

## How to Formulate an Exact Proof of the Riemann Hypothesis?

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### Introduction

My proof of the celebrated Riemann hypothesis is simple and direct. The history of the Riemann zeta function goes back to Euler. Euler noted that for any real number  $S > 1$  the series  $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$  is convergent and in fact, for every  $S > 1$  it is uniformly convergent on the half line extending from  $S$  to infinity. Thus it defines a function for  $S$  between one and infinity which is continuous and differentiable. This function is called the zeta function. Euler formulated a product which expresses the unique factorization of integers as product of primes. This product is the link between the zeta function and the prime numbers with the same idea used in his proof of the existence of infinitely many primes, Euler proved also that the sum of the inverses of prime numbers is divergent. Riemann went beyond Euler by defining the zeta function for complex numbers  $s$  having real parts greater than one. The Euler product formula still holds for every complex  $s$  with  $\text{Re}(s) > 1$ . Formally speaking, following Riemann, the Riemann zeta function is the function of the complex variable  $s = a + bi$  ( $i = \sqrt{-1}$ ), defined in the half plane  $a > 1$  by an absolute convergent series and in the whole complex plane by analytic continuation. The Riemann zeta function is every where holomorphic except at  $s=1$ , where it has a simple pole with residue 1. The Riemann zeta function has zeros at the negative even integers  $-2, -4, \dots$ . These zeros are known as the trivial zeros. The Riemann hypothesis states that the nontrivial zeros of the Riemann zeta function have real part equal to 0.5 [1,2]. I assume that any such zero is of the form  $s = a + bi$ , that is because it is well known that the nontrivial zeros of the Riemann zeta function are all complex and their real parts lie between zero and one [3]. I use the integral definition of the Riemann zeta function as the starting point of my proof [4,5]. I carry out the integral through many steps by resorting to many rules and techniques: e.g. separating the real and imaginary parts, applying the functional equation and using the rules of double integral. For instance, if the limits of integration do not involve variables, the product of

integrals can be expressed as a double integral to carry out the integral, I consider the region of integration as the union of three subregions. This results in a bounded integral form. In the second part of the proof, I employ variational calculus as it is applied in field quantization [6]. The variation of the integral due to a variation of the integrand is estimated. In general, the functional derivative tells how the value of the functional changes, if the function is changed at a given point. For example, if the functional carries an additional dependence on an index (i.e., parameter and therefore has a parametric dependence) then the derivative with respect to the function simply gives the integral kernel. Through a sequence of equations I treat (a) as a fixed exponent and verify that  $a=0.5$ . From this result onward I consider (a) as a parameter ( $a < 0.5$ ) (a parametric dependence) and use variational calculus and simple methods of integration to arrive at a contradiction. At the end of the proof and employing the assumption that (a) is a parameter, I verify again that  $a=0.5$ . At last I consider the case  $a > 0.5$ . This case is also rejected, since according to the functional equation, if the Riemann zeta function has a root with  $a > 0.5$ , then it must have another root with another value of  $a < 0.5$  [1]. But this last case with  $a < 0.5$  has already been rejected. All in all, what we are left with is the only possible value of which is  $a=0.5$ . Therefore  $a=0.5$  and the Riemann hypothesis is proved.

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