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Lie Triple Derivations of the Lie Algebra of Strictly Block Upper Triangular Matrices

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Abstract

Let $\mathcal{N}$ be the Lie algebra of all $n \times n$ strictly block upper triangular matrices over a field $\mathbb{F}$. In this paper, we explicitly describe all Lie triple derivations of $\mathcal{N}$ when char($\mathbb{F}$)≠2.

Keywords: Lie triple derivation; Triangular matrix; Lie algebra

Introduction

This paper is a study of the Lie triple derivations of the Lie algebra of strictly block upper triangular matrices over a field $\mathbb{F}$ when char($\mathbb{F}$)≠2. Let ($\mathfrak{g}$,[,]) be a Lie algebra over a field $\mathbb{F}$ or a ring R. Recall that an $\mathbb{F}$-linear map (resp.an$R$–*linearmap*) $\delta : \mathfrak{g} \to \mathfrak{g}$ is called a derivation of $\mathfrak{g}$ if:

$$\delta([X,Y])=[\delta(X),Y]+[X,\delta(Y)] \tag{1}$$

for all $X,Y \in \mathfrak{g}$. A Lie triple derivation is an $\mathbb{F}$-linear map (resp.an R–*linearmap*) $f : \mathfrak{g} \to \mathfrak{g}$ which satisfies:

$$f([X,[Y,Z]])=[f(X),[Y,Z]]+[X[f(Y),Z]]+[X,[Y,f(Z)]] \tag{2}$$

for all $X,Y,Z \in \mathfrak{g}$. Clearly, any derivation is a Lie triple derivation. However, the converse statement is, in general, not true [1]. The following elements of $End(\mathfrak{g})$ are typical examples of Lie triple derivations:

1. For any $X \in \mathfrak{g}$, the linear map $ad\,X$ defined by $ad\,X(Y):=[X,Y]$ for all $Y \in \mathfrak{g}$ is a derivation which is called an inner derivation. Clearly, an inner derivation is a Lie triple derivation.

2. Any linear map f that maps $\mathfrak{g}$ to the center Z($\mathfrak{g}$) of $\mathfrak{g}$ and $[\mathfrak{g},[\mathfrak{g},\mathfrak{g}]]$ to zero is a Lie triple derivation, called a central Lie triple derivation.

In recent years, significant progress has been made in studying the derivations and Lie triple derivations of matrix Lie algebras over a field or a ring. Wang and Li characterized the Lie triple derivations of the Lie algebra of strictly upper triangular matrices over a commutative ring [2]. Lie, Cao, and Li described the Lie triple and generalized triple derivations of the parabolic subalgebras of the general linear Lie algebra over a computational ring [3]. Benkovic determined the Lie triple derivations on triangular matrices [4]. More recently, Benkovic described the Lie derivations and Lie triple derivations of upper triangular matrix algebras over a unital algebra [5]. Some other results on the derivations of certain matrix Lie algebras are given in refs. [6-12].

Fix a field $\mathbb{F}$. Let $M_{m,n}$ be the set of all $m \times n$ matrices over $\mathbb{F}$, and put $M_n:=M_{n,n}$. Let $\mathcal{N}$ (resp. $\mathcal{B}$) denotes the set of all strictly block upper triangular matrices (resp. block upper triangular matrices) in M_n relative to a given partition. Then $\mathcal{N}$ and $\mathcal{B}$ are Lie subalgebras of $\mathfrak{gl}(n, \mathbb{F})$, i. e. M_n with the standard Lie bracket. In this paper, we explicitly determine the Lie triple derivations of $\mathcal{N}$ which are as follows:

- When char($\mathbb{F}$)≠2, Theorem 3.1 shows that every Lie triple derivation of $\mathcal{N}$ is a sum of the adjoint action of a block upper triangular matrix in $\mathcal{B}$, a central Lie triple derivation, and two special linear maps.

The main motivation of this work comes from Wang and Li's work on the Lie triple derivation of the Lie algebra of strictly upper triangular matrices over a commutative ring [2], and authors' study of derivations of the Lie algebra of strictly block upper triangular matrices over a field [9]. Our work on the Lie triple derivations of $\mathcal{N}$ not only generalizes the main result of Wang and Li over a field, but also use a new approach that is promising to find the Lie triple derivations of other matrix Lie algebras with appropriate block forms. In disclosing the Lie triple derivation action on $\mathcal{N}$, we factor out the effects of adjoint action of block upper triangular matrices and those of central Lie triple derivations of $\mathcal{N}$, to explore the remaining Lie triple derivations.

Section 2 gives a basic introduction and determines some linear maps between matrix spaces that will be useful to describe the Lie triple derivations of $\mathcal{N}$ in section 3. Section 3 describes the Lie triple derivations of $\mathcal{N}$ over $\mathbb{F}$ when char($\mathbb{F}$)≠2.

Preliminary

In this section, we describe some linear maps between matrix spaces, and introduce some basic definitions and notations. The result in this section will be useful to prove the main result in Section 3.

Let $[n]:=\{1,2,\ldots,n\}$. Let $E_{pq}^{(mn)} \in M_{m,n}$ denote the matrix with the only nonzero entry 1 on the (p,q) position for $(p,q) \in [m] \times [n]$.

Lemma 2.1

Suppose $\mathbb{F}$ is an arbitrary field. If $X \in M_m$ and $Y \in M_n$ satisfy that:

$$XA=AY \tag{3}$$

For all $A \in M_{mn}$, then $X=\lambda I_m$ and $Y=\lambda I_n$ for certain $\lambda \in \mathbb{F}$.

Proof: Suppose $X=(x_{ip}) \in M_m$ and $Y=(y_{qj}) \in M_n$. For any $(i, j) \in [m] \times [n]$, by eqn (3),

$$XE_{ij}^{(mn)} = E_{ij}^{(mn)}Y. \tag{4}$$

Comparing the (i, j) entry of the matrices in eqn (4), we get $x_{ii}=y_{jj}$. Similarly, comparing the (p, j) entry for $p \neq i$, we get $x_{pi}=0$ and

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comparing the (i, q) entry for $q\neq j$, we get $0=y_{jq}$. Therefore, $X=\lambda I_m$ and $Y=\lambda I_n$ for some $\lambda\in\mathbb{F}$.

Lemma 2.2

If linear maps $\phi: M_{m,p}\to M_{m,q}$ and $\varphi: M_{n,p}\to M_{n,q}$ satisfy that:

$$\phi(AB)=A\varphi(B) \tag{5}$$

for all $A\in M_{m,n}$, $B\in M_{n,p}$, then there is $X\in M_{p,q}$ such that $\phi(C)=CX$ for $C\in M_{m,p}$ and $\varphi(D)=DX$ for $D\in M_{n,p}$.

Proof: For any $j\in[n]$ and $B\in M_{n,p}$, by eqn (5),

$$\phi(E_{1j}^{(mn)}B)=E_{1j}^{(mn)}\varphi(B). \tag{6}$$

All such $E_{1j}^{(mn)}B$ span the first row space of $M_{m,p}$. So φ sends the first row of $M_{m,p}$ to the first row of $M_{m,q}$. There exists a unique $X\in M_{p,q}$ such that:

$$E_{1j}^{(mn)}\varphi(B)=\phi(E_{1j}^{(mn)}B)=E_{1j}^{(mn)}BX,\quad \text{forall } j\in[n], B\in M_{n,p}, \tag{7}$$

where the first equality in eqn (7) is by eqn (6). Therefore, $\varphi(B)=BX$. Hence $\phi(AB)=A\varphi(B)=ABX$ for any $A\in M_{m,n}$ and $B\in M_{n,p}$. All such AB span $M_{m,p}$. So $\phi(C)=CX$ for all $C\in M_{m,p}$.

