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Jordan Triple Derivation on Alternative Rings

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Abstract

Let $\mathcal{D}$ be a mapping from an alternative ring $\mathcal{R}$ into itself satisfying $\mathcal{D}(a\cdot ba)=\mathcal{D}(a)\cdot ba+a\cdot\mathcal{D}(b)a+a\cdot b\mathcal{D}(a)$ for all a, $b \in \mathcal{R}$. Under some conditions on $\mathcal{R}$, we show that $\mathcal{D}$ is additive.

Keywords: Alternative ring; Idempotent element; Maps; Additivity

Introduction

In this paper, $\mathcal{R}$ will be a ring not necessarily associative or commutative and consider the following convention for its multiplication operation: $xy\cdot z=(xy)z$ and $x\cdot yz=x(yz)$ for $x, y, z \in \mathcal{R}$, to reduce the number of parentheses. For $x; y; z \in \mathcal{R}$ we denote the associator by $(x, y, z)=(xy)z-x(yz)$.

A ring $\mathcal{R}$ is called k-torsion free if $kx=0$ implies $x=0$; for any $x \in \mathcal{R}$; where $k \in \mathbb{Z}$, $k>0$, prime if $IJ\neq 0$ for any two nonzero ideals $I, J \subseteq \mathcal{R}$ and semiprime if it contains no nonzero ideal whose square is zero.

A ring $\mathcal{R}$ is said to be alternative if,

$(x, x, y)=0=(y, x, x)$, for all $x, y \in \mathcal{R}$,

and flexible if,

$(x, y, x)=0$, for all $x, y \in \mathcal{R}$,

One easily sees that any alternative ring is flexible.

Theorem 1.1 Let $\mathfrak{R}$ be a 3-torsion free alternative ring. So $\mathfrak{R}$ is a prime ring if and only if $a\mathfrak{R}\cdot b=0$ (or $a\cdot\mathfrak{R}\, b=0$) implies $a=0$ or $b=0$ for $a, b \in \mathfrak{R}$.

Proof: Clearly all alternative rings satisfying the properties $a\mathfrak{R}b=0$ (or $a\cdot\mathfrak{R}b=0$) are prime rings. Suppose $\mathfrak{R}$ is a prime ring by [1] Lemma 2:4, Theorem A and Proposition 3:5] we have $\mathfrak{R}=\mathcal{A}_0 \supseteq \mathcal{A}_1 \supseteq \ldots \supseteq \mathcal{A}_n=\mathcal{A} \neq (0)$ is a chain of subrings of $\mathfrak{R}$. If $a\mathfrak{R}\cdot b=0$ (or $a\cdot\mathfrak{R}\, b=0$) hence $a\mathcal{A}\cdot b=0$ (or $a\cdot\mathcal{A}b=0$) follows [1] Proposition 3.5 (e)] that $a=0$ or $b=0$.

A mapping $\mathcal{D}$: $\mathcal{R} \to \mathcal{R}$ is Jordan triple multiplicative derivation if,

$\mathcal{D}(a\cdot ba)=\mathcal{D}(a)\cdot ba+a\mathcal{D}(b)a+a\cdot b\mathcal{D}(a)$,

for all a, $b \in \mathcal{R}$. It is worth noting that by the flexible identity of alternative rings we can write,

$\mathcal{D}(aba)=\mathcal{D}(a)\cdot ba+a\mathcal{D}(b)a+a\cdot b\mathcal{D}(a)$,

for all $a, b \in \mathcal{R}$. Let us consider $\mathcal{R}$ an alternative ring and let us x a nontrivial idempotent $e_1 \in \mathcal{R}$, i.e, $e_1^2=e_1$; $e_1 \neq 0$ and e_1 is not an unity element. Let e_2: $\mathcal{R} \to \mathcal{R}$ and $e_2' = \mathcal{R} \to \mathcal{R}$ be given by $e_2a=a-e_1a$ and $e_2'a=a-ae_1$. We shall denote $e_2'a$ by ae_2. Note that $\mathcal{R}$ need not have an identity element. The operation $x(1-y)$ for $x, y \in \mathcal{R}$ is understood as $x-xy$: It is easy to see that $(e_ia)e_j=e_i(ae_j)$ for all $a\in\mathcal{R}$ and $i, j=1, 2$. Then $\mathcal{R}$ has a Peirce decomposition $\mathcal{R}=\mathcal{R}_{11}\oplus\mathcal{R}_{12}\oplus\mathcal{R}_{21}\oplus\mathcal{R}_{22}$, where $\mathcal{R}_{ij}=e_i\mathcal{R}e_j$ ($i, j=1, 2$), satisfying the multiplicative relations:

