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Lie Group for MHD and Reaction Porosity Effects of Variable Viscosity on Heat Generation and Mass Transfer Fluid

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Abstract

Magnetohydrodynamic and chemical reaction effects of variable viscosity on heat generation and mass transfer viscous dissipation fluid in the presence of suction/injection through porosity is solved numerically. Lie symmetry group transformations are used to convert the boundary layer equations into non-linear ordinary differential equations. Finally, numerical results are presented for velocity, temperature and concentration profiles for different parameters of the problem are studied. In addition, the effects of the pertinent parameters on the skin friction, the rate of heat transfer and mass fluxes are also discussed numerically.

Keywords: Lie group; Thermal reaction; Porous medium; Heat generation; Temperature depended fluid viscosity

Nomenclature

A: Non-dimensional fluid viscosity of temperature,

A_1: Non-dimensional fluid viscosity of concentration,

A: Positive constant,

B_0: Uniform magnetic field,

b: Positive constant,

c: Positive constant,

C_p: Heat capacity at constant pressure,

C_w: surface concentration,

C_∞: Full stream concentration,

D: Diffusional coefficient,

E_c: Eckert number $(=U_2/(T_w-T_\infty)c_p)$,

g: Temperature profile,

g^*: Gravity field,

h: Concentration profile,

L_e: Liwes number $(=v_0/D)$,

K: Permeability of the porous medium,

k^*: Mean absorption coefficient,

k_1: Rate of chemical reaction,

M: Magnetic field parameter, $\left(=\sigma B_0^2/\rho\right)$,

m: Power law exponent, is also constant,

Q_1: Volumetric heat source/sink rate,

P_r: Prandtl number $(=\mu_0 c_p/K)$,

q_r: Radiative heat flux,

T: Temperature profile,

T_w: Wall temperature,

$T\infty$: Full stream temperature,

S: Porosity parameter (=0 1/k),

R: Thermal radiation parameter (= * * 2 /4 ∞ k k σ T),

u: Components velocity in the x-direction,

v: Components velocity in the y-direction,

V: Velocity of suction fluid,

V_0: Constant velocity

Greek symbols

α_1: Temperature buoyancy parameter $(=g^*\beta/bv_0)$,

α_2: Concentration buoyancy parameter $(=g^*\beta^*/bv_0)$,

δ_1: Fluid viscosity of temperature $(=Av_0)$,

δ_2: Positive constant $(=av_0)$,

δ_3: Fluid viscosity of concentration $(=A_1v_0)$,

β: Volumetric coefficient of thermal expansion,

β^*: Volumetric coefficient of concentration expansion,

Γ: Heat generation/absorption parameter $(=Q_0/\mu_0c_p)$,

γ: Chemical reaction parameter $(=\Omega/v_0)$,

θ: Dimensional temperature profile,

ϕ: Dimensional concentration profile,

η: Similarity variable,

λ: Suction/injection parameter $(=4V_0/3v_0)$,

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K: Thermal conductivity,

μ: Viscosity of the fluid,

μ_0: Constant value viscosity of the fluid,

ρ: Density of the fluid,

v_0: kinematic viscosity of the fluid ($=\mu_0/\rho$),

σ: Electrical conductivity,

σ^*: Stephan–Boltzmann constant

Subscripts

w, ∞: Stand for the wall and free stream conditions

Introduction

In many engineering processes and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed bed catalytic reactors and cooling of nuclear reactors. Many practical diffusive operations involve the molecular diffusion of species in the presence of a chemical reaction within or at the boundary layer.

Lie group transformation, also called symmetry analysis, was developed by Sophus Lie to find point transformations which map a given partial differential equation to it. This method has been used by many researchers to solve some nonlinear problems in fluid mechanics [1-3]. Heat transfer in a liquid film on an unsteady stretching surface by Andersson et al. [4], the production of sheeting material arises in a number of industrial manufacturing processes and includes both metal and polymer sheets. It is well known that the flow in a boundary layer separates in the regions of adverse pressure gradient and the occurrence of separation has several undesirable effects in so far as it leads to increase in the drag on the body immersed in the flow and adversely affects the heat transfer from the surface of the body. Several methods have been developed for the purpose of artificial control of flow separation. Separation can be prevented by suction as the low-energy fluid in the boundary layer is removed [5,6].

Heat transfer in a porous medium over a stretching surface with internal heat generation and suction or injection by Elbashbeshy et al. [7], Mukhopadhyay et al. [8] studied of MHD boundary layer flow over a heated stretching sheet with variable Viscosity, Mukhopadhyay et al. [9] investigated effects of thermal radiation and variable fluid viscosity on free convective flow and heat transfer past a porous stretching surface.

Very recently, Loganathan et al. [10] studied Lie group analysis for the effects of variable fluid viscosity and thermal radiation on free convective heat and mass transfer with variable stream condition. Our aim in this analysis is to consider the effect of the Magneto hydrodynamic and chemical reaction of variable viscosity on heat generation and mass transfer viscous dissipation fluid in the presence of suction/injection through porosity. And the influence various physical parameters on the numerical results will also be discussed. In addition, the effects of the pertinent parameters on the skin friction, the rate of heat transfer and mass fluxes are also discussed.