Lemma 2.3

If linear maps $\phi: M_{m,p}\to M_{n,p}$ and $\varphi: M_{m,q}\to M_{n,q}$ satisfy that:

$$\phi(BA)\varphi(B)A \tag{8}$$

For all $A\in M_{q,p}$, $B\in M_{m,q}$, then there is $X\in M_{n,m}$ such that $\phi(C)=XC$ for $C\in M_{m,p}$ and $\varphi(D)=XD$ for $D\in M_{m,q}$.

Proof: The proof (omitted) is similar to that of Lemma 2.2.

Lemma 2.4

If linear maps $\phi: M_{m,p}\to M_{m,q}$ and $\varphi: M_{q,n}\to M_{p,n}$ satisfy that:

$$\phi(A)B=A\varphi(B) \tag{9}$$

For all $A\in M_{m,p}$, $B\in M_{q,n}$, then there is $X\in M_{p,q}$ such that $\phi(C)=CX$ for $C\in M_{m,p}$ and $\varphi(D)=XD$ for $D\in M_{q,n}$.

Proof: For any $j\in[P]$ and any $E_{kl}^{(qn)}\in M_{q,n}$, by eqn (9),

$$\phi(E_{1j}^{(mp)})E_{kl}^{(qn)}=E_{1j}^{(mp)}\varphi(E_{kl}^{(qn)}) \tag{10}$$

which shows that the only possibly nonzero row of $\phi(E_{1j}^{(mp)})$ is the first row. So maps the first row of $M_{m,p}$ to the first row of $M_{m,q}$. There exists a unique $X\in M_{p,q}$ such that:

$$E_{1j}^{(mp)}\varphi(E_{kl}^{(qn)})=\phi(E_{1j}^{(mp)})E_{kl}^{(qn)}=E_{1j}^{(mp)}XE_{kl}^{(qn)}, \text{forall } j\in[p],\ E_{kl}^{(qn)}\in M_{q,n}, \tag{11}$$

where the first equality in eqn (11) is by eqn (10). Therefore, $\varphi(E_{kl}^{(qn)})=XE_{kl}^{(qn)}$ for all $E_{kl}^{(qn)}\in M_{q,n}$. So $\varphi(B)=XB$ for $B\in M_{q,n}$. Then $\phi(A)B=AXB$ for any $A\in M_{m,p}$ and $B\in M_{q,n}$. Hence $\phi(A)=AX$ for all $A\in M_{m,p}$.

Lemma 2.5

If linear maps $f: M_{p,r}\to M_{p,r}$, $g: M_{p,q}\to M_{p,q}$, and $h: M_{q,r}\to M_{q,r}$ satisfy that:

$$f(AB)=g(A)B+Ah(B)\quad \text{forall}\quad A\in M_{p,q},\ B\in M_{q,r}, \tag{12}$$

then there exist $X\in M_p$, $Y\in M_r$, and $Z\in M_q$ such that:

$$f(C)=XC+CY\qquad \text{for } C\in M_{p,r}, \tag{13}$$

$$g(A)=XA+AZ\qquad \text{for } A\in M_{p,q}, \tag{14}$$

$$h(B)=BY-ZB\qquad \text{for } B\in M_{q,r}. \tag{15}$$

Proof: For any $n\in[p]$, $j,k\in[q]$, $m\in[r]$, $E_{nj}^{(pq)}\in M_{p,q}$ and $E_{km}^{(qr)}\in M_{q,r}$, by eqn (12),

$$f(E_{nj}^{(pq)}E_{km}^{(qr)})=g(E_{nj}^{(pq)})E_{km}^{(qr)}+E_{nj}^{(pq)}h(E_{km}^{(qr)}). \tag{16}$$

We further discuss eqn (16) in two cases:

1. $j\neq k$: the left side of eqn (16) is zero and

$$g(E_{nj}^{(pq)})E_{km}^{(qr)}=-E_{nj}^{(pq)}h(E_{km}^{(qr)}). \tag{17}$$

2. $j=k$: the left side of eqn (16) is $f(E_{nm}^{(pr)})$, and according to eqn (16), the only possibly nonzero entries of $f(E_{nm}^{(pr)})$ are:

$$f(E_{nm}^{(pr)})_{im}=g(E_{nk}^{(pq)})_{ik}\qquad \text{forall } i\in[p], i\neq n; \tag{18}$$

$$f(E_{nm}^{(pr)})_{n\ell}=h(E_{km}^{(qr)})_{k\ell}\qquad \text{forall } \ell\in[r], \ell\neq m; \tag{19}$$

$$f(E_{nm}^{(pr)})_{nm}=g(E_{nk}^{(pq)})_{nk}+h(E_{km}^{(qr)})_{km}. \tag{20}$$

Next we define a linear map $f': M_{p,r}\to M_{p,r}$ such that property eqn (12) still holds. For $C\in M_{p,r}$, let:

$$f'(C):=\left[\sum_{i,j\in[p]} f(E_{j1}^{(pr)})_{i1}E_{ij}^{(pp)}\right]C+C\left[\sum_{k,\ell\in[r]} f(E_{1k}^{(pr)})_{1\ell}E_{k\ell}^{(rr)}\right]-f(E_{11}^{(pr)})_{11}C. \tag{21}$$

Then for any $n\in[p]$, $m\in[r]$and $E_{nm}^{(pr)}\in M_{p,r}$, by eqn (21),

$$f'(E_{nm}^{(pr)})=\sum_{i\in[p]} f(E_{n1}^{(pr)})_{i1}E_{im}^{(pr)}+\sum_{\ell\in[r]} f(E_{1m}^{(pr)})_{1\ell}E_{n\ell}^{(pr)}-f(E_{11}^{(pr)})_{11}E_{nm}^{(pr)},$$

which implies that the only possibly nonzero entries of $f'(E_{nm}^{(pr)})$ are:

$$f'(E_{nm}^{(pr)})_{im}=f(E_{n1}^{(pr)})_{i1}=f(E_{nm}^{(pr)})_{im}\qquad \text{for } i\in[p], i\neq n, \tag{22}$$

$$f'(E_{nm}^{(pr)})_{n\ell}=f(E_{1m}^{(pr)})_{1\ell}=f(E_{nm}^{(pr)})_{n\ell}\qquad \text{for } \ell\in[r], \ell\neq m, \tag{23}$$

$$f'(E_{nm}^{(pr)})_{nm}=f(E_{n1}^{(pr)})_{n1}+f(E_{1m}^{(pr)})_{1m}-f(E_{11}^{(pr)})_{11}=f(E_{nm}^{(pr)})_{nm}, \tag{24}$$

where the last equality in eqns (22), (23) and (24) is by eqns (18), (19) and (20) respectively. Therefore, $f'=f$ on each $E_{nm}^{(pr)}\in M_{p,r}$ and thus on the whole $M_{p,r}$. Denote:

$$X:=\sum_{i,j\in[p]} f(E_{j1}^{(pr)})_{i1}E_{ij}^{(pp)}-f(E_{11}^{(pr)})_{11}I_p,\qquad Y:=\sum_{k,\ell\in[r]} f(E_{1k}^{(pr)})_{1\ell}E_{k\ell}^{(rr)}. \tag{25}$$

We get $f(C)=f'(C)=XC+CY$ for $C\in M_{p,r}$. So eqn (13) is done. Now for $A\in M_{p,q}$ and $B\in M_{q,r}$, by eqns (12) and (13),

$$g(A)B+Ah(B)=f(AB)=XAB+ABY\Rightarrow (g(A)-XA)B=A(BY-h(B)). \tag{26}$$

Applying Lemma 2.4 to $\phi: M_{p,q}\to M_{p,q}$ defined by $\phi(A)=g(A)-XA$ and $\varphi: M_{q,r}\to M_{q,r}$ defined by $\varphi(B)=BY-h(B)$ in eqn (26), we find $Z\in M_q$ such that:

$$g(A)-XA=\phi(A)=AZ\qquad \text{for } A\in M_{p,q},$$

$$BY-h(B)=\varphi(B)=ZB\qquad \text{for } B\in M_{q,r},$$

which imply eqns (14) and (15).

