that measurement is *subjective*; which clock is slower or which distance is shorter is merely a matter of the perspective of the observer.

At the same time the LT implies that the previously longstanding belief in the simultaneity of remote events must be discounted. It is seen from Equation 1a that, with the LT value of  $\epsilon=1$ , a value of  $\Delta t=0$  guarantees that  $\Delta t' \neq 0$  if both  $v$  and  $\Delta x/\Delta t$  have non-zero values. Poincare [3,4] raised the question of whether there was sufficient experimental data to rule out the possibility of *remote non-simultaneity* of events. More than a century later the majority opinion of physicists clearly rejects simultaneity as a universal principle because it is inconsistent with the predictions of the LT. The same equation implies that space and time are inextricably mixed, unlike the view of classical physicists. As a consequence, the concept of "space-time" has since become an essential tool in cosmological science [5].

The LT also has a problem with the causality principle if remote non-simultaneity is not allowed. This can be seen upon dividing Equation 1a with  $\Delta t$ :

$$\Delta t'/\Delta t = \gamma (1 - v c^{-2} \Delta x/\Delta t) = \gamma \eta^{-1}. \quad (3)$$

This equation implies that the ratio of time differences measured by the two observers in  $S$  and  $S'$  may depend on the value of the spatial separation of the corresponding two events. Assuming remote simultaneity forces the conclusion that  $\Delta t'/\Delta t$  is also the ratio of proper clock rates, however, which therefore would amount to a clear violation of causality since neither clock rate can be affected by the event separation. On the contrary, the causality principle implies that the rates of proper clocks in inertial systems are strictly constant since no external force exists which would alter their values over the course of time. This conclusion therefore leads one to believe that the ratio of proper clock rates in a given pair of inertial systems is also a constant, leading to the simple proportionality relationship:

$$\Delta t' = \Delta t/Q, \quad (4)$$

Where  $Q$  may only depend on the respective states of motion of the two inertial systems. Since Equation 4 is incompatible with Equations 2a and 2b, it is clear that the LT can only be valid if the remote non-simultaneity of events is an essential characteristic of physical reality.

The above discussion points out the need to carry out experiments to test whether time dilation is symmetric or asymmetric. The first study of this type was carried out by Hay et al. [6]. They employed a high-speed rotor to measure the transverse Doppler Effect using the Mössbauer technique. An x-ray source and a corresponding absorber/detector were mounted on the rotor axis. The LT predicts that a moving clock always runs slower than its identical counterpart at rest in the laboratory, in accordance with Equations 2a and 2b. The effect of time dilation is therefore expected to be perfectly symmetric, i.e. the emitted frequency  $\nu_e$  must be greater at the source than that ( $\nu_r$ ) measured by the x-ray absorber [7]. Specifically,

$$\nu_r = \gamma^{-1} \nu_e. \quad (5)$$

The empirical findings [6] for the shift in frequency  $\Delta \nu/\nu$  are summarized by the formula:

$$\Delta \nu/\nu = (R_a^2 - R_s^2) \omega^2 / 2c^2, \quad (6)$$

where  $R_a$  and  $R_s$  are the respective distances of the absorber and x-ray source from the rotor axis ( $\omega$  is the circular frequency of the rotor). It shows that a shift to higher frequency (*blue shift*) is observed when  $R_a$  is greater than  $R_s$ , as in the experimental arrangement actually employed. The corresponding result expected from Equation (5) would be:

$$\Delta \nu/\nu = \gamma^{-1} (|R_a - R_s| \omega - 1) \approx -(R_a - R_s)^2 \omega^2 / 2c^2, \quad (7)$$

i.e. a *red shift* should be observed in all cases, in accordance with the symmetric interpretation of time dilation. However, the results shown in Equation 6 indicate on the contrary that the effect is *anti-symmetric*, in clear contradiction to both Equation 7 and the [6], nonetheless declared that their results were consistent with Einstein's theory [1] without mentioning the difficulty with the LT prediction of symmetry. One can only assume [8] that they based this conclusion strictly on the magnitude of the observed shift, totally ignoring its direction/sign. Hay et al. also noted that Equation 6 can be derived from Einstein's equivalence principle [9], which equates centrifugal force and the effects of gravity. Subsequent experiments by Kündig [10] and Champeney et al. [11] also found that their results were summarized by Equation 6. Kündig stated explicitly that the findings confirmed the position that it is *the accelerated clock that is slowed by time dilation*, thereby asserting that the measurement process is *objective* in this experiment, contrary to the prediction of Equations 5-7) and of the LT itself.