Formulation of the Problem

Consider the steady, viscous dissipating laminar flow and heat generation and mass transfer fluid over a vertical stretching sheet emerging out of a slit at origin in the presence chemical reaction through a porous medium and moving with non-uniform velocity $U(x)$ under the influence of a transversely applied magnetic field B is considered. The temperature dependent fluid viscosity in the form $\mu=\mu_0[a+b(Tw-T)]$ [11]. The governing equation for the flow by using incompressible fluid is:

Continuity Eq.:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum Eq.:

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right)+g^*\beta(T-T_\infty)+g^*\beta(C-C_\infty)-\frac{\sigma B^2}{\rho}u-\frac{\mu(T)}{\rho K}u \tag{2}$$

Energy Eq.:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{Q_1}{\rho c_p}(T-T_\infty)+\frac{\mu(T)}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Diffusion Eq.:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\mathrm{D}\frac{\partial^2 C}{\partial y^2}-k_1(C-C_\infty) \tag{4}$$

The boundary conditions are:

$$u=U(x),\ v=-V_0,\ T=T_w,\ C=C_w \text{ at } y=0 \tag{5}$$

Using the Rossel and approximation (Rashed [12]), the radiative heat flux q_r is given by

$$\mathrm{qr}=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

Assuming that the temperature difference within the flow is sufficiently small such that T^4 could be approached as the linear function of temperature;

$$T^4 \cong 4T_\infty^3 T-3T_\infty^4 \tag{7}$$

We introduce the following relations for u, v, θ and ϕ as

$$u=\frac{\partial \psi}{\partial y}, v=\frac{\partial \psi}{\partial x}, T=T_\infty+(T_w-T_\infty)\theta \text{ and } C=C_\infty+(C_w-C_\infty)\varphi \tag{8}$$

where ψ is the stream function. The stream wise velocity and the suction/injection velocity are:

$$U(x)=cx^m,\ V(x)=V_0 x^{\frac{m-1}{2}} \tag{9}$$

We assumed the form of the magnetic field $B(x)=B_0x^{-1/4}$ and the permeability of the porous medium $K(x)=k_0x^{1/2}$.

Eqn. (1) is satisfied automatically and Eqns. (2)-(5) we get:

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y}-\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2}=-Av_0\frac{\partial \theta}{\partial y}\frac{\partial^2 \psi}{\partial y^2}+v_0\left[a+A(1-\theta)\right]\frac{\partial^3 \psi}{\partial y^3}+\frac{g^* A\beta}{b}\theta+\frac{g^* A_1\beta^*}{b}\varphi$$
$$-x-1/2\left[\frac{\sigma B_0^2}{\rho}+\frac{v_0}{k_0}\left(a+A(1-\theta)\right)\right]\frac{\partial \psi}{\partial y} \tag{10}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x}-\frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 \theta}{\partial y^2}+\frac{16\sigma^* T_\infty^3}{3k^*\rho c_p}\frac{\partial^2 \theta}{\partial y^2}+\frac{\mathrm{Q1}}{\rho c_p}\theta+\frac{v_0}{(T_w-T_\infty)c_p}\left[a+A(1-\theta)\right]\left(\frac{\partial^3 \psi}{\partial y^3}\right)^2, \tag{11}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x}-\frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y}=D\frac{\partial^2 \varphi}{\partial y^2}-k_1\varphi\cdot \tag{12}$$

and

$$\frac{\partial \psi}{\partial y}=cx^m,\ \frac{\partial \psi}{\partial x}V_0 x^{\frac{m-1}{2}},\ \theta=2,\ \varphi=1 \text{ at } y=0$$

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$$\frac{\partial\psi}{\partial y}\to 0,\ \theta\to 0,\ \varphi\to 0 \text{ as } y\to\infty \tag{13}$$

where $A=b(T_w-T_\infty)$ and $A_1=b(C_w-C_\infty)$.

Lie Group Transformations

The simplified form of Lie group transformations namely, the scaling group of transformations [8],

$$x^*=xe^{\varepsilon c_1},\ y^*=ye^{\varepsilon c_2},\ u^*=ue^{\varepsilon c_3},\ v^*=ve^{\varepsilon c_4},\ \psi^*=\psi e^{\varepsilon c_5},\ \theta^*=\theta e^{\varepsilon c_6},\ \varphi^*=\varphi e^{\varepsilon c_7}, \tag{14}$$

Eqn. (14) may be considered as point transformation which transform coordinates $(x, y, u, v,\psi,\theta,\phi)$ to the

Coordinates $(x^*, y, u^*, v^*,\psi^*,\theta^*,\phi^*)$.

Substituting eqn. (14) in eqns. (10)-(13) we get,

$$\begin{aligned} e^{\varepsilon(c_1+2c_2-2c_5)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial^2\psi^*}{\partial x^*\partial y^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial^2\psi^*}{\partial y^{*2}}\right) &= -Av_0e^{\varepsilon(3c_2-c_5-c_6)}\frac{\partial\theta^*}{\partial y^*}\frac{\partial^2\psi^*}{\partial y^{*2}}+v_0e^{\varepsilon(3c_2-c_5)}\left[a+A\left(1-\theta^* e^{-\varepsilon c_6}\right)\right]\frac{\partial^3\psi^*}{\partial y^{*3}}+ \\ &\quad \frac{g^*A\beta}{b}\theta^* e^{-\varepsilon c_6}+\frac{g^*A_1\beta^*}{b}\varphi^* e^{-\varepsilon c_7}-e^{\varepsilon\left(\frac{1}{2}c_1+c_2-c_5\right)}\left[\frac{\sigma B_0^2}{\rho}+\frac{v_0}{k_0}\left(a+A\left(1-\theta^* e^{-\varepsilon c_6}\right)\right)\right]\frac{\partial\psi^*}{\partial y^*} \end{aligned} \tag{15}$$