(i) $\mathcal{R}_{ij}\mathcal{R}_{jl}\subseteq\mathcal{R}_{il}$ ($i, j, l=1, 2$);

(ii) $\mathcal{R}_{ij}\mathcal{R}_{ij} \subseteq \mathcal{R}_{ji}$ ($i, j=1, 2$; $i \neq j$);

(iii) $\mathcal{R}_{ij}\mathcal{R}_{kl}=0$, if $j \neq k$ and $(i, j) \neq (k, l)$; ($i, j, k, l=1, 2$);

(iv) $x_{ij}^2=0$ for all $x_{ij} \in \mathcal{R}_{ij}$ ($i; j=1, 2$; $i \neq j$).

Remark 1.1

By the linearization of (iv) we obtain,

$x_{ij}y_{ij}+y_{ij}x_{ij}=0$

if $i \neq j$. This identity is very useful for the main result to be verified.

The study of the relationship between the multiplicative and the additive structures of a ring has become an interesting and active topic in ring theory. In non-associative ring theory we can mention recent works such as [2-5] where the authors generalized the results for a class of non-associative rings, namely alternative rings. The present paper we investigate the problem of when a Jordan triple multiplicative derivation must be an additive map for the class of alternative rings. The hypotheses of the main Theorem allow the author to make its proof based on calculus using the Peirce decomposition notion for Alternative rings. But it is worth noting that the notion of Peirce decomposition for the alternative rings is similar to the notion of Peirce decomposition for the associative rings. However, the similarity of this notion is only in its written form, but not in its theoretical structure because the Peirce decomposition for alternative rings is the generalization of the Peirce decomposition for associative rings. The symbol "$\cdot$", as defined in the introduction section of our article, is essential to elucidate how the non-associative multiplication should be done, and also the symbol "$\cdot$" is used to simplify the notation. Therefore, the symbol "$\cdot$" is crucial to the logic, characterization and generalization of associative results to the alternative results. In this paper we shall continue the line of research introduced in refs. [6,7] where its authors demonstrate the following results.

Theorem 1.2

Let $\mathcal{R}$ be an alternative ring containing a non-trivial idempotent e_1 and $\mathcal{R}=\mathcal{R}_{11}\oplus\mathcal{R}_{12}\oplus\mathcal{R}_{21}\mathcal{R}_{22}$, the Peirce Decomposition of $\mathcal{R}$; relative to e_1, satisfying:

(i) If $((e_ia)e_j)x_{jk}=0$ for all $x_{jk}\in\mathcal{R}_{jk}$; then $((e_ia)e_j)=0$;

(ii) If $x_{ij}((e_ja)e_i)=0$ for all $x_{ij} \in \mathcal{R}_{ij}$; then $((e_ja)e_i)=0$,

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for $i, j, k \in \{1, 2\}$. If $D:\mathcal{R}\to\mathcal{R}$ is a multiplicative derivation, then D is additive.

And,

Theorem 1.3

Let $\mathcal{R}$ be an alternative ring containing a non-trivial idempotent e_1 and $\mathcal{R}=\mathcal{R}_{11}\oplus\mathcal{R}_{12}\oplus\mathcal{R}_{21}\mathcal{R}_{22}$, the Peirce Decomposition of $\mathcal{R}$, relative to e_1, satisfying:

(i) If $(e_iae_j)x_{jk=}0$ for all $x_{jk} \in \mathcal{R}_{jk}$, then $(e_iae_j)=0$;

(ii) If $x_{ij}(e_jae_k)=0$ for all $x_{ij} \in \mathcal{R}_{ij}$, then $(e_jae_k)=0$;

(iii) If $(e_iae_i)x_{ii}+x_{ii}(e_iae_i)=0$ for all $x_{ii} \in \mathcal{R}_{ii}$, then $(e_iae_i)=0$.

for $i, j, k \in\{1, 2\}$. If $D: \mathcal{R} \to\mathcal{R}$ is a Jordan multiplicative derivation, then D is additive.