Lemma 2.6

If linear maps $\phi: M_{p,s}\to M_{p,s}$, $\alpha: M_{p,q}\to M_{p,q}$, $\beta: M_{q,r}\to M_{q,r}$, and $\gamma: M_{r,s}\to M_{r,s}$ satisfy that:

$$\phi(ABC)=\alpha(A)BC+A\beta(B)C+AB\gamma(C)\ \text{forall } A\in M_{p,q}, B\in M_{q,r}, C\in M_{r,s}, \tag{27}$$

then there exists $X\in M_p$, $Y\in M_q$, $Z\in M_r$, $W\in M_s$ such that:

$$\phi(D)=XD+DW\qquad \text{for } D\in M_{p,s} \tag{28}$$

$$\alpha(A) = XA - AY \quad \text{for } A \in M_{p,q} \tag{29}$$

$$\beta(B) = YB + BZ \quad \text{for } B \in M_{q,s} \tag{30}$$

$$\gamma(C) = CW - ZC \quad \text{for } C \in M_{r,s} \tag{31}$$

Proof: We first define a linear map $f: M_{p,r} \to M_{p,r}$ by:

$$f'(AB) = \alpha(A)B + A\beta(B) \quad \text{for } A \in M_{p,q}, B \in M_{q,r}. \tag{32}$$

By eqns (27) and (32), for $C \in M_{r,s}$

$$\phi(ABC) = f'(AB)C + AB\gamma(C). \tag{33}$$

Applying Lemma 2.5 to $f: M_{p,s} \to M_{p,s}$ defined by $f(D)=\varphi(D)$ for $D \in M_{p,s}$, $g: M_{p,r} \to M_{p,r}$ defined by $g(F)=f'(F)$ for $F \in M_{p,r}$, and $h: M_{r,s} \to M_{r,s}$ defined by $h(G)=\gamma(G)$ for $G \in M_{r,s}$ in eqn (33), we find $X \in M_p$, $Z \in M_r$, $W \in M_s$, such that:

$$\phi(D) = XD + DW \quad \text{for } D \in M_{p,s} \tag{34}$$

$$\gamma(C) = CW - ZC \quad \text{for } C \in M_{r,s} \tag{35}$$

$$f'(F) = XF + FZ \quad \textit{for } F \in M_{p,r} \tag{36}$$

So eqns (28) and (31) are done. Again, applying Lemma 2.5 to $f: M_{p,r} \to M_{p,r}$ defined by $f(F)=f'(F)$ for $F \in M_{p,r}$, $g: M_{p,q} \to M_{p,q}$ defined by $g(A)=\alpha(A)$ for $A \in M_{p,q}$, and $h: M_{q,r} \to M_{q,r}$ defined by $h(B)=\beta(B)$ for $B \in M_{q,r}$ in eqn (32), we find $X' \in M_p$, $Y' \in M_q$, and $Z' \in M_r$ such that:

$$\alpha(A) = X'A + AY' \quad \textit{for } A \in M_{p,q} \tag{37}$$

$$\beta(B) = BZ' - Y'B \quad \text{for } B \in M_{q,r}. \tag{38}$$

By eqns (33), (36), (35), and (38),

$$X'AB + AY'B + ABZ' - AY'B = XAB + ABZ \Rightarrow (X' - X)AB = AB(Z - Z'). \tag{39}$$

Applying Lemma 2.1 in eqn (39), we find $\lambda \in \mathbb{F}$, $I_p \in M_p$, and $I_r \in M_r$ such that:

$$X' - X = \lambda I_p, \quad Z - Z' = \lambda I_r. \tag{40}$$

Therefore, by eqns (37), (38), and (40),

$$\alpha(A) = XA + A(\lambda I_q + Y') \quad \text{for } A \in M_{M_{pq}}$$

$$\beta(B) = BZ - (\lambda I_q + Y')B \quad \text{for } B \in M_{M_{qr}}.$$

Define $Y := -(\lambda I_q + Y')$, we get eqns (29) and (30).

We make some notations that will be frequently used later. A sequence $(n_1, n_2, \ldots, n_t)$ is called an ordered partition of n if $t, n_1, \ldots, n_t \in \mathbb{Z}^+$ and $n_1 + n_2 + \ldots + n_t = n$. The $t \times t$ block matrix form associate with an order partition$(n_1, n_2, \ldots, n_t)$ is an expression of $n \times n$ matrices $A=[A_{ij}]_{t\times t}$ where the (i,j) block $A_{ij} \in M_{n_i,n_j}$ for $i,j \in [t]$.

From now on, let us fix an order partition $(n_1, n_2, \ldots, n_t)$ of n and the corresponding block matrix form. Given $A \in M_n$, let A^{ij} denote the matrix in M_n that has the same (i, j) block as A and 0's elsewhere. Given a subset $\mathcal{A} \subseteq M_n$, let $\mathcal{A}_{ij} \subseteq M_{n_i,n_j}c$ (resp. $\mathcal{A}^{ij} \subseteq M_n$) denote the set of A_{ij} (resp. A^{ij}) for all $A \in \mathcal{A}$; for examples, we will use $\mathcal{N}_{ij}$ and $\mathcal{N}^{ij}$ in this manner for $\mathcal{N}$ defined in Definition 2.7. Let $E_{pq}^{ij} \in M_n$ denote the matrix that has the only nonzero entry 1 on the (p,q) position of the (i,j) block for $(i, j) \in [t] \times [t]$ and $(p,q) \in [n_i] \times [n_j]$. Any notation of double index, say A_{ij} (resp. A^{ij}), may be written as $A_{i,j}$ (resp. $A^{i,j}$) for clarity purpose. If $A \in M_n$ is not given in advance, A_{ij} (resp. A^{ij}) may refer to an arbitrary matrix in $M_{n_i \times n_j}$ (resp. M_n).

An $n \times n$ matrix A is called block upper triangular (resp. strictlyblockuppertriangular) if $A_{ij}=0$ for all $1 \le j < i \le t$ (resp. $1 \le j \le i \le t$).

Definition 2.7: Consider the matrices in M_n corresponding to the $t \times t$ block matrix form associate with an ordered partition $(n_1, n_2, \ldots, n_t)$ of n.

1. Let $\mathcal{B}(n_1, n_2, \ldots, n_t)$ (abbr. $\mathcal{B}$) denote the Lie Theory algebra of all block upper triangular matrices in M_n.

2. Let $\mathcal{N}(n_1, n_2 \ldots, n_t)$ (abbr. $\mathcal{N}$) denote the Lie algebra of all strictly block upper triangular matrices in M_n.