$$\begin{aligned} e^{\varepsilon(c_1+c_2-c_5-c_6)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial\theta^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial\theta^*}{\partial y^*}\right) &= e^{\varepsilon(2c_2-c_6)}\left(\frac{\kappa}{\rho c_p}\frac{\partial^2\theta^*}{\partial y^{*2}}+\frac{16\sigma^* T_\infty^3}{3k^*\rho c_p}\frac{\partial^2\theta^*}{\partial y^{*2}}\right)+\frac{Q_1}{\rho c_p}e^{-\varepsilon c_6}\theta^*+ \\ &\quad \frac{v_0}{(T_w-T_\infty)c_p}e^{\varepsilon(4c_2-2c_5)}\left(a+A\left(1-\theta^* e^{-\varepsilon c_6}\right)\right)\left(\frac{\partial^2\psi^*}{\partial y^{*2}}\right)^2 \end{aligned} \tag{16}$$

$$e^{\varepsilon(c_1+c_2-c_5-c_7)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial\varphi^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial\varphi^*}{\partial y^*}\right)=De^{\varepsilon(2c_2-c_7)}\frac{\partial^2\varphi^*}{\partial y^{*2}}k_1e^{-\varepsilon c_7}\varphi^* \tag{17}$$

and

$$e^{\varepsilon(c_2-c_5)}\frac{\partial\psi^*}{\partial y^*}=cx^{*m}e^{-\varepsilon c_1 m},\ e^{\varepsilon(c_2-c_5)}\frac{\partial\psi^*}{\partial y^*}=V_0x^{*\frac{m-1}{2}}e-^{\varepsilon\left(\frac{m-1}{2}\right)c_1},\ \theta^*=1,\ \varphi^*=1 \text{ at } y^*=0$$

$$\frac{\partial\psi^*}{\partial y^*}\to 0,\ \theta^*\to 0,\ \varphi^*\to 0 \text{ as } y^*\to\infty \tag{18}$$

The system will remain invariant under the group of transformations, we have

$$c_1+2c_2-2c_5=3c_2-c_5-c_6=3c_2-c_5=-c_6=-c_7=\frac{1}{2}c_1+c_2-c_5,$$
$$c_1+c_2-c_5-c_6=2c_2-c_6=-c_6=4c_2-2c_5=4c_2-2c_5-4c_2-2c_5-c_6,$$
$$c_1+c_2-c_5-c_7=2c_2-c_7=-c_7,$$
$$c_2-c_5=-c_1m \text{ and } c_1-c_5=-c_1\left(\frac{m-1}{2}\right).$$

In this study we take $m=\frac{1}{2}$ and c=1, we get:

$$c_2=\frac{c_1}{4},\ c_3=\frac{c_1}{2},\ c_4=\frac{c_1}{4},\ c_5=\frac{3c_1}{4},\ c_6=0 \text{ and } c_7=0 \tag{19}$$

In view of these, the boundary conditions become

$$\begin{aligned} &\frac{\partial\psi^*}{\partial y^*}=x^{*\frac{1}{2}},\ \frac{\partial\psi^*}{\partial x^*}=V_0x^{*\frac{-1}{4}},\ \theta^*=1,\ \varphi^*=1 \text{ at } y^*=0 \\ &\frac{\partial\psi^*}{\partial y^*}\to 0,\ \theta^*\to 0,\ \varphi^*\to 0 \text{ as } y^*\to\infty. \end{aligned} \tag{20}$$

We also assume volumetric heat source/sink rate $Q_1=Q_0x^{*-1/2}$ and the thermal conductivity $k_1=\Omega x^{*-1/2}$. The translation transformation in powers of ε and keeping terms up to the order ε, we have

$$x^*=x+\varepsilon xc_1,\ y^*=y+\frac{1}{4}\varepsilon yc_1,\ u^*=u+\frac{1}{2}\varepsilon uc_1,$$

$$\psi^*=\psi+\frac{3}{4}\varepsilon\psi c_1,\ \theta^*=\theta,\ \varphi^*=\varphi.$$

In terms of differentials these yield

$$\frac{dx}{xc_1}=\frac{dy}{\frac{1}{4}yc_1}=\frac{du}{\frac{1}{2}uc_1}=-\frac{dv}{\frac{1}{4}vc_1}=\frac{d\psi}{\frac{3}{4}\psi c_1}=\frac{d\theta}{0}=\frac{d\varphi}{0} \tag{21}$$

Solving (21), we get

$$\psi^*=x^{*3/4}f(\eta),\ \ \theta^*=g(\eta),\ \ \varphi^*=h(\eta),\ \ \eta=y^*x^{*-1/4} \tag{22}$$

Substituting (22) into (15-17) and (20), we get

$$\begin{aligned} &4\left[\delta_2+\delta_1(1-g)\right]f'''-4\delta_1g'f''-2f'^2+3ff''+4\delta_1\alpha_1g+4\delta_3\alpha_2h-4 \\ &\left[M+S\left(\delta_2+\delta_1(1-g)\right)\right]f'=0, \end{aligned} \tag{23}$$

$$4(3R+4)g''+3RP_r\left(3fg'+4\Gamma g\right)+3RP_rE_c\left(\delta_2+\delta_1(1-g)\right)f''=0, \tag{24}$$

$$4h''+L_e\left(3fh'-4\gamma h\right)=0, \tag{25}$$

with:

$$f(0)=\lambda,\ f'(0)=1,\ g(0)=1,\ h(0)=1,\ f'(\infty)\to 0,\ g(\infty)\to 0,\ h(\infty)\to 0 \tag{26}$$

Here λ is the suction ($\lambda>0$) and it is injection ($\lambda<0$).