There is no question as to which clock is slower, as Sherwin pointed out [12] shortly after the Hay et al. experimental data [6] became available. "It is the completely unambiguous nature of the result in the 'clock paradox' which is, perhaps, its most unique feature. Here for the first time, one is comparing a proper time interval in one inertial frame to what might be described as the sum of proper time intervals which were collected by the traveling clock in several different inertial frames. The result is completely unambiguous: One particular clock certainly runs fast, and the other certainly runs slow. By contrast, in experiments involving uniform translation (where one is comparing a proper time interval in one inertial frame with a nonproper time interval in another inertial frame) the clock rates (as determined by the prescribed operational procedures) are ambiguous, that is, the observers in each frame measure the *other* clock to be running slow." There are two aspects of Sherwin's summary that require special comment [8]. First, even to the present day, no such experiments have been reported "involving uniform translation" in which the symmetry of clock rates that is expected from STR is actually observed. Secondly, the fact that the latter predictions are not verified in the rotor experiments constitutes a clear contradiction of the LT. In essence Sherwin has concluded that the range of applicability for the LT does not extend to clocks which are accelerated during the time of measurement.

A decade later, Hafele and Keating carried out experiments with atomic clocks located on circumnavigating airplanes [13,14]. They also found that the symmetry of clock rates expected from the LT did not occur in practice. Instead, they found that the clocks on the eastward-flying airplane (after making corrections for the effects of the gravitational red shift) ran slower than those left behind at the airport. On the other hand, the westward-flying clocks were observed to run faster than the airport clocks, again after taking into account the effects of gravity. They could explain these results by assuming that the rate of a given clock varies in inverse proportion to  $\gamma(v)$ , where  $v$  is its speed relative to the "non-rotating polar axes" of the earth. Since the earth was turning beneath a given clock, this meant that the effective speed of the eastward clock was greater than that of the airport clock, which

in turn was traveling faster than the westward clock. The predicted elapsed times after the airplanes had returned to their point of origin agreed within 10% of the observed results.

The fact that the clock speeds had to be computed relative to the hypothetical polar axis could be explained by the fact that the latter constituted a uniquely inertial system in the experiment [13]. One should also recognize, however, that the earth's center of mass (ECM) is stationary on this axis, so one might just as well assume that the relevant speeds of the clocks are to be computed relative to this specific point. The ECM also plays a critical role in the evaluation of the gravitational red shift for each clock, so there is a certain consistency in the two effects by looking at the results in this way.

The results of both the rotor and atomic clock experiments are seen to have many common characteristics. In both cases the fractional amount of time dilation is found to be proportional to a  $\gamma(v)$  factor, where  $v$  is the relative speed of the clock to some specific reference system. In previous work [15], this rest frame has been referred to as the objective rest system (ORS). One can summarize both sets of results in the following equation:

$$\tau_1 \gamma(v_{10}) = \tau_2 \gamma(v_{20}). \quad (8)$$

The speed of a given clock relative to the ORS is defined as  $v_{i0}$  in each case. The corresponding elapsed time in the Hafele-Keating study [13,14] is  $\tau_i$ . In the rotor experiments [6,10,11]  $\tau_i$  is equal to the reciprocal of the observed frequency  $\nu_i$  of a given clock [10] and the rotor axis serves as the ORS. In the example of asymmetric time dilation cited by Einstein in his original work, the ORS is the rest frame where a force is applied to an electron. As in the other two cases, the amount of time dilation is assumed to be inversely proportional to the  $\gamma$  factor computed on the basis of the electron's speed relative to this reference. For this reason it is appropriate to refer to Equation 8 as the universal law of time dilation (ULTD) [16]. *There are no known exceptions to this equation.* It assumes that time dilation is always asymmetric, in contradiction to the LT prediction of symmetry between the results of different observers in relative motion to one another.