3. Call $\{(i, j) \in \mathbb{Z}^2 | 1 \le i < j \le t\}$ the block index set of $\mathcal{N}$. Given a subset Δ of $\{(i, j) \in \mathbb{Z}^2 | 1 \le i < j \le t\}$, we denote:

$$\Delta^c := \{(i, j) \in \mathbb{Z}^2 \,|\, 1 \le i < j \le t\} \setminus \Delta \tag{41}$$

$$\mathcal{N}^\Delta := \bigoplus_{(i,j)\in\Delta} \mathcal{N}^{ij}. \tag{42}$$

4. $\Omega := \{(i, j) \in [t] \times [t] \,|\, j \in \{i+1, i+2\}\}$

The normalizer of Lie algebra $\mathcal{N}$ in M_n is $\mathcal{B}$. For any $X \in \mathcal{B}$, the adjoint action of $\mathcal{N}$.

$$ad\, X : \mathcal{N} \to \mathcal{N}, \quad Y \mapsto [X, Y]$$

is a derivation (resp. Lie triple derivation) of $\mathcal{N}$.

Lie triple derivations of $\mathcal{N}$ for char($\mathbb{F}$) ≠2

The goal of this section is to describe the Lie triple derivations of the Lie algebra $\mathcal{N}$ of strictly block upper triangular $n \times n$ matrices over a field $\mathbb{F}$ with char($\mathbb{F}$)≠2. The cases $1 \le t \le 4$ are trivial. So we consider $t \ge 5$. Here is the main result for $t \ge 5$.

Theorem 3.1

Suppose char($\mathbb{F}$)≠2. When $t \ge 5$, every Lie triple derivation f of the Lie algebra $\mathcal{N} = \mathcal{N}(n_1, \ldots, n_t)$ can be written (notuniquely) as:

$$f = ad\, X + \varphi_{1t} + \phi_{2t}^{\Omega} + \phi_{1,t-1}^{\Omega} \tag{43}$$

where the summand components are given below:

1. $X \in \mathcal{B}$.

2. $\varphi_{1t} \in End(\mathcal{N})$ is a central Lie triple derivation in:

$$Cen(\mathcal{N}) = \{\varphi \in End(\mathcal{N}) \,|\, \ker\varphi \supseteq \mathcal{N}^{\Omega^c}, \;\; Im\,\varphi \subseteq \mathcal{N}^{1t}\}.$$

3. $\phi_{2t}^{\Omega} \in End(\mathcal{N})$ satisfies that $\ker\phi \supseteq \mathcal{N}^{\Omega^c}$ and $Im\,\phi \subseteq \mathcal{N}^{2t}$.

4. $\phi_{1,t-1}^{\Omega} \in End(\mathcal{N})$ satisfies that $\ker\phi \supseteq \mathcal{N}^{\Omega^c}$ and $Im\,\phi \subseteq \mathcal{N}^{1,t-1}$.

Before proving Theorem 3.1, we first prove several results on the images $f(\mathcal{N}^{ij})$ for a Lie triple derivation f and $\mathcal{N}^{ij} \subseteq \mathcal{N}$. The next lemma describes the image of f on $\mathcal{N}^{12}$ and $\mathcal{N}^{t-1,t}$.

Lemma 3.2

Suppose char($\mathbb{F}$)≠2. Then for any Lie triple derivation f of $\mathcal{N}$:

$$f(\mathcal{N}^{12}) \subseteq \sum_{q=2}^{t} \mathcal{N}^{1q} + \mathcal{N}^{2t}, \tag{44}$$

$$f(\mathcal{N}^{t-1,t}) \subseteq \sum_{p=1}^{t-1} \mathcal{N}^{pt} + \mathcal{N}^{1,t-1}. \tag{45}$$

Proof: To get eqn (44), first we prove that $f(A^{12})_{ij}=0$ for any $A^{12} \in \mathcal{N}^{12}$, $1<i<j$, and $(i,j) \notin \{(2,t),(3,t)\}$. Either $i>3$ or $j<t$.

1. Suppose $j<t$. Then for any $A^{1i} \in \mathcal{N}^{1i}$ and $A^{jt}\mathcal{N}^{jt}$,

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$$0 = f([A^{1i},[A^{12},A^{jt}]])_{1t}$$
$$= [f(A^{1i}),[A^{12},A^{jt}]]_{1t} + [A^{1i},[f(A^{12}),A^{jt}]]_{1t} + [A^{1i},[A^{12},f(A^{jt})]]_{1t}$$
$$= (A^{1i})_{1i} f(A^{12})_{ij} (A^{jt})_{jt}.$$

Therefore, $0=(A^{1i})_{1i} f(A^{12})_{ij}(A^{jt})_{jt}$ for any $A^{jt}\in\mathcal{N}^{jt}$. So:

$$0 = (A^{1i})_{1i} f(A^{12})_{ij} \tag{46}$$

For $A^{1i}\in\mathcal{N}^{1i}$. Now we further discuss eqn (46) in the following two cases:

- If $i>2$, $0=(A^{1i})_{1i} f(A^{12})_{ij}$ for any $A^{1i}\mathcal{N}^{1i}$. So $f(A^{12})_{ij}=0$.

- If $i=2$, it suffices to show that $f(E_{k\ell}^{12})_{2j}=0$ for any $k\in[n_1]$, $\ell\in[n_2]$. Given $E_{k\ell}^{12}\in\mathcal{N}^{12}$,

$$0 = (E_{k\ell}^{12})_{12} f(E_{k\ell}^{12})_{2j} \tag{47}$$

Comparing the k-th row in the equality eqn (47), we see that the ℓ-th row of $f(E_{k\ell}^{12})_{2j}$ is zero. Since $\ell\in[n_2]$ is arbitrary, $f(E_{k\ell}^{12})_{2j}=0$.

2. Suppose $i>3$. Then for any $A^{13}\in\mathcal{N}^{13}$ and $A^{3i}\mathcal{N}^{3i}$,

$$0 = f([A^{13},[A^{12},A^{3i}]])_{1j}$$
$$= [f(A^{13}),[A^{12},A^{3i}]]_{1j} + [A^{13},[f(A^{12}),A^{3i}]]_{1j} + [A^{13},[A^{12},f(A^{3i})]]_{1j}$$
$$= -(A^{13})_{13}(A^{3i})_{3i} f(A^{12})_{ij}.$$

Therefore, $0=(A^{13})_{13}(A^{3i})_{3i} f(A^{12})_{ij}$ for any $A^{13}\in\mathcal{N}^{13}$ and $A^{3i}\in\mathcal{N}^{3i}$. So $f(A^{12})_{ij}=0$.

Next we show that $f(A^{12})_{3t}=0$. For any $A^{23}\in\mathcal{N}^{23}$,

$$0 = f([A^{12},[A^{12},A^{23}]])_{1t}$$
$$= [f(A^{12}),[A^{12},A^{23}]]_{1t} + [A^{12},[f(A^{12}),A^{23}]]_{1t} + [A^{12},[A^{12},f(A^{23})]]_{1t} \tag{48}$$
$$= -2(A^{12})_{12}(A^{23})_{23} f(A^{12})_{3t}.$$

Since char$\trianglelefteq\mathbb{F}\not\supset\neq 2$, by eqn (48), $0=(A^{12})_{12}(A^{23})_{23} f(A^{12})_{3t}$. Given A^{12}, the matrix $(A^{12})_{12}(A^{23})_{23}$ for any $A^{23}\in\mathcal{N}^{23}$ could be any matrix in $\mathcal{N}_{13}$ with the rank no more than rank $(A^{12})_{12}$. Therefore, $f(A^{12})_{3t}=0$. Overall, we have proved eqn (44). The proof of eqn (45) (omitted) is similar to that of eqn (44).

Now we consider the image of a Lie triple derivation f on $\mathcal{N}^{23}$ and $\mathcal{N}^{t-2,t-1}$.