The Main Theorem

We shall prove the following result:

Theorem 2.1

Let $\mathcal{R}$ be an alternative ring containing a non-trivial idempotent e_1 and $\mathcal{R}=\mathcal{R}_{11}\oplus\mathcal{R}_{12}\oplus\mathcal{R}_{21}\mathcal{R}_{22}$, the Peirce Decomposition of $\mathcal{R}$; relative to e_1, satisfying:

(i) If $(e_1ae_1)x_{12}=0$ for all $x_{12} \in \mathcal{R}_{12}$; then $(e_1ae_1)=0$;

(ii) If $x_{12}(e_2ae_2)=0$ for all $x_{12} \in \mathcal{R}_{12}$; then $(e_2ae_2)=0$;

(iii) If $x_{ij}ax_{ij}=0$ for all $x_{ij} \in \mathcal{R}_{ij}$; then $(e_jae_i)=0$.

for i, j 2 {1; 2}. If $\mathcal{D}$: $\mathcal{R} \to\mathcal{R}$ is a Jordan triple multiplicative derivation, then $\mathcal{D}$ is additive.

The proof of the Theorem is organized as a series of Lemmas.

We begin with the following Lemma with a simple proof.

Lemma 2.1: $\mathcal{D}(0)=0$:

Proof: $\mathcal{D}(0)=\mathcal{D}(000)=\mathcal{D}(0)\cdot 00+0\mathcal{D}(0)0+0\cdot 0\mathcal{D}(0)=0$.

Lemma 2.2: $\mathcal{D}(a_{11}+b_{12}+c_{21}+d_{22})=\mathcal{D}(a_{11})+\mathcal{D}(b_{12})+\mathcal{D}(c_{21})+\mathcal{D}(d_{22})$.

Proof: For any $x_{ij} \in \mathcal{R}$, i, j=1, 2, on one hand, we have,

$$\begin{aligned}\mathcal{D}[x_{ij}(a_{11}+b_{12}+c_{21}+d_{22})x_{ij}] = {}&\mathcal{D}(x_{ij})\cdot(a_{11}+b_{12}+c_{21}+d_{22})x_{ij}\\ &+x_{ij}\mathcal{D}(a_{11}+b_{12}+c_{21}+d_{22})x_{ij}\\ &+x_{ij}\cdot(a_{11}+b_{12}+c_{21}+d_{22})\mathcal{D}(x_{ij}).\end{aligned}$$

On the other hand,

These imply that,

$$\begin{aligned}&\mathcal{D}[x_{ij}(a_{11}+b_{12}+c_{21}+d_{22})x_{ij}] - \mathcal{D}(x_{ij}a_{11}x_{ij})\\ &-\mathcal{D}(x_{ij}b_{12}x_{ij}) -\mathcal{D}(x_{ij}c_{21}x_{ij}) - \mathcal{D}(x_{ij}d_{22}x_{ij})\\ &= x_{ij}[\mathcal{D}(a_{11}+b_{12}+c_{21}+d_{22}) - \mathcal{D}(a_{11}) - \mathcal{D}(b_{12}) - \mathcal{D}(c_{21}) - \mathcal{D}(d_{22})]x_{ij},\end{aligned}$$

where we use the flexible identity. By the flexible identity we note that for any i, j=1, 2, we have,

$$\begin{aligned}&\mathcal{D}[x_{ij}(a_{11}+b_{12}+c_{21}+d_{22})x_{ij}]-\mathcal{D}(x_{ij}a_{11}x_{ij})\\ &-\mathcal{D}(x_{ij}b_{12}x_{ij})-\mathcal{D}(x_{ij}c_{21}x_{ij})-\mathcal{D}(x_{ij}d_{22}x_{ij}) = 0.\end{aligned}$$