The ULTD is compatible with the proportionality relation of Equation 4. By changing notation so that the speeds of the clocks in S and S' relative to their common ORS are  $v_0$  and  $v_0'$ , respectively, one obtains the following value for the proportionality constant:

$$Q = \gamma(v_0') / \gamma(v_0). \quad (9)$$

Thus, the constant ratio of clock rates in different inertial systems expected from the causality principle is seen to be consistent with experiment, i.e., asymmetric time dilation. The ULTD allows one to compute the value of Q quantitatively provided the pertinent ORS is known as well as the speeds of the two stationary clocks in S and S' relative to it.

The global positioning system (GPS) operates on the assumption of a strict proportionality between the rate of a proper clock located on a satellite and that on the earth's surface. The former clock is adjusted prior to launch so that it will run at the same rate as the ground clock upon reaching orbit [17]. This procedure also implies that events on the satellite occur simultaneously for the ground observer, thereby excluding the possibility of remote non-simultaneity. If the times of two events in one rest frame are equal ( $T_1 = T_2$ ), it follows that the times in the other rest frame must be equal as well ( $QT_1 = QT_2$ ). Both the LT predictions of symmetric time dilation and remote non-simultaneity are therefore seen to be contradicted by all the considerable experimental evidence which has as yet become available.

## Incorporating Clock-Rate Proportionality in the Lorentz Transformation

The LT has enjoyed unwavering support from the physics community for over a century despite its inability to anticipate the asymmetric nature of time dilation. There are probably two main reasons for this state of affairs. First, it has an excellent record of predicting other experimental results. Second, it is widely believed that no other space-time transformation is capable of satisfying both the LSP and the RP. The question, therefore, is whether a different transformation can be devised which retains the above advantages while still being consistent with asymmetric time dilation and the ULTD.

To investigate this possibility it is best to return to the general Lorentz transformation of Equations 1a-1d. It will be recalled that any choice of the normalization factor  $\epsilon$  therein will satisfy the LSP. The clock-rate proportionality relation of Equation 4 guarantees the prediction of asymmetric time dilation, and it is a simple matter to choose a value of  $\epsilon$  to satisfy this condition. It is merely necessary to combine Equation 4 with Equation 1a to obtain the corresponding value of  $\epsilon$ :

$$\Delta t' = \gamma \epsilon (\Delta t - v \Delta x / c^2) = \gamma \epsilon \eta^{-1} \Delta t = \Delta t / Q. \quad (10)$$

The solution is:

$$\epsilon = [\gamma Q (1 - v c^{-2} \Delta x / \Delta t)]^{-1} = \eta / \gamma Q. \quad (11)$$

Substitution of this value of  $\epsilon$  in the general Lorentz transformation leads to the following alternative set of equations:

$$\Delta t' = \Delta t / Q \quad (12a) \quad \Delta x' = (\eta / Q) (\Delta x - v \Delta t) \quad (12b) \quad \Delta y' = \eta \Delta y / \gamma Q \quad (12c) \quad \Delta z' = \eta \Delta z / \gamma Q. \quad (12d)$$

The first of these relations is by construction Equation 4, guaranteeing that the time dilation is asymmetric. The proportionality between elapsed times in S and S' ensures that all events occur simultaneously for the respective observers in each rest frame. It is clear that the equations satisfy the LSP but the question remains as to whether they also satisfy Einstein's other postulate of relativity, the RP.