Lemma 3.3

Let f be a Lie triple derivation of . Then:

$$f(\mathcal{N}^{23}) \subseteq \mathcal{N}^{13} + \sum_{q=3}^{t}\mathcal{N}^{2q} + \mathcal{N}^{1,t-1} + \mathcal{N}^{1,t}, \tag{49}$$

$$f(\mathcal{N}^{t-2,t-1}) \subseteq \mathcal{N}^{t-2,t} + \sum_{p=1}^{t-2}\mathcal{N}^{p,t-1} + \mathcal{N}^{2,t} + \mathcal{N}^{1,t}. \tag{50}$$

Proof: To get eqn (49), first we prove that $f(A^{23})_{ij}=0$ for any $A^{23}\in\mathcal{N}^{23}$, $i<j$, $i\neq 2$, $j\neq 3$, and $(i,j)\notin\{(1,t-1),(1,t)\}$. Either $i>2$ or $j<t-1$.

1. Suppose $j<t-1$. Then for any $A^{j,j+1}\in\mathcal{N}^{j,j+1}$ and $A^{j+1,t}\in\mathcal{N}^{j+1,t}$

$$0 = f([A^{23},[A^{j,j+1},A^{j+1,t}]])_{it}$$
$$= [f(A^{23}),[A^{j,j+1},A^{j+1,t}]]_{it} + [A^{23},[f(A^{j,j+1}),A^{j+1,t}]]_{it} + [A^{23},[A^{j,j+1},f(A^{j+1,t})]]_{it} \tag{51}$$
$$= f(A^{23})_{ij}(A^{j,j+1})_{j,j+1}(A^{j+1,t})_{j+1,t}.$$

Now we further discuss eqn (51) in the following two cases:

- When $j=2$, $0=f(A^{23})_{i2}(A^{23})_{i2}(A^{23})_{23}(A^{3t})_{3t}$. Given A^{23}, the matrix $(A^{23})_{23}(A^{3t})_{3t}$ for any $A^{3t}\in\mathcal{N}^{3t}$ could be any matrix in $\mathcal{N}_{2t}$ with rank no more then rank $(A^{23})_{23}$. Therefore, $f(A^{23})_{i2}=0$.

- When $j>3$, $0=f(A^{23})_{ij}(A^{i,i+1})_{j,j+1}(A^{j+1,t})_{j+1,t}$ for any $A^{j,j+1}\in\mathcal{N}^{j,j+1}$ and $A^{j+1,t}\mathcal{N}^{j+1,t}$. Therefore, $f(A^{23})_{ij}=0$.

2. Suppose $i>2$. Then for any $A^{12}\in\mathcal{N}^{12}$ and any $A^{2i}\in\mathcal{N}^{2i}$,

$$0 = f([A^{23},[A^{12},A^{2i}]])_{1j}$$
$$= [f(A^{23}),[A^{12},A^{2i}]]_{1j} + [A^{23},[f(A^{12}),A^{2i}]]_{1j} + [A^{23},[A^{12},f(A^{2i})]]_{1j} \tag{52}$$
$$= -(A^{12})_{12}(A^{2i})_{2i} f(A^{23})_{ij}.$$

Now we further discuss eqn (52) in the following two cases:

- When $i=0$, $0=(A^{12})_{12}(A^{23})_{23} f(A^{23})_{3j}$. Given A^{23}, the matrix $(A^{12})_{12}(A^{23})_{23}$ for any $A^{12}\in\mathcal{N}^{12}$ could be any matrix in $\mathcal{N}_{13}$ with rank no more then rank $(A^{23})_{23}$. Therefore, $f(A^{23})_{3j}=0$.

- When $i>3$, $0=(A^{12})_{12}(A^{2I})_{2i} f(A^{23})_{ij}$ for any $A^{12}\in\mathcal{N}^{12}$ and $A^{2i}\mathcal{N}^{2i}$. Therefore, $f(A^{23})_{ij}=0$

Overall, we have proved eqn (49). The proof of eqn (49) is similar to that of eqn (50).

Now we consider the image of a Lie triple derivation f on $\mathcal{N}^{34},\ldots,\mathcal{N}^{t-3,t-2}$.

Lemma 3.4

Let f be a Lie triple derivation of and $3\le k\le t-3$. Then:

$$f(\mathcal{N}^{k,k+1}) \subseteq \sum_{p=1}^{k-1}\mathcal{N}^{p,k+1} + \sum_{q=k+1}^{t}\mathcal{N}^{k,q} + \mathcal{N}^{1,t-1} + \mathcal{N}^{1,t} + \mathcal{N}^{2,t} \tag{53}$$

Proof: Given any $A^{k,k+1}\in\mathcal{N}^{k,k+1}$, it suffices to prove that $f(A^{k,k+1})_{ij}=0$ for $i<j$, $i\neq k$, $j\neq k+1$, and $(i,j)\notin\{(1,t-1),(1,t),(2,t)\}$ to get eqn (53). Either $i>2$ or $j<t-1$.

1. Suppose $j<t-1$. Then for any $A^{j,j+1}\in\mathcal{N}^{j,j+1}$ and $A^{j+1,t}\mathcal{N}^{j+1,t}$

$$0 = f([A^{k,k+1},[A^{j,j+1},A^{j+1,t}]])_{it}$$
$$= [f(A^{k,k+1}),[A^{j,j+1},A^{j+1,t}]]_{it} + [A^{k,k+1},[f(A^{j,j+1}),A^{j+1,t}]]_{it} + [A^{k,k+1},[A^{j,j+1},f(A^{j+1,t})]]_{it} \tag{54}$$
$$= f(A^{k,k+1})_{ij}(A^{j,j+1})_{j,j+1}(A^{j+1,t})_{j+1,t}.$$

Now we further discuss eqn (54) in the following two cases:

- When $j=k$, $0=f(A^{k,k+1})_{ik}(A^{k,k+1})_{k,k+1}(A^{k+1,t})_{k+1,t}$. Given $A^{k,k+1}$, the matrix $(A^{k,k+1})_{k,k+1}(A^{k+1,t})_{k+1,t}$ for any $A^{k+1,t}\in\mathcal{N}^{k+1,t}$ could be any matrix in $\mathcal{N}_{kt}$ with rank no more then rank $(A^{k,k+1})_{k,k+1}$. Therefore, $f(A^{k,k+1})_{ij}=0$.

- When $j\neq k$, $0=f(A^{k,k+1})_{ij}(A^{j,j+1})_{i,j+1}(A^{j+1,t})_{j+1,t}$ for any $A^{j,j+1}\in\mathcal{N}^{j,j+1}$ and $A^{j+1,t}\mathcal{N}^{j+1,t}$. Therefore, $f(A^{k,k+1})_{ij}=0$.

2. Suppose $i>2$. Then for any $A^{12}\in\mathcal{N}^{12}$ and any $A^{2i}\in\mathcal{N}^{2i}$,

$$0 = f([A^{k,k+1},[A^{12},A^{2i}]])_{1j}$$
$$= [f(A^{k,k+1}),[A^{12},A^{2i}]]_{1j} + [A^{k,k+1},[f(A^{12}),A^{2i}]]_{1j} + [A^{k,k+1},[A^{12},f(A^{2i})]]_{1j}$$
$$= -(A^{12})_{12}(A^{2i})_{2i} f(A^{k,k+1})_{ij}.$$

Therefore $i=0$, $0=(A^{12})_{12}(A^{2i})_{2i} f(A^{k,k+1})_{ij}$ for any $A^{12}\in\mathcal{N}^{12}$ and $A^{2i}\in\mathcal{N}^{2i}$. So $f(A^{k,k+1})_{ij}=0$.