Skin-Friction Coefficient, Nusselt Number and Sherwood Number

The parameters of engineering interest for the present problem are the skin-friction coefficient, Nusselt number and the Sherwood number which indicate the physical wall shear stress, rate of heat transfer and rate of mass transfer, respectively. Which are defined as:

$$C_f=\tau_w/\rho U^{1/2},\quad N_u=U^{1/2}\rho c_pq_w/\kappa(T_w-T_\infty) \text{ and } Sn=U^{1/2}J_w/D(C_w-C_\infty) \tag{27}$$

Where (τ_w) is the shear stress along the stretching sheet, (q_w) is the heat flux from the sheet and the mass flux (J_w) from the sheet and those are defined as

$$\tau_w=\mu(T)\left(\frac{\partial u}{\partial y}\right)_{y=0},\quad q_w=-\frac{\kappa}{\rho cp}\left(\frac{\partial T}{\partial y}\right)_{y=0} \text{ and } J_w=-D\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{28}$$

Hence, skin-friction coefficient C_f, the Nusselt numbers N_u and the Sherwood number S_n as follows:

$$C_f=(\delta_2+\delta_1(1-g(0)))f''(0),\ N_u=-g'(0) \text{ and } S_n=-h'(0) \tag{29}$$

Result and Discussion

Eqns. (23)-(26) are coupled nonlinear boundary value problems, these equations are solved numerically by fourth order mono-implicit Runge-Kutta method. It is difficult to study the influence of all parameters involved in the present problem.

In Figures 1-4(a, b and c), respectively; we have found that velocity profile increases, while temperature and concentration profiles decrease with the increase of each of fluid viscosity of temperature, fluid viscosity of concentration, temperature buoyancy and concentration buoyancy parameters. In absence of both magnetic field and porosity parameters effects on velocity, temperature and concentration which are illustrated in Figures 5 and 6(a, b and c), respectively; the velocity profile decreases, but the temperature and the concentration profiles increase with the increase of each of magnetic field and porosity parameters, and this is due to the fact that the thermal boundary layer increases with magnetic field parameter and porosity parameter.

The effects of the heat generation ($\Gamma>0$) and the heat absorption ($\Gamma<0$) on velocity, temperature and concentration profiles in Figure 7a-7c, respectively. It is noticed that the velocity and the temperature

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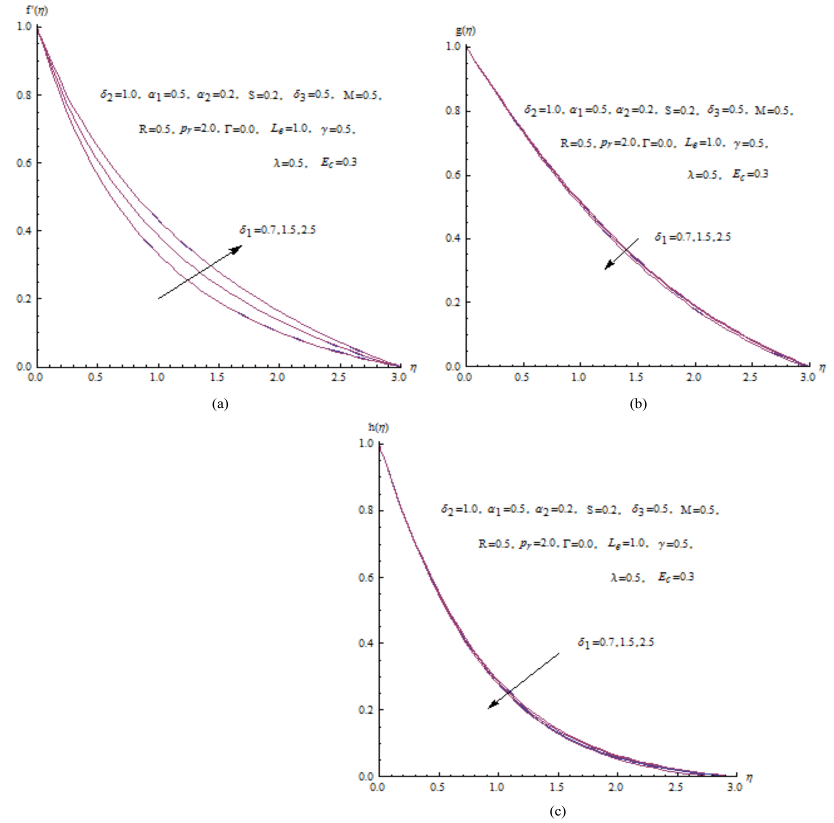

Figure 1: Influence of the fluid viscosity of temperature parameter on (a) velocity, (b) temperature and (c) concentration profiles.