In order to investigate this point, it is helpful to form the squares of Equations 1a-1d) of the general Lorentz transformation and sum them. The result is

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = \epsilon^2 (x^2 + y^2 + z^2 - c^2 t^2). \quad (13)$$

The inverse of these equations is obtained in the usual way by interchanging the primed and unprimed symbols for the two rest frames and changing the sign of their relative speed  $v$ . Carrying out the same operations for the inverse equations leads to the corresponding result:

$$x^2 + y^2 + z^2 - c^2 t^2 = \epsilon'^2 (x'^2 + y'^2 + z'^2 - c^2 t'^2). \quad (14)$$

Algebraic manipulation of Equations 13 and 14 shows that there is a clear condition for satisfying the RP, namely  $\epsilon'^2$  must be equal to  $\epsilon^{-2}$ , or more simply:

$$\epsilon \epsilon' = 1. \quad (15)$$

The LT, with its value of  $\epsilon=1$  obviously has the desired symmetry between the two rest frames since coordinate inversion leads to the result of  $\epsilon'=1$ . This fact has led to a strong belief in the physics community that the LT is unique in this respect. For example, the value of  $\epsilon=\gamma^{-1}$  originally proposed by Voigt [18] does not satisfy Equation 15 because  $\epsilon'^2=\gamma^{-1}$  as well. In order to satisfy the RP, the alternative transformation

in Equations 12a-12d) with the value of  $\epsilon$  given in Equation 11 requires instead that

$$\eta\eta' = \gamma^2 Q Q' \quad (16)$$

However, comparison of Equation 12a with its inverse,  $\Delta t = \Delta t' / Q'$ , shows that  $Q Q' = 1$ , so the condition for satisfying the RP reduces to:

$$\eta\eta' = \gamma^2 \quad (17)$$

The definition of  $\eta$  has been given above in connection with Equation 1a of the GLT in terms of the ratio  $\Delta x / \Delta t$ . The corresponding value for the inverse function  $\eta'$  is obtained as

$$(1 + v c^{-2} \Delta x' / \Delta t')^{-1}.$$

Before proceeding further, however, it is useful to note that the relativistic velocity transformation (RVT) can be obtained from the GLT by dividing the three spatial equations by the corresponding equation of the time variables. Since  $\epsilon$  appears on the right-hand side of each of the four GLT equations, it is simply canceled out in the divisions. The RVT is therefore obtained in this manner from any of the specific transformations for a given value of  $\epsilon$ . This is a key observation since it shows that effects such as the aberration of starlight at the zenith [19] and the Fresnel light-drag phenomenon [20] which are derived on the basis of the RVT do not constitute direct evidence for the validity of any particular space-time transformation. Therefore, they can just as well be seen as successful applications of the alternative set of relations in Equations 12a-12d as they can for the LT.

The RVT is given below for the three velocity components, whereby  $u_x = \Delta x / \Delta t$ ,  $u_x' = \Delta x' / \Delta t'$  etc.:

$$u_x' = (1 - v u_x c^{-2})^{-1} (u_x - v) = \eta (u_x - v) \quad (18a)$$

$$u_y' = \gamma^{-1} (1 - v u_x c^{-2})^{-1} u_y = \eta \gamma^{-1} u_y \quad (18b)$$

$$u_z' = \gamma^{-1} (1 - v u_x c^{-2})^{-1} u_z = \eta \gamma^{-1} u_z \quad (18c)$$

In the following derivation it is convenient to use the definitions of  $\eta$  and  $\eta'$  in terms of  $u_x$  and  $u_x'$  as defined in the RVT. Hence, from Equation 17,

$$\begin{aligned} \eta\eta' &= [(1 - v u_x c^{-2}) (1 + v u_x' c^{-2})]^{-1} = (1 - v u_x c^{-2})^{-1} [1 + v \eta (u_x - v) c^{-2}]^{-1} \\ &= c^4 (c^2 - v u_x)^{-1} [c^2 + v (u_x - v) (1 - v u_x c^{-2})]^{-1} \\ &= c^4 (1 - v c^{-2} u_x) [(c^2 - v u_x)(c^2 - v u_x + v u_x - v^2)]^{-1} \\ &= (c^4 - v c^2 u_x) [(c^2 - v u_x)(c^2 - v^2)]^{-1} \\ &= (c^2 - v u_x) [(c^2 - v u_x)(1 - v^2 c^2)]^{-1} \\ &= (1 - v^2 c^2)^{-1} = \gamma^2. \end{aligned} \quad (19)$$

The condition of Equation 17 is indeed satisfied. Note that the RVT is used in the first step of Equation 19 to eliminate  $u_x'$  so that only the unprimed velocity component  $u_x$  remains. The space-time transformation of Equations 12a-12d) is thus shown to satisfy the RP as well as the other of Einstein's relativity postulates, the LSP. In short, from a purely theoretical point of view there is no reason to favor the LT over this alternative set of equations.