It remains to show that $f(A^{k,k+1})_{2,t-1}=0$. For any $A^{12}\in\mathcal{N}^{12}$ and $A^{t1,t}\mathcal{N}^{t-1,t}$

$$0 = f([A^{12},[A^{k,k+1},A^{t-1,t}]])_{1t}$$
$$= [f(A^{12}),[A^{12},A^{t-1,t}]]_{1t} + [A^{12},[f(A^{k,k+1}),A^{t-1,t}]]_{1t} + [A^{k,k+1},[A^{12},f(A^{t-1,t})]]_{1t}$$
$$= (A^{12})_{12} f(A^{k,k+1})_{2,t-1}(A^{t-1,t})_{t-1,t}.$$

Therefore, $f(A^{k,k+1})_{2,t-1}=0$. Overall, we have proved eqn (53).

Next we consider the image of a Lie triple derivation f on $\mathcal{N}^{13}$, $\mathcal{N}^{24},\ldots,\mathcal{N}^{t-2,t}$

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Lemma 3.5

Suppose f be a Lie triple derivation of $\mathcal{N}$ and $1\le k\le t-1$. Then:

$$f(\mathcal{N}^{k,k+2}) \subseteq \sum_{i=1}^{k-1}\mathcal{N}^{i,k+2} + \sum_{j=k+2}^{t}\mathcal{N}^{kj} + \mathcal{N}^{1,t-1} + \mathcal{N}^{1,t} + \mathcal{N}^{2,t} \tag{55}$$

Proof: Given any $A^{k,k+2}\in\mathcal{N}^{k,k+2}$, it suffices to prove that $f(A^{k,k+2})_{k,k+1}=0$, $f(A^{k,k+2})_{k+1,k+2}=0$, and $f\ (A^{k,k+2})_{ij}=0$ for $i<j$, $i\neq k$, $j\neq k+2$, $(i,j)\notin\{(1,t-1),(1,t),(2,t)\}$ to get eqn (55).

We first prove that $f\ (A^{k,k+2})_{ij}=0$ for $i<j$, $i\neq k$, $j\neq k+2$, $(i,j)\notin\{(1,t-1),(1,t),(2,t)\}$. Either $i>2$ or $j<t-1$.

1. Suppose $j<t-1$. Then for any $A^{j,j+1}\in\mathcal{N}^{j,j+1}$ and $A^{j+1,t}\mathcal{N}^{j+1,t}$

$$\begin{aligned} 0 &= f([A^{k,k+2},[A^{j,j+1},A^{j+1,t}]])_{it} \\ &= [f(A^{k,k+2}),[A^{j,j+1},A^{j+1,t}]]_{it} + [A^{k,k+2},[f(A^{j,j+1}),A^{j+1,t}]]_{it} + [A^{k,k+2},[A^{j,j+1},f(A^{j+1,t})]]_{it} \\ &= f(A^{k,k+2})_{ij}(A^{j,j+1})_{j,j+1}(A^{j+1,t})_{j+1,t}. \end{aligned}$$

Therefore, $0=f(A^{k,k+2})_{ij}(A^{j,j+1})_{j,j+1}(A^{j+1,t})_{j+1,t}$ for any $A^{j,j+1}\in\mathcal{N}^{j,j+1}$ and $A^{j+1,t}\mathcal{N}^{j+1,t}$. So $f(A^{k,k+2})_{ij}=0$.

2. Suppose $i>2$. Then for any $A^{12}\in\mathcal{N}^{12}$ and any $A^{2i}\in\mathcal{N}^{2i}$,

$$\begin{aligned} 0 &= f([A^{k,k+2},[A^{12},A^{2i}]])_{1j} \\ &= [f(A^{k,k+2}),[A^{12},A^{2i}]]_{1j} + [A^{k,k+2},[f(A^{12}),A^{2i}]]_{1j} + [A^{k,k+2},[A^{12},f(A^{2i})]]_{1j} \\ &= -(A^{12})_{12}(A^{2i})_{2i}f(A^{k,k+2})_{ij}. \end{aligned}$$

Therefore, $0=(A^{12})_{12}(A^{2i})_{2i}f(A^{k,k+2})_{ij}$ for any $A^{12}\in\mathcal{N}^{12}$ and $A^{2i}\mathcal{N}^{2i}$. So $f(A^{k,k+2})_{ij}=0$.

Next we show that $f(A^{k,k+2})_{2,t-1}=0$ for $(k,k+2)\neq\{(2,4),(t2,t-2)\}$. For any $A^{12}\in\mathcal{N}^{12}$ and $A^{t-1,t}\in\mathcal{N}^{t1,t}$,

$$\begin{aligned} 0 &= f([A^{12},[A^{k,k+2},A^{t-1,t}]])_{1t} \\ &= [f(A^{12}),[A^{k,k+2},A^{t-1,t}]]_{1t} + [A^{12},[f(A^{k,k+2}),A^{t-1,t}]]_{1t} + [A^{k,k+2},[A^{12},f(A^{t-1,t})]]_{1t} \\ &= (A^{12})_{12}f(A^{k,k+2})_{2,t-1}(A^{t-1,t})_{t-1,t}. \end{aligned}$$

Therefore, $f(A^{k,k+2})_{2,t-1}=0$. It remains to show that $f(A^{k,k+2})_{k,k+1}=0$, $f(A^{k,k+2})_{k+1,k+2}=0$. Now we show that $f(A^{k,k+1})_{k,k+1}=0$ (similarlyfor $f(A^{k,k+2})_{k+1,k+2}=0$). For any $A^{k+1,k+2}\in\mathcal{N}^{k+1,k+2}$ and $A^{k+2,t}\mathcal{N}^{k+2,t}$,

$$\begin{aligned} 0 &= f([A^{k,k+2},[A^{k+1,k+2},A^{k+2,t}]])_{kt} \\ &= [f(A^{k,k+2}),[A^{k+1,k+2},A^{k+2,t}]]_{kt} + [A^{k,k+2},[f(A^{k+1,k+2}),A^{k+2,t}]]_{kt} \\ &\quad + [A^{k,k+2},[A^{k+1,k+2},f(A^{k+2,t})]]_{kt} \\ &= f(A^{k,k+2})_{k,k+1}(A^{k+1,k+2})_{k+1,k+2}(A^{k+2,t})_{k+2,t}. \end{aligned}$$

Therefore, $f(A^{k,k+2})_{k,k+1}=0$.

Overall, we have proved eqn (55).

The following lemma shows that any $f\in End(\mathcal{N})$ that satisfies $Ker\ f\supseteq\mathcal{N}^{\Omega^c}$, $Im\ f\subseteq\mathcal{N}^{2t}$ is a Lie triple derivation.

Lemma 3.6

Suppose $f\in End(\mathcal{N})$ satisfies that:

$$(\quad ^{ij}) \subseteq \begin{cases} 0 & \text{for } \mathcal{N}^{ij}\subseteq\mathcal{N},(i,j)\notin\Omega; \\ & \text{otherwise.} \end{cases} \tag{56}$$

Then f is a Lie triple derivation of $\mathcal{N}$.

Proof: The f satisfying eqn (56) also satisfies the Lie triple derivation property:

$$f([\mathcal{N},[\mathcal{N},\mathcal{N}]]) = [f(\mathcal{N}),[\mathcal{N},\mathcal{N}]] + [\mathcal{N},[f(\mathcal{N}),\mathcal{N}]] + [\mathcal{N},[\mathcal{N},f(\mathcal{N})]].$$

Similarly, we have the following lemma.