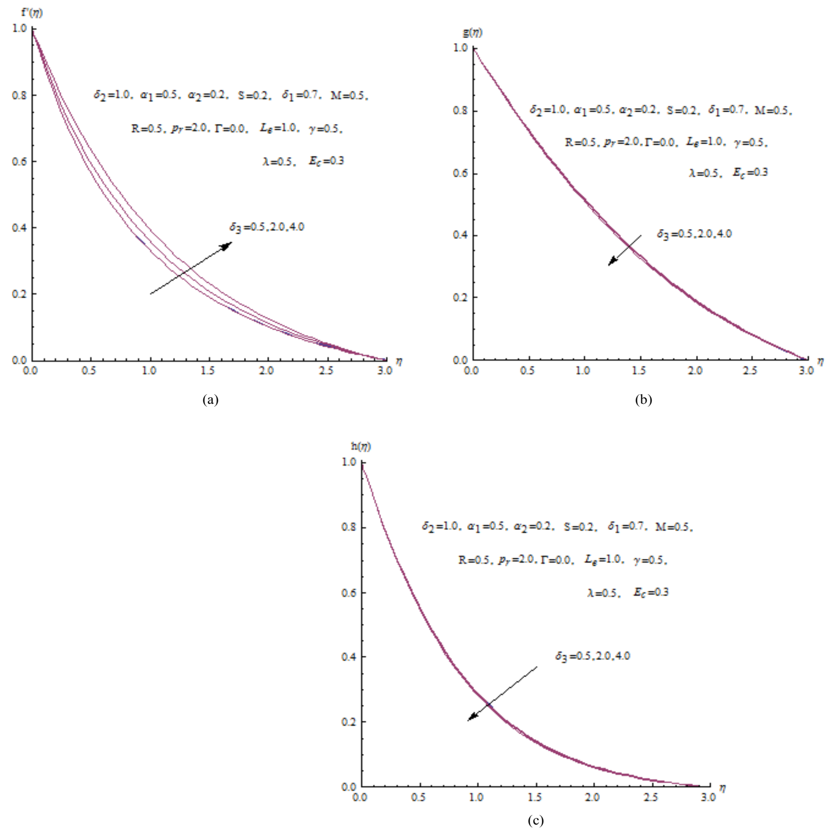

Figure 2: Influence of the fluid viscosity of concentration parameter on (a) velocity, (b) temperature and (c) concentration profiles.

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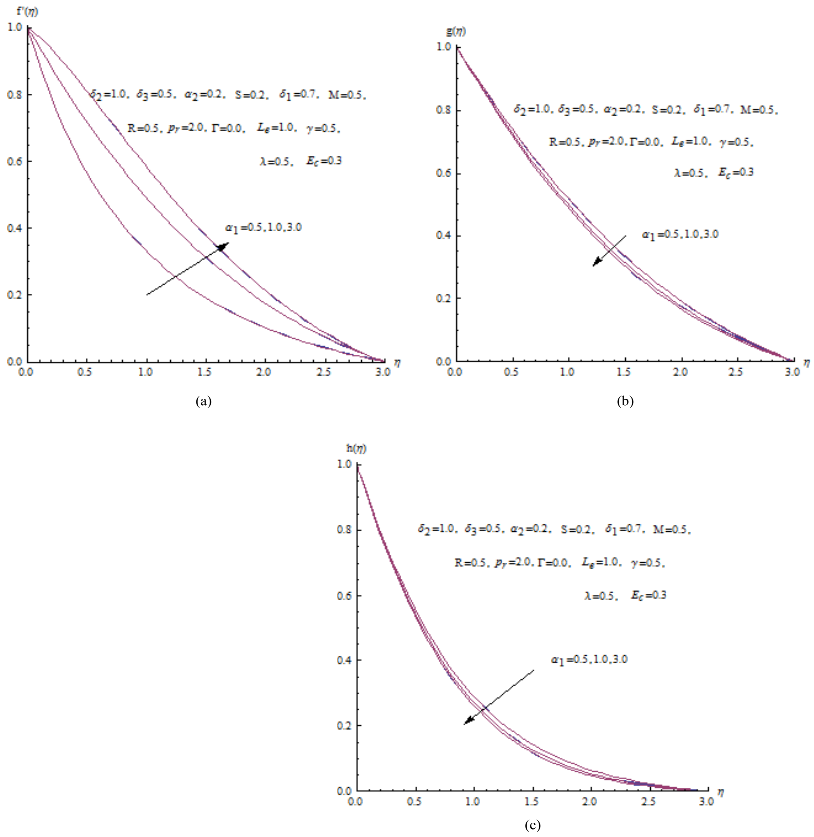

Figure 3: Influence of the temperature buoyancy parameter on (a) velocity, (b) temperature and (c) concentration profiles.