The previous discussion has shown, however, that the predictions of the two transformations are quite different. The LT famously requires that time dilation and other phenomena have a distinctly symmetric characteristic, as indicated in Equations 2a and 2b. The transformation of Equations 12a-12d), by contrast, predicts that only asymmetric time dilation can occur, i.e. it is always possible in principle to determine which of two clocks has the slower rate.

Moreover, the proportionality relation of Equation 12a precludes any possibility of remote non-simultaneity, as discussed in Sect. II., again in definite contrast to the LT.

Experiment must be the ultimate arbiter of whether the LT or the transformation of Equations 12a-12d) is correct. As discussed in Sect. II, it has invariably been found that time dilation is an asymmetric phenomenon; one clock is slower and one is faster. The type of symmetry in clock rates demanded by the LT has never been observed. The evidence from all such observations is therefore unanimous; the LT does not agree with experiment whereas the other transformation does. The proportional relationship of Equation 12a between elapsed times measured for the same event in S and S' is consistent with the empirical relationship given by the ULTD of Equation 8.

The GPS technology is based on the principle that a proper clock on a satellite can be adjusted by a constant factor so that it runs at exactly the same rate as its counterpart on the earth's surface, so asymmetric time dilation has become a staple of everyday modern life. For this reason it is appropriate to refer to Equations 12a-12d) as the GPS Lorentz transformation (GPS-LT).

### Length variations in relativity theory

Thus far in the discussion, attention has been centered exclusively on comparisons of the elapsed times measured by different observers for the same event. Standard relativity theory holds [21] that the lengths of objects contract at the same time that clock rates slow (Fitzgerald-Lorentz length contraction). Furthermore, the amount of the contraction is predicted from the LT to vary with the orientation of the object to the relative velocity of the two rest frames, decreasing by a factor of  $\gamma$  in the parallel but remaining the same in the perpendicular direction.

The corresponding predictions of the GPS-LT are quite different. To see this, consider a metal rod located on a satellite. The definition of the meter is the distance light travels in free space in  $c^{-1}$  s. If light passes between the two ends of the rod in time  $\Delta t'$  from the vantage point of an observer who is stationary on the satellite (S'), it therefore follows that its length is  $c \Delta t'$  in his units. Let us assume that a proper clock on the satellite runs  $\gamma$  times slower than for its counterpart on the earth's surface (S), i.e. the observer there finds the corresponding elapsed time to be  $\Delta t = \gamma \Delta t'$  (Equation 12a,  $Q = \gamma$ ). However, according to the LSP, the latter observer must find that the speed of light on the satellite is also equal to  $c$  in all directions. From the definition of speed it follows that the distance traveled by the light is equal to  $c$  times the corresponding elapsed time, i.e.  $L = c \Delta t$ , for the ground observer. One therefore obtains the following relation between the values of the rod's length measured by the two observers:

$$L = c \Delta t = c (\gamma \Delta t') = \gamma c \Delta t' = \gamma L' \quad (20)$$

In accordance with the LSP, this result is seen to be clearly independent of the orientation of the rod. Since  $\gamma > 1$ , it therefore follows that the length of the rod has *increased in all directions* by exactly the same factor as for the time dilation on the satellite. It is therefore found on the basis of the GPS-LT that isotropic length expansion accompanies time dilation in a given rest frame, not the type of anisotropic length contraction expected from the LT. The Ives-Stilwell study [22,23] of the transverse Doppler Effect provides a straightforward test of Equation 20. The wavelength of light emanating from an accelerated source (S') was measured in the laboratory (S) on a photographic plate. The first-order Doppler effect was eliminated by averaging the two wavelengths coming from opposing directions. The

remaining second-order effect is tantamount to the shift in wavelength that occurs by virtue the acceleration of the light source. *It is found to be independent of the direction in which the opposing wavelengths are measured in a given experiment.* The observed wavelength  $\lambda$  is found to have the following relationship to the standard value  $\lambda_0$  obtained when the light source is at rest in the laboratory:

$$\lambda = \gamma \lambda_0. \quad (21)$$

In other words, the wavelength of light in the accelerated rest frame has increased by the same factor of  $\gamma$  in all directions relative to its standard value in the laboratory, consistent with what is predicted from Equation 20.