Then, for i, j=1, 2, we get,

$$x_{ij}[\mathcal{D}(a_{11}+b_{12}+c_{21}+d_{22}) - \mathcal{D}(a_{11}) - \mathcal{D}(b_{12}) - \mathcal{D}(c_{21}) - \mathcal{D}(d_{22})]x_{ij}=0.$$

By Condition (iii), we see that,

$$[\mathcal{D}(a_{11}+b_{12}+c_{21}+d_{22}) - \mathcal{D}(a_{11}) - \mathcal{D}(b_{12}) - \mathcal{D}(c_{21}) - \mathcal{D}(d_{22})]_{ji}=0,\ i, j=1, 2.$$

Equivalently,

$$\mathcal{D}(a_{11}+b_{12}+c_{21}+d_{22}) - \mathcal{D}(a_{11}) - \mathcal{D}(b_{12}) - \mathcal{D}(c_{21}) - \mathcal{D}(d_{22})=0.$$

Lemma 2.3: 1. $\mathcal{D}(a_{12}+b_{12}c_{22})=\mathcal{D}(a_{12})+\mathcal{D}(b_{12}c_{22})$,

2. $\mathcal{D}(a_{21}+b_{22}c_{21})=\mathcal{D}(a_{21})+\mathcal{D}(b_{22}c_{21})$.

Proof: We will prove only (1) because the proof of (2) is similar to (1). By Remark 1.1 we note that,

$$e_1+a_{12}+b_{12}a_{12}+a_{12}b_{12}+b_{12}c_{22}=(e_1+a_{12}+c_{22})(e_1+b_{12})(e_1+a_{12}+c_{22}),$$

By applying Lemma 2.2, Remark 1.1 and flexible identity we have,

$$\begin{aligned}\mathcal{D}(e_1)+\mathcal{D}(a_{12}+b_{12}c_{22}) &= \mathcal{D}(e_1+a_{12}+b_{12}c_{22})\\ &= \mathcal{D}[(e_1+a_{12}+c_{22})(e_1+b_{12})(e_1+a_{12}+c_{22})]\\ &= \mathcal{D}(e_1+a_{12}+c_{22})\cdot(e_1+b_{12})(e_1+a_{12}+c_{22})\\ &\quad+(e_1+a_{12}+c_{22})\mathcal{D}(e_1+b_{12})(e_1+a_{12}+c_{22})\\ &\quad+(e_1+a_{12}+c_{22})\cdot(e_1+b_{12})\mathcal{D}(e_1+a_{12}+c_{22})\\ &= [\mathcal{D}(e_1)+\mathcal{D}(a_{12})+\mathcal{D}(c_{22})]\cdot(e_1+b_{12})(e_1+a_{12}+c_{22})\\ &\quad+(e_1+a_{12}+c_{22})[\mathcal{D}(e_1)+\mathcal{D}(b_{12})](e_1+a_{12}+c_{22})\\ &\quad+(e_1+a_{12}+c_{22})\cdot(e_1+b_{12})[\mathcal{D}(e_1)+\mathcal{D}(a_{12}+\mathcal{D}(c_{22})]\\ &= [\mathcal{D}(e_1)+\mathcal{D}(a_{12})+\mathcal{D}(c_{22})]\cdot e_1(e_1+a_{12}+c_{22})+(e_1+a_{12}+c_{22})\mathcal{D}(e_1)(e_1+a_{12}+c_{22})\\ &\quad+(e_1+a_{12}+c_{22})\cdot e_1[\mathcal{D}(e_1)+\mathcal{D}(a_{12})+\mathcal{D}(c_{22})]+[\mathcal{D}(e_1)+\mathcal{D}(a_{12})+\mathcal{D}(c_{22})]\cdot b_{12}(e_1+a_{12}+c_{22})\\ &\quad+(e_1+a_{12}+c_{22})\mathcal{D}(b_{12}(e_1+a_{12}+c_{22})+(e_1+a_{12}+c_{22})\cdot b_{12}[\mathcal{D}(e_1)+\mathcal{D}(a_{12})+\mathcal{D}(c_{22})]\\ &= \mathcal{D}[(e_1+a_{12}+c_{22})e_1(e_1+a_{12}+c_{22})]+\mathcal{D}[(e_1+a_{12}+c_{22})b_{12}(e_1+a_{12}+c_{22})]\\ &= \mathcal{D}(e_1+a_{12})+\mathcal{D}(b_{12}a_{12}+b_{12}c_{22}+a_{12}b_{12}+a_{12}b_{12}a_{12})\\ &= \mathcal{D}(e_1)+\mathcal{D}(a_{12})+\mathcal{D}(b_{12}c_{22}).\end{aligned}$$

