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Short Communication

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Improvement Bryc's Approximation to the Cumulative Distribution Function

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Abstract

We develop two news approximations to the cumulative distribution function. We begin by improving the accuracy of Cadwell's approximation. We reduce the accuracy from 0.006466 to 1.6635e-004. For the second approximation we reduce the accuracy of Bryc's approximation from 7.062e-004 to 2.072e-005. As a performance, we use a Maximum Absolute Error (M.A.E.). We recommend these two new approximations for their high accuracy.

Keywords: Cumulative distribution function; Maximum absolute error; Cadwell's approximation; Bryc's approximation

Introduction

Several domains of engineering, statistics or applied mathematics need the Cumulative Distribution Function (C.D.F.). In statistics, the Probability Density Function (P.D.F.) defined by:

$$\varphi(\mathrm{t})=\frac{e-\frac{t^2}{2}}{\sqrt{2\pi}};\quad -\infty<t<+\infty \tag{1}$$

The Cumulative Distribution Function (C.D.F.) given by:

$$\phi(Z)=\int_{-\infty}^{Z}\varphi(t)\,dt \tag{2}$$

This C.D.F. does not have a closed form. Most books in probability and statistics insert tables of C.D.F. For each value of the variable Z non-in this tables, we need a computer for estimate Φ (Z) by elementary methods [1]. For this raison, in the literature we find many approximations to the C.D.F. with closed forms. In section two, we present Cadwell's approximation with his original form and we improve the accuracy of this approximation with four decimal places. In section three, we introduce an approximation with four decimals places and we improve his accuracy until five decimals places. In section four, we conclude our paper.

Improving Cadwell's Formula

In 1951, Cadwell presents his new formula for approximate the cumulative normal distribution [2]. This formula given by:

$$\phi_{cadwell}(\mathrm{z})\approx\frac{1}{2}\left\{1+\sqrt{1-e^{-Z^2\left(\frac{2}{\pi}+\frac{2(\pi-3)}{3\pi^2}Z^2\right)}}\right\} \tag{3}$$

The M.A.E. is:

$$E_{cadwell}=\max_{-6\le Z\le 6}|\Phi_{cadwell}(\mathrm{z})-\Phi(\mathrm{z})|\cong 0.00646 \tag{4}$$

Our new formula defined by:

$$\phi_{Malki}(\mathrm{z})\approx\frac{1}{2}\left\{1+\sqrt{1-e^{-z^2\left(0.6349114-0.0073962z^2\right)}}\right\} \tag{5}$$

$$E_{Malki}=\max_{-6\le Z\le 6}|\Phi_{Malki}(\mathrm{z})-\Phi(\mathrm{z})|=1.663503182035564e-004$$

We have then $E_{cadwell}\cong 38.87\times E_{Malki}$ (6) (Figure 1).

The M.A.E. for Cadwell is about 40 times that Malki.

Improving Bryc's Formula

In, 2002, Bryc presents in his paper the following formula [3].

$$\Phi_{Bryc}(\mathrm{z})=1-\frac{z+3.333}{2\times 3.333+7.32z+\sqrt{2\pi z^2}}e^{-\frac{Z^2}{2}} \tag{7}$$

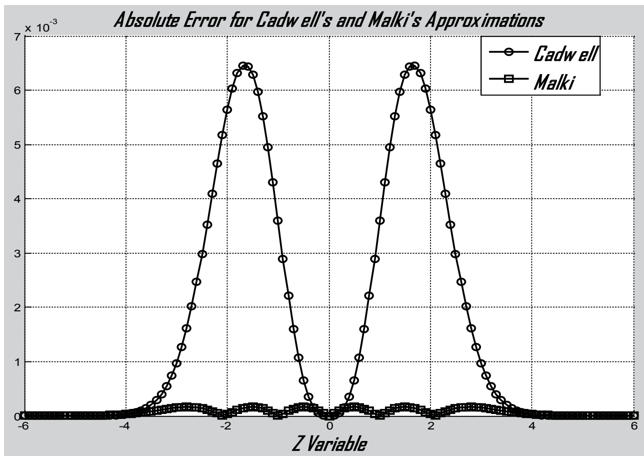

Figure 1: Shows the curves of absolute error for Cadwell and our new formula Malki.

The maximum absolute error is:

$$E_0=\max_z|\phi_{Bryc}(\mathrm{z})-\Phi(\mathrm{z})|=7.062660715300151e-004 \tag{8}$$

We can write formula (8) as

$$\Phi_{Bryc}(\mathrm{z})=1-\frac{\frac{z}{\sqrt{2\pi}}+\frac{3.333}{\sqrt{2\pi}}}{2\times\frac{3.333}{\sqrt{2\pi}}+\frac{7.32}{\sqrt{2\pi}}z+z^2}e^{-\frac{Z^2}{2}}$$

This formula has the form

$$\Phi_{Bryc}(\mathrm{z})=1-\frac{a\,z+b}{2\times b+cz+z^2}e^{-\frac{Z^2}{2}} \tag{9}$$

We improve the accuracy of this formula by our new formula

$$\Phi_{Malki2017}(\mathrm{z})=1-\frac{0.3656123z+1.4578862}{2\times 1.4578862+3.054682z+z^2}e^{-\frac{Z^2}{2}} \tag{10}$$

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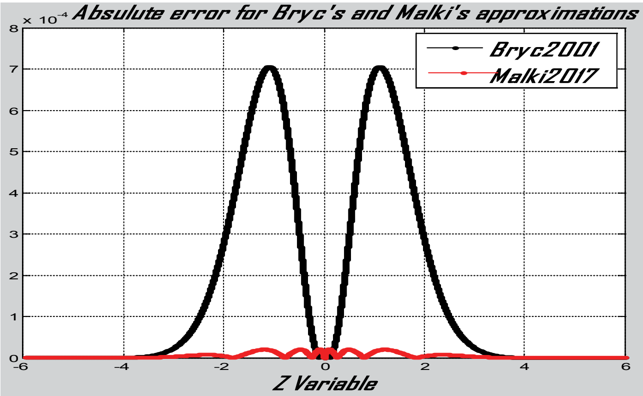

Figure 2: Shows the comparison of maximum absolute error for Bryc2001 and our new formula.

The maximum absolute error is:

$$E_1 = \max_z |\phi_{Malki2017}(z) - \Phi(z)| = 2.072294382748918e-005 \quad (11)$$

Hence, we have a ratio

$$E_0 \cong 34.1 \times E_1 \quad (12)$$ (Figure 2)

Conclusion

This paper presents two approximations to the cumulative normal distribution and their improving approximations. The first approximation is two decimals places; we improve it from two decimals places to four decimals places. In the second approximation, we improve the Bryc's formula by our second new formula that reduce the accuracy is about 34 times. We recommend these two new approximations.

References

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