It is interesting that the authors [23] did not comment on this result directly, but rather used it to infer from the LSP that the corresponding frequency of the light must have decreased in  $S'$  by virtue of its motion relative to the laboratory. No consideration was given to the fact that the increase in wavelength itself is not consistent with the prediction of length contraction expected from the LT. It has been argued by various authors that the wavelength increase does not constitute a violation of the length-contraction prediction because the latter only applies to material objects and not to radiation. However, this position fails to take into account the following inference from the RP. The standard value  $\lambda_0$  of the wavelength must be measured in the rest frame of the light source, even though a larger value is obtained in the laboratory. The only rational explanation for this fact is that *the dimensions of the measuring apparatus (diffraction grating) must have increased by the same factor  $\gamma$  in all directions in the accelerated rest frame*; hence, no change in wavelength can be observed there.

Another well-known experiment that demonstrates that distances increase as clocks slow down is the muon-decay study carried out by Rossi et al. [24]. It was found that the average range before decay  $L$  of the particles created by cosmic radiation increased with their speed  $v$  relative to their initial location in the earth's atmosphere according to the empirical formula ( $\tau_0$  is the proper lifetime of the particles):

$$L = \gamma v \tau_0 = v \tau. \quad (22)$$

This result is easily understood by assuming that the lifetime  $\tau$  of the particles increases in direct proportion to  $\gamma(v)$ . In accord with the ULTD of Equation 8, it is found that the elapsed time measured with the accelerated muon-clocks decreases in inverse proportion to  $\gamma$ .

The RP can again be used to obtain the value of the average range of decay  $L_0$  that must be measured by an observer co-moving with the accelerated particles, namely:

$$L_0 = v \tau_0. \quad (23)$$

In accord with the ULTD of Equation 8, it is found that the elapsed time measured with the accelerated muon-clocks decreases in inverse proportion to  $\gamma$ . The average range before decay ( $L_0$ ) also decreases by the same factor for this observer because it is assumed in accord with the LSP that the speed of the particles is the same in both rest frames. The conclusion from the RP has been misinterpreted by some authors [25,26] to be a confirmation of length contraction because distances measured in the accelerated rest frame are smaller. What it shows instead is that the unit of distance is greater in the latter rest frame, just as is the unit of time there, i.e. a meter stick there is  $\gamma$  times larger than on the earth's surface. The effect is independent of direction, so once again it is seen that isotropic length expansion of stationary objects accompanies time dilation in any given rest frame. In subsequent experiments carried out at CERN [27,28] it has further been shown that

the degree of acceleration has no effect on the amount of time dilation, but rather only the speed  $v$  of the particles relative to their ORS. This result is also in agreement with the ULTD and contradicts Sherwin's speculation [12] that symmetric time dilation (ambiguity of relative clock rates) will ensue in the absence of applied forces on the clocks, as discussed in Sect. II.

Examination of previous claims of length-contraction observations [29] shows that they involve distributions of a large ensemble of particles such as electrons. As such, these claims ignore the effects of de Broglie wave-particle duality [29], which is known to produce a decrease in the wavelength of the distribution in inverse proportion to the momentum of the particles ( $p = h\lambda^{-1}$ ). It should be noted that Fitzgerald-Lorentz length contraction of STR has a substantially different dependence on the speed of particles than does the de Broglie duality. For example, doubling  $v$  in the latter case leads to a reduction in the de Broglie wavelength of the particles by 50%, where if the STR length contraction is invoked, a much smaller decrease is expected, namely by a maximum factor of  $\frac{\gamma(2v)}{\gamma(v)} \approx 1 + 1.5v^2c^{-2}$ . No contraction could be expected to be observed on this basis at the speeds employed in the Josephson effect experiments cited [29].