To Prove (2) just use the identity.

$$e_1+a_{21}+b_{22}c_{21}=(e_1+a_{21}+b_{22})\cdot(e_1+c_{21})(e_1+a_{21}+b_{22}).$$

Lemma 2.4: $\mathcal{D}(a_{ij}+b_{ij})=\mathcal{D}(a_{ij})+\mathcal{D}(b_{ij})$, $i \neq j$.

Proof: For any $x_{ij} \in \mathcal{R}_{ij}$, i, j=1, 2, from,

$$\mathcal{D}[x_{ij}(a_{ij}+b_{ij})x_{ij}]=0=\mathcal{D}(x_{ij}a_{ij}x_{ij})+\mathcal{D}(x_{ij}b_{ij}x_{ij}),$$

we can get,

$$x_{ij}[\mathcal{D}(a_{ij}+b_{ij}) - \mathcal{D}(a_{ij}) - \mathcal{D}(b_{ij})]x_{ij=}0.$$

This implies, by conditions of the Theorem 2.1 that,

$$[\mathcal{D}(a_{ij}+b_{ij}) - \mathcal{D}(a_{ij}) \subseteq \mathcal{D}(b_{ij})]_{ii}=[\mathcal{D}(a_{ij}+b_{ij}) - \mathcal{D}(a_{ij}) - \mathcal{D}(b_{ij})]_{ji}=0.$$

This completes the proof.

Lemma 2.5: $\mathcal{D}(a_{kk}+b_{kk})=\mathcal{D}(a_{kk})+\mathcal{D}(b_{kk})$.

Proof: Let a_{kk} and b_{kk} be arbitrary elements of $\mathcal{R}_{kk}$, k=1; 2. By considering $\mathcal{D}[x_{ij}(a_{kk}+b_{kk})x_{ij}]$, $\mathcal{D}(x_{ij}a_{kk}x_{ij})$, and $\mathcal{D}(x_{ij}b_{kk}x_{ij})$ for the cases of $i \neq j$ and $i=j$ respectively, one can easily get that,

$$[\mathcal{D}(a_{kk}+b_{kk}) - \mathcal{D}(a_{kk}) - \mathcal{D}(b_{kk})]_{ji}=0,$$

$$[\mathcal{D}(a_{kk}+b_{kk}) - \mathcal{D}(a_{kk}) - \mathcal{D}(b_{kk})]_{jj}=0,$$

where in the second identity $i=j \neq k$: Now we have only to prove that,

$$[\mathcal{D}(a_{kk}+b_{kk}) - \mathcal{D}(a_{kk}) - \mathcal{D}(b_{kk})]_{jj}=0,$$

with $k=j$. For any $x_{12}\in\mathcal{R}_{12}$, $r_{11}\in\mathcal{R}_{11}$ and $r_{22}\in\mathcal{R}_{22}$, from,

$$r_{11}x_{12}=(e_1+r_{11})x_{12}(e_1+r_{11}) \tag{1}$$

and,

$$x_{12}r_{22}=(e_1+r_{22})x_{12}(e_1+r_{22}) \tag{2}$$

can check, by (1) and (2) that,

$$\mathcal{D}(r_{11}x_{12})=\mathcal{D}(e_1)\cdot r_{11}x_{12}+e_1\cdot\mathcal{D}(r_{11})x_{12}+r_{11}\mathcal{D}(x_{12})+\mathcal{D}(x_{12})r_{11}+x_{12}\cdot\mathcal{D}(r_{11})e_1; \tag{3}$$

$$\mathcal{D}(x_{12}r_{22})=\mathcal{D}(e_1)\cdot x_{12}r_{22}+e_1\cdot\mathcal{D}(x_{12})r_{22}+r_{22}\cdot\mathcal{D}(x_{12})e_1+x_{12}\mathcal{D}(r_{22}). \tag{4}$$