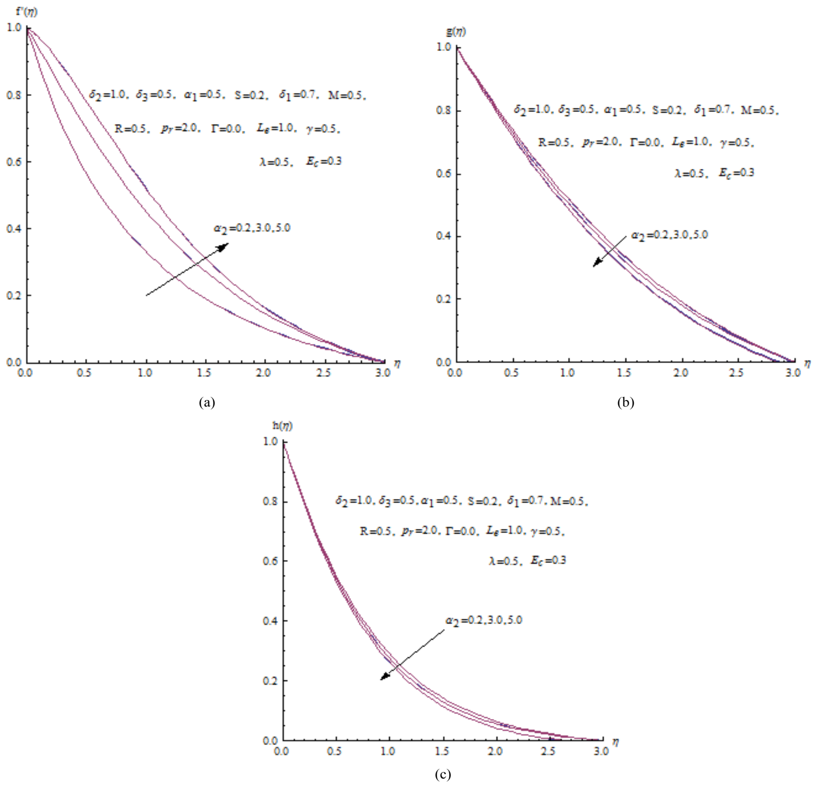

Figure 4: Influence of the concentration buoyancy parameter on (a) velocity, (b) temperature and (c) concentration profiles.

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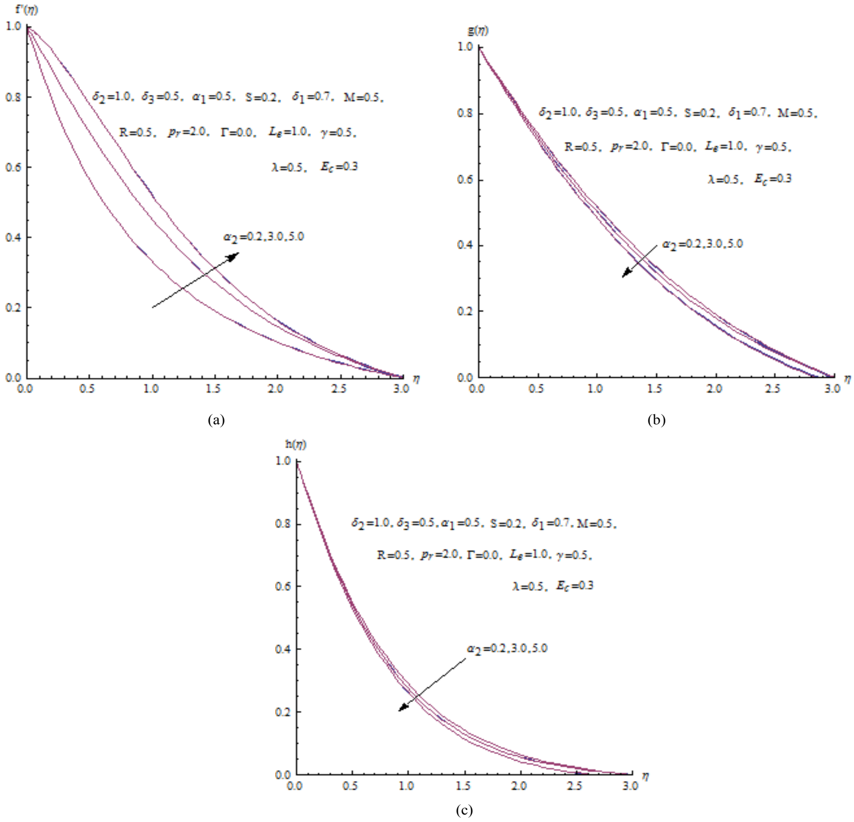

Figure 5: Influence of the magnetic field parameter on (a) velocity, (b) temperature and (c) concentration profiles.