## Conclusion

The Lorentz transformation (LT) predicts a *symmetric* characteristic for time dilation,

Where by two observers in relative motion must disagree as to whose clock runs slower. It therefore conforms to a subjective view of the measurement process. By contrast, with *asymmetric* time dilation there is no question, at least in principle, which clock is slower and which is faster. It is therefore a thoroughly objective phenomenon, quite distinct from its symmetric counterpart. Unfortunately for the LT and for STR in general, every experiment that has as yet been carried out to try and confirm that time dilation is symmetric has failed in this crucial respect. This includes studies of the transverse Doppler effect with high-speed rotors [6, 10, 11] and of the rates of atomic clocks carried onboard circumnavigating airplanes [13, 14]. In all cases it is possible to fit the data to a simple empirical formula given in Equation 8 and referred to as the universal law of time dilation (ULTD). To evaluate this formula it is necessary to identify a specific rest frame (ORS) from which to measure the speed  $v_{10}$  of the clocks. The ORS is the earth's center of mass in the case of the airplane tests and the axis of the rotor in the Hay et al. experiment [6]. Accordingly, one finds that elapsed times are inversely proportional to  $\gamma(v_{10})$ . There is never any question about which of two clocks runs slower, proving that time dilation is asymmetric, contrary to the predictions of the LT and STR.

Nonetheless, the physics community has held steadfast to its belief in the LT because of the broadly held assumption that it is the only space-time transformation that satisfies both of Einstein's two postulates of relativity [1]. This conclusion requires that one subscribe to the position that space and time are inexorably mixed and are simply different parts of a single entity. Experiment indicates unequivocally, by contrast, that the rates of clocks in different inertial systems are strictly proportional to one another. This fact is used on an everyday basis in the GPS methodology to adjust satellite clocks so that they run synchronously with their counterparts on the earth's surface.

The present work shows that the space-time transformation of Equations 12a-12d also satisfies both postulates of relativity while assuming a strict proportionality between the elapsed times for a

given event measured by two observers in different rest frames. It has been referred to as the GPS-LT to emphasize its relationship to both the asymmetric time dilation of the navigation system and also to the original LT. It is compatible with the relativistic velocity transformation (RVT) first derived by Einstein in his original work [1]. The GPS-LT is therefore responsible for a number of the successes of STR (for example, the explanations for the Fresnel light-drag phenomenon and the aberration of starlight at the zenith) that require only this relationship between measured velocities for their actual justification, and not specifically, as is often erroneously assumed, the LT itself. Indeed, there is no known experiment at the present time that cannot be explained either directly or indirectly through the RVT that is not consistent with the predictions of the GPS-LT. This includes first and foremost the various observations of asymmetric time dilation that are not anticipated by the LT.

The standard treatment of length variations is also based on the LT (FitzGerald-Lorentz length contraction). It is assumed thereby that the lengths of objects in motion decrease by varying amounts depending on their orientation to the observer. Use of the LSP by itself indicates on the contrary that an object's dimensions expand isotropically as it is accelerated, i.e. that isotropic length expansion accompanies asymmetric time dilation. This is also the conclusion of the GPS-LT. One notes, for example, that the lengths of objects can be determined by measuring the elapsed time for light to traverse the full length of the object and multiplying this result with  $c$ . It follows therefore that the observer with the larger value of the elapsed time will also obtain a proportionately greater value for the length of the object.

Finally, although it is fundamentally impossible to directly measure lengths in rest frames that are moving with respect to the observer, it is a simple matter to deduce such changes using the RP. For example, in the Ives-Stilwell study of the transverse Doppler effect [22,23], the fact that the laboratory observer measures an increase in wavelength while his counterpart co-moving with the light-source does not, clearly implies that the latter's measuring device has also increased by the same fraction. The observed increase in wavelength was taken as proof of time dilation (frequency decrease because of the LSP) in the accelerated rest frame, but the corresponding deduction of length expansion based on the wavelength measurements themselves was ignored in the authors' presentation. This was presumably because it did not fit in with their expectations of relativistic length contraction based on the LT. By contrast, everything is consistent if the theoretical predictions are based on the GPS-LT [30,31].

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