Lemma 3.7

Suppose $f\in End(\mathcal{N})$ satisfies that:

$$f(\mathcal{N}^{ij}) \subseteq \begin{cases} 0 & \text{for } \mathcal{N}^{ij}\subseteq\mathcal{N},(i,j)\notin\Omega; \\ \mathcal{N}^{1,t-1} & \text{otherwise.} \end{cases} \tag{57}$$

Then f is a Lie triple derivation of $\mathcal{N}$.

Next we consider the image of a Lie triple derivation f on other $\mathcal{N}^{ij}$.

Lemma 3.8

Suppose char$(\mathbb{F})\neq2$. For a Lie triple derivation f of $\mathcal{N}$, $i,j\in[t]$ and $j>i+2$, the image $f(\mathcal{N}^{ij})$ satisfies that:

$$f(\mathcal{N}^{ij}) \subseteq \sum_{p=1}^{i-1}\mathcal{N}^{pj} + \sum_{q=j}^{t}\mathcal{N}^{iq} \tag{58}$$

Proof: Let $j=i+k$, $k\ge3$. We prove eqn (58) by induction on k.

1. $k=3$: $\mathcal{N}^{i,i+3} = \mathcal{N}^{i,i+1}\mathcal{N}^{i+1,i+2}\mathcal{N}^{i+2,i+3} = [\mathcal{N}^{i,i+1},[\mathcal{N}^{i+1,i+2},\mathcal{N}^{i+2,i+3}]]$.

For $A^{i,i+1}\in\mathcal{N}^{i,i+1}$, $A^{i+1,i+2}\in\mathcal{N}^{i+1,i+2}$, and $A^{i+2,i+3}\in\mathcal{N}^{i+2,i+3}$,

$$\begin{aligned} & f([A^{i,i+1},[A^{i+1,i+2},A^{i+2,i+3}]]) = [f(A^{i,i+1}),[A^{i+1,i+2},A^{i+2,i+3}]] + [A^{i,i+1},[f(A^{i+1,i+2}),A^{i+2,i+3}]] \\ & + [A^{i,i+1},[A^{i+1,i+2},f(A^{i+2,i+3})]] \in \sum_{p=1}^{i-1}\mathcal{N}^{p,i+2} + \sum_{q=i+3}^{t}\mathcal{N}^{iq} \end{aligned} \tag{59}$$

where the last relation in eqn (59) is by lemmas 3.2, 3.3 and 3.4. So we done for $k=3$.

2. $k=\ell>3$: Suppose eqn (58) holds for all $k<\ell$ where $\ell>3$. Now $\mathcal{N}^{i,i+\ell} = [\mathcal{N}^{i,i+2},[\mathcal{N}^{i+2,i+3},\mathcal{N}^{i+3,i+\ell}]]$.

For $A^{i,i+2}\in\mathcal{N}^{i,i+2}$, $A^{i+2,i+3}\in\mathcal{N}^{i+2,i+3}$, and $A^{i+3,i+}\in\mathcal{N}^{i+3,i+}$,

$$\begin{aligned} & f([A^{i,i+2},[A^{i+2,i+3},A^{i+3,i+\ell}]]) = [f(A^{i,i+2}), \\ & [A^{i+2,i+3},A^{i+3,i+\ell}]] + [A^{i,i+2},[f(A^{i+2,i+3}),A^{i+3,i+\ell}]] \\ & + [A^{i,i+2},[A^{i+2,i+3},f(A^{i+3,i+\ell})]] \in \sum_{p=1}^{i-1}\mathcal{N}^{p,i+\ell} + \sum_{q=i+\ell}^{t}\mathcal{N}^{iq} \end{aligned} \tag{60}$$

where the last relation in eqn (60) is by induction hypothesis, the case $k=3$, and lemmas 3.2, 3.3, 3.4, and 3.5. So eqn (58) is true for $k=\ell$.

So far all possible nonzero blocks of $f(\mathcal{N}^{ij})$ for $\mathcal{N}^{ij}\mathcal{N}$ have been located. The next lemma explicitly describes most of those nonzero blocks of $f(\mathcal{N}^{ij})$. It essentially implies that the f-images on these blocks are the same as the images of an adjoint action of a block upper triangular matrix. Denote the index set:

$$\Omega' := \{(i,j)\in \quad | 1\le i<j\le t\}\setminus\{(1,t-1),(1,t),(2,t-1),(2,t)\}.$$

Lemma 3.9

Let f be a Lie triple derivation of . Then for any $(p,q)\in\Omega'$, there exists $X_{pq}\in\mathcal{N}_{pq}$ such that:

$$f(A^{ip})_{iq} = -(A^{ip})_{ip}X_{pq} \quad \text{for all } A^{ip}\in\mathcal{N}^{ip}\subseteq\mathcal{N} \tag{61}$$

$$f(A^{qj})_{pj} = X_{pq}(A^{qj})_{qj} \quad \text{for all } A^{qj}\in\mathcal{N}^{qj}\subseteq\mathcal{N} \tag{62}$$

Proof: Given $(p,q)\in\Omega'$, we prove eqns (61) and (62) by the following steps:

1. We prove eqn (62) for $(q,j)=(t-1,t)$. Then $2<p<t-1$. For $A^{12}\in\mathcal{N}^{12}$, $A^{2p}\in\mathcal{N}^{2p}$, and $A^{t-1,t}\in\mathcal{N}^{t1,t}$,

$$\begin{aligned} & 0 = f([A^{12},[A^{2p},A^{t-1,t}]])_{1t} = [f(A^{12}),[A^{2p},A^{t-1,t}]]_{1t} + [A^{12},[f(A^{2p}),A^{t-1,t}]]_{1t} \\ & + [A^{12},[A^{2p},f(A^{t-1,t})]]_{1t} = (A^{12})_{12}f(A^{2p})_{2,t-1}(A^{t-1,t})_{t-1,t} + (A^{12})_{12}(A^{2p})_{2,p}f(A^{t-1,t})_{p,t} \end{aligned}$$

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Therefore,

$$-f(A^{2p})_{2,t-1}(A^{t-1,t})_{t-1,t}=(A^{2p})_{2,p}f(A^{t-1,t})_{p,t} \quad (63)$$

Applying lemma 2.4 to $\varphi:\mathcal{N}_{2p}\to\mathcal{N}_{2,t-1}$ defined by $\varphi(C)=-f(C^{2p})_{2,t-1}$ and $\varphi:\mathcal{N}_{t-1,t}\to\mathcal{N}_{p,t}$ defined by $\varphi(D)=f(D^{t1,t})_{pt}$ in eqn (63), we find $X_{p,t-1}\in\mathcal{N}_{p,t-1}$ such that $f(A^{t-1,t})_{pt}=X_{p,t-1}(A^{t-1,t})_{t-1,t}$ for all $A^{t-1,t}\in\mathcal{N}^{t1,t}$.

2. Similarly, we can prove eqn (61) for $(i,p)=(1,2)$ via lemma 2.4. In other words, for $2<q<t-1$, there is $-Y_{2q}\in\mathcal{N}_{2q}$ such that $f(A^{12})_{1q}=-(A^{12})_{12}Y_{2q}$ for $A^{12}\in\mathcal{N}^{12}$.