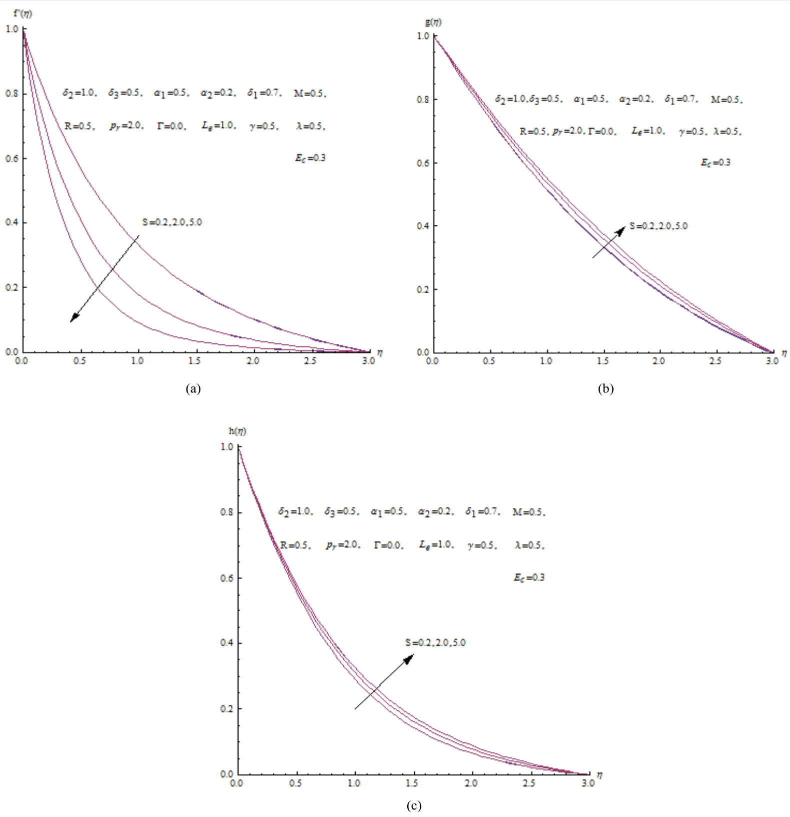

Figure 6: Influence of the porosity parameter on (a) velocity, (b) temperature and (c) concentration profiles.

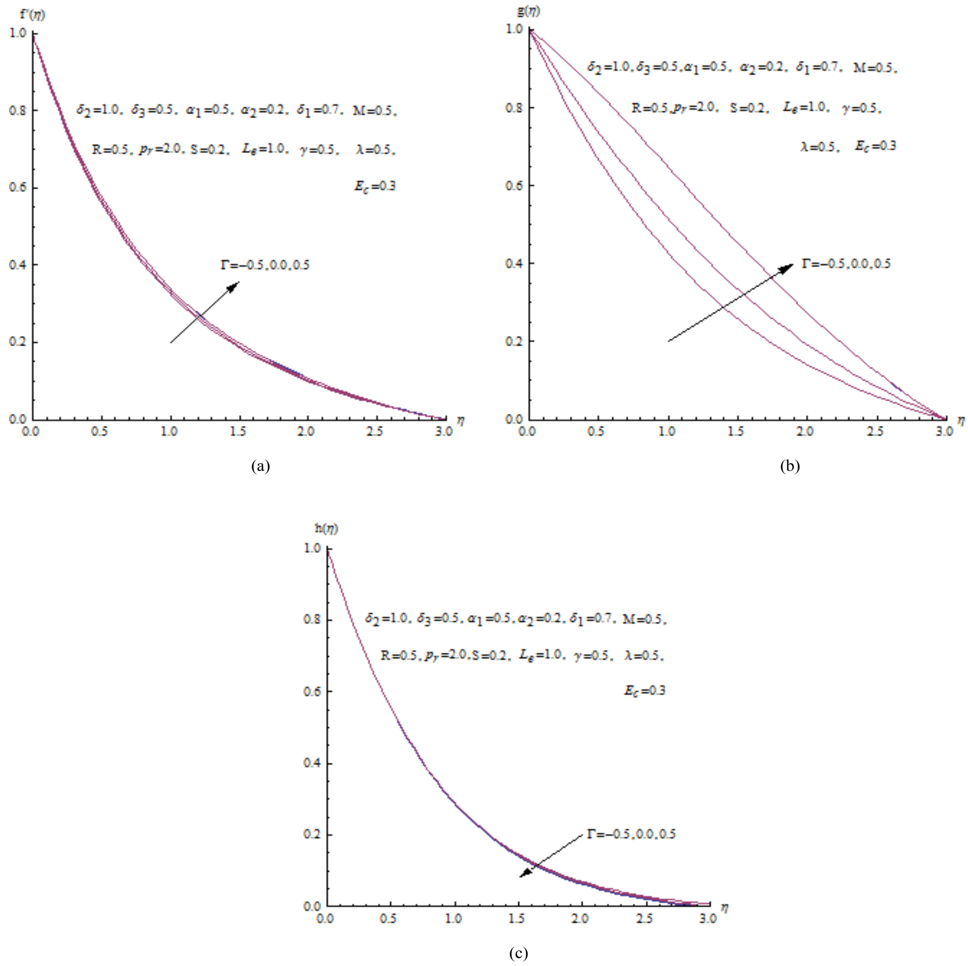

Figure 7: Influence of the heat generation/absorption parameter on (a) velocity, (b) temperature and (c) concentration profiles.

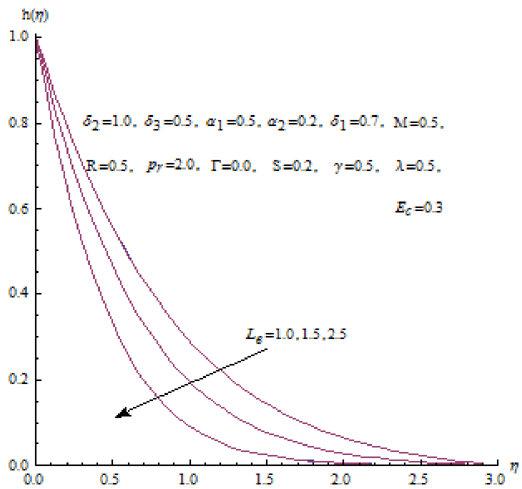

Figure 8: Influence of the Liwes number on concentration profile.