Now, applying equality (3) for $r_{11}=a_{11}+b_{11}$, $r_{11}=a_{11}$ and $r_{11}=b_{11}$ and applying equality (4) for $r_{22}=a_{22}+b_{22}$, $r_{22}=a_{22}$ and $r_{22}=b_{22}$, we can get,

$[\mathcal{D}(a_{11}+b_{11}) - \mathcal{D}(a_{11}) - \mathcal{D}(b_{11})]x_{12}=0,$

$x_{12}[\mathcal{D}(a_{22}+b_{22}) - \mathcal{D}(a_{22}) - \mathcal{D}(b_{22})]=0.$

It follows from Condition (i) and (ii) of the Theorem 2.1 that $[\mathcal{D}(a_{kk}+b_{kk}) - \mathcal{D}(a_{kk}) - \mathcal{D}(b_{kk})]_{jj}=0$, with $k=j$, which completes the proof.

Now we are ready to prove our main result.

Proof of the Theorem 2.1: For any a; $b \in \mathcal{R}$, we write $a=a_{11}+a_{12}+a_{21}+a_{22}$ and $b=b_{11}+b_{12}+b_{21}+b_{22}$. Applying the previous Lemmas, we have,

$$\begin{aligned}\mathcal{D}(a+b) &= \mathcal{D}(a_{11}+a_{12}+a_{21}+a_{22}+b_{11}+b_{12}+b_{21}+b_{22})\\ &= \mathcal{D}[(a_{11}+b_{11})+(a_{12}+b_{12})+(a_{21}+b_{21})+(a_{22}+b_{22})]\\ &= \mathcal{D}(a_{11}+b_{11})+\mathcal{D}(a_{12}+b_{12})+\mathcal{D}(a_{21}+b_{21})+\mathcal{D}(a_{22}+b_{22})\\ &= \mathcal{D}(a_{11})+\mathcal{D}(b_{11})+\mathcal{D}(a_{12})+\mathcal{D}(b_{12})+\mathcal{D}(a_{21})+\mathcal{D}(b_{21})+\mathcal{D}(a_{22})+\mathcal{D}(b_{22})\\ &= \mathcal{D}(a_{11}+a_{12}+a_{21}+a_{22})+\mathcal{D}(b_{11}+b_{12}+b_{21}+b_{22})\\ &= \mathcal{D}(a)+\mathcal{D}(b),\end{aligned}$$

i. e., $\mathcal{D}$ is additive.

Applications in Prime Alternative Rings

In the case of an unital alternative ring we have.

Corollary 3.1

Let $\mathcal{R}$ be an unital alternative ring containing a non-trivial idempotent e_1 and $\mathcal{R}=\mathcal{R}_{11}\oplus\mathcal{R}_{12}\oplus\mathcal{R}_{21}\oplus\mathcal{R}_{22}$; the Peirce Decomposition of $\mathcal{R}$; relative to e_1; satisfying:

(i) If $(e_1ae_1)x_{12}=0$ for all $x_{12}\in\mathcal{R}_{12}$; then $(e_1ae_1)=0$;

(ii) If $x_{12}(e_2ae_2)=0$ for all $x_{12}\in\mathcal{R}_{12}$; then $(e_2ae_2)=0$;

(iii) If $x_{ij}ax_{ij}=0$ for all $x_{ij}\in\mathcal{R}_{ij}$; $(i\neq j)$ then $(e_jae_i)=0$.

for $i; j \in \{1; 2\}$. If $\mathcal{D}: \mathcal{R}\rightarrow\mathcal{R}$ is a Jordan triple multiplicative derivation, then $\mathcal{D}$ is additive.

As a last result of our paper follows the Corollary.

Corollary 3.2

Let $\mathcal{R}$ be a 3-torsion free prime unital alternative ring with a non-trivial idempotent. If mapping $\mathcal{D}:\mathcal{R}\rightarrow\mathcal{R}$ satisfies,

$\mathcal{D}(aba)=\mathcal{D}(a)\cdot ba+a\mathcal{D}(b)a+a\cdot b\mathcal{D}(a)$.

for all $a, b \in \mathcal{R}$, then $\mathcal{D}$ is additive.

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