On Isaiah Kantor (1936–2006)

Leo Landau once said that all physicists can be divided into physicists-composers and physicists-performers. Issai Kantor is, in my opinion, a mathematician-composer. Here are two his compositions on the theme of the Jordan algebras.

Jacques Tits in his study of models for exceptional Lie algebras made the following observation. Let L be a Lie algebra over a field $\mathbb{F}$, which contains the Lie algebra

$$\mathrm{sl}(2, \mathbb{F}) = \mathbb{F}e \oplus \mathbb{F}f \oplus \mathbb{F}h$$
$$[e, f] = h, \quad [h, f] = -2f, \quad [h, e] = 2e$$

If the adjoint operator $\mathrm{ad}(h)$ acting on L has only the eigenvalues $-2, 2, 2$, then the 2-eigenspace $L(2)$ with the operation $x \bullet y := [[x, f], y]$ is a Jordan algebra.

Issai Kantor generalized it in the following way. Let L be a $\mathbb{Z}$-graded Lie algebra,

$$L = L(-1) \oplus L(0) \oplus L(1)$$

Then for an arbitrary element a from $L(-1)$ the operation $x \bullet y := [[x, a], y]$ defines a structure of a Jordan algebra on $L(1)$. This lead to the notions of the *Jordan triple systems and pairs* (M. Koecher came to the same structure from the side of Hermitian symmetric spaces).

Moreover, Kantor and Koecher independently showed that every Jordan algebra arises in this way from a $\mathbb{Z}$-graded Lie algebra via the *Tits-Kantor-Koecher construction.*

Kantor's observation about the operation $[[x, a], y]$ played a crucial role in my proof of the Restricted Burnside problem.

Another brilliant example of Kantor's insight is the discovery that every Poisson bracket leads to a *Jordan superalgebra.* Let R be an associative commutative algebra with a bilinear bracket $[\cdot, \cdot] : R \times R \to R$. The bracket is said to be a *Poisson bracket* if it satisfies the product rule and $(R, [\cdot, \cdot])$ is a Lie algebra. Kantor noticed that the superalgebra $J := R \oplus \overline{R}$, where

$$a\overline{b} = \overline{ab}, \quad \overline{a}\overline{b} = [a, b]$$

is a Jordan superalgebra, which is now called the *Kantor double* of R. The application of this construction to the supercommutative Grassmann algebras yielded the first examples of finite dimensional Jordan superalgebras with nonsemisimple even parts.

Kantor had fantastic intuition and a sense of what is important.

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