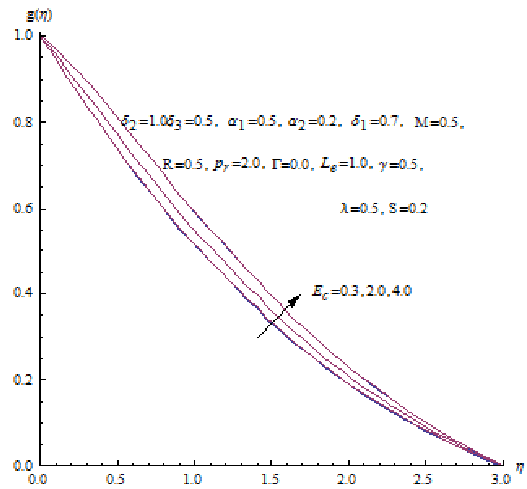

Figure 9: Influence of the chemical reaction parameter on concentration profile.

profiles increase, while the concentration profile decreases with increases of heat generation/absorption. The concentration profile decreases with increase of each of Liwes number and chemical reaction parameter in Figures 8 and 9, respectively. In Figure 10, the temperature profile is an increasing function of the increasing Eckert number. In addition, at the sheet, the temperature also is different because of the existence of convection heat.

From Table 1, we found that, the skin-friction coefficient, Nusselt number and Sherwood Number increase with the increase of each

δ_1	δ_3	α_1	α_2	M	S	Γ	L_e	E_c	γ	C_f	N_u	S_n
0.7										-1.1885	0.548	1.111
1.5										-1.0789	0.5567	1.1199
2.5										-0.96	0.5666	1.1312
	0.5									-1.1885	0.548	1.111
	2									-1.0601	0.555	1.1169
	4									-0.8916	0.5637	1.1244
		0.5								-1.1885	0.548	1.111
		2								-0.6116	0.5833	1.1412
		3								-0.2487	0.6013	1.1586
			0.5							-1.1885	0.548	1.111
			3							-0.6027	0.5767	1.1365
			5							-0.2176	0.6039	1.158
				0.5						-1.1885	0.548	1.111
				2						-1.7501	0.5157	1.0859
				4						-2.298	0.4916	1.0658
					0.2					-1.1885	0.548	1.111
					2					-1.8988	0.5053	1.0784
					5					-2.6598	0.4691	1.0534
						-0.5				-1.2282	0.758	1.1098
						0				-1.1885	0.548	1.111
						0.5				-1.1311	0.2709	1.1112
							1			-1.1885	0.548	1.111
							1.5			-1.1947	0.5473	1.4395
							2.5			-1.2017	0.5468	2.0288
								0.3		-1.1885	0.548	1.111
								2		-1.1711	0.4218	1.1114
								4		-1.1578	0.2718	1.1116
									0.5	-1.1885	0.548	1.111
									1.5	-1.1964	0.5471	1.5482
									2.5	-1.1993	0.5463	1.8778

Table 1: Numerical of the Skin-friction coefficient, Nusselt number and Sherwood Number for different δ_1, δ_3, α_1, α_2, Γ, M, S, L_e, E_c and γ.

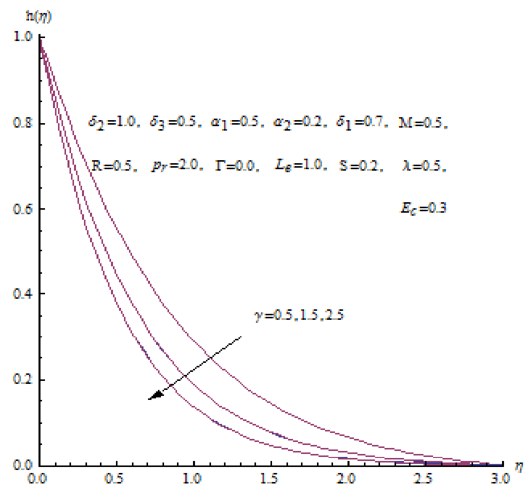

Figure 10: Influence of the Eckert number on temperature profile.

of fluid viscosity of temperature, fluid viscosity of concentration, temperature buoyancy and concentration buoyancy parameters. The skin-friction coefficient, Nusselt number and Sherwood Number decrease with the increase of each of the magnetic field and the porosity parameters.

The skin-friction coefficient and Sherwood Number increase while Nusselt number decreases with the increase of each of heat generation/absorption and Eckert number. And in case of increasing of each of the Liwes number and chemical reaction parameter, it is observed that skin-friction coefficient and Nusselt number decrease but Sherwood Number increases (Table 1 and Figures 1-7).

Conclusions

In the present study, Magnetohydrodynamic and chemical reaction effects of variable viscosity on heat generation and mass transfer viscous dissipation fluid in the presence of suction/injection through porosity is solved numerically. It is observed that:

(1) The velocity profile, the skin-friction coefficient, Nusselt number and Sherwood Number increase while the temperature and concentration profiles decrease with the increase of each of fluid viscosity of concentration, temperature buoyancy and concentration buoyancy parameters (Figures 8-10).

(2) The velocity profile, the skin-friction coefficient, Nusselt number and Sherwood Number decrease, but the temperature and the concentration profiles increase with the increase of each of magnetic field and porosity parameters.

(3) The velocity, the temperature profiles, the skin-friction coefficient and Sherwood Number increase, while the concentration profile and Nusselt number decrease with increases of heat generation/absorption.

(4) The concentration profile, the skin-friction coefficient and Nusselt number decrease but Sherwood

Number increases with increase of each of Liwes number and chemical reaction.

(5) The temperature profile, the skin-friction coefficient and Sherwood Number increase but Nusselt number decreases with increase of Eckert number.

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