3. We prove eqn (62) for $(q,j)=(t-2,t-1)$. For $1<p<t-2$, $A^{1p}\in\mathcal{N}^{1p}$, $A^{t-2,t-1}\in\mathcal{N}^{t-2,t-1}$. By a similar computation as eqn (63), for $A^{1p}\mathcal{N}^{1p}$, $A^{t2,t1}\in\mathcal{N}^{t-2,t-1}$, and $A^{t-1,t}\in\mathcal{N}^{t1,t}$,

$$-f(A^{1p})_{1,t-2}(A^{t-2,t-1})_{t-2,t-1}=(A^{1p})_{1p}f(A^{t-2,t-1})_{p,t-1}. \quad (64)$$

Applying lemma 2.4 to ϕ: $\mathcal{N}_{1p}\to\mathcal{N}_{1,t-1}$ defined by $\phi(C)=-f(C^{1p})_{1,t-1}$ and $\phi:\mathcal{N}_{t-2,t-1}\to\mathcal{N}_{p,t-1}$ defined by $\phi(D)=f(D^{t2,t1})_{p,t-1}$ in eqn (64), we find $X_{p,t2}\in\mathcal{N}_{p,t-2}$ such that $f(A^{t-2,t-1})_{p,t-1}=X_{p,t-2}(A^{t-2,t-1})_{t-2,t-1}$ for all $A^{t-2,t-1}\in\mathcal{N}^{t-2,t-1}$.

4. Similarly, we can prove eqn (61) for $(i,p)=(2,3)$ via lemma 2.4. In other words, for $3<q<t$, there is $-Y_{3q}\in\mathcal{N}_{3q}$ such that $f(A^{23})_{2q}=-(A^{23})_{23}Y_{3q}$ for $A^{23}\in\mathcal{N}^{23}$.

5. Now we prove eqn (61) for $(q,j)=(t-2,t)$. For $2<p<t-2$, $A^{2p}\in\mathcal{N}^{2p}$, $A^{t2,t1}\in\mathcal{N}^{t-2,t-1}$. By a similar computation as eqn (62), for $A^{12}\in\mathcal{N}^{12}$, $A^{2p}\mathcal{N}^{2p}$, and $A^{t-2,t}\in\mathcal{N}^{t-2,t}$,

$$-f(A^{2p})_{2,t-2}(A^{t-2,t})_{t-2,t}=(A^{2p})_{2p}f(A^{t-2,t})_{p,t}. \quad (65)$$

Applying lemma 2.4 to $\phi:\mathcal{N}_{2p}\to\mathcal{N}_{2,t-2}$ defined by $\phi(C)=-f(C^{1p})_{2,t-2}$ and $\phi:\mathcal{N}_{t-2,t}\to\mathcal{N}_{p,t}$ defined by $\phi(D)=f(D^{t2,t})_{p,t}$ in eqn (65), we find $X_{p,t-2}\in\mathcal{N}_{p,t-2}$ such that $f(A^{t-2,t})_{p,t}=X_{p,t-2}(^{At-2,t-1})_{t-2,t}$ for all $A^{t2,t}\mathcal{N}^{t-2t}$.

6. Similarly, we can prove eqn (62) for $(i,p)=(1,3)$ via lemma 2.4. In other words, for $3<q<t-1$, there is $-Y_{2q}\in\mathcal{N}_{2q}$ such that $f(A^{13})_{1q}=-(A^{13})_{13}Y_{3q}$ for $A_{13}\in\mathcal{N}_{13}$.

7. Next we prove eqn (62) for $(q,j)=(t-3,t-1)$. For $1<p<t-3$, $A^{1p}\in\mathcal{N}^{1p}$, $A^{t-3,t-1}\in\mathcal{N}^{t-3,t-1}$. Then by a similar computation as eqn (63), for $A^{1p}\mathcal{N}^{1p}$, $A^{t3,t1}\in\mathcal{N}^{t-3,t-1}$, and $A^{t-1,t}\in\mathcal{N}^{t1,t}$,

$$-f(A^{1p})_{1,t-3}(A^{t-3,t-1})_{t-3,t-1}=(A^{1p})_{1p}f(A^{t-3,t-1})_{p,t-1}. \quad (66)$$

Applying lemma 2.4 to $\phi:\mathcal{N}_{1p}\to\mathcal{N}_{1,t-3}$ defined by $\phi(C)=-f(C^{1p})_{1,t-3}$ and $\varphi:\mathcal{N}_{t-3,t-1}\to\mathcal{N}_{p,t-1}$ defined by $\varphi(D)=f(D^{t3,t1})_{p,t-1}$ in eqn (66), we find $X_{p,t3}\in\mathcal{N}_{p,t-3}$ such that $f(A^{t-3,t-1})_{p,t-1}=X_{p,t-3}(A^{t-3,t-1})_{t-3,t-1}$ for all $A^{t-3,t-1}\in\mathcal{N}^{t-3,t-1}$.

8. Similarly, we can prove eqn (61) for $(i,p)=(2,4)$ via lemma 2.4. In other words, for $4<q<t$, there is $-Y_{4q}\in\mathcal{N}_{4q}$ such that $f(A^{24})_{2q}=-(A^{24})_{24}Y_{4q}$ for $A^{24}\in\mathcal{N}^{24}$.

9. Now we prove eqn (62) for $(q,j)\notin\{(t-1,t),(t-2,t),(t-2,t-1),(t-3,t-1)\}$. Then $q<t-3$. Given any $j''>j'>j$ in $[t]$, we have $\mathcal{N}^{qj''}=\mathcal{N}^{qj}\mathcal{N}^{jj'}\mathcal{N}^{j'j''}=[\mathcal{N}^{qj},[\mathcal{N}^{jj'},\mathcal{N}^{j'j''}]]$. Then for $A^{qj}\mathcal{N}^{qj}$, $A^{jj'}\in\mathcal{N}^{jj'}$, and $A^{j'j''}\in\mathcal{N}^{jj''}$,

$$f(A^{qj}A^{jj'}A^{j'j''})_{pj''}=f([A^{qj},[A^{jj'},A^{j'j''}]])_{pj''}=[f(A^{qj}),[A^{jj'},A^{j'j''}]]_{pj''}$$

$$+[A^{qj},[f(A^{jj'}),A^{j'j''}]]_{pj''}+[A^{qj},[A^{jj'},f(A^{j'j''})]]_{pj''}=f(A^{qj})_{pj}A^{jj'}A^{j'j''}. \quad (67)$$

Applying Lemma 2.3 to $\phi:\mathcal{N}_{qj}\to\mathcal{N}_{pj}$ defined by $\phi(C)=f(C^{qj''})_{pj''}$ and: $\mathcal{N}_{qj}\to\mathcal{N}_{pj}$ defined by $\varphi(D)=f(D^{qj})_{pj}$ in eqn (67), we will find $X_{pq}\in\mathcal{N}_{pq}$ such that $f(A^{qj})_{pj}=X_{pq}(A^{qj})_{qj}$ for all $A^{qj}\in\mathcal{N}^{qj}$ and $(q,j)\notin\{(t-1,t),(t-2,t),(t-2,t-1),(t-3,t-1)\}$.

10. Similarly, we can prove eqn (67) for $(i,p)\notin\{(1,2),(1,3),(2,3),(2,4)\}$ via Lemma 2.2. In other words, there exists $-Y_{pq}\in\mathcal{N}_{pq}$ such that $f(A^{ip})_{iq}=-(A^{ip})_{ip}Y_{pq}$ for all $A^{ip}\in\mathcal{N}^{ip}$ and $(i,p)\notin\{(1,2),(1,3),(2,3)(2,4)\}$.

11. Now we prove that $X_{pq}=Y_{pq}$ for $(p,q)\in\Omega'$. We prove it by the following two steps:

(a) For any $A^{i'i}\mathcal{N}^{i'i}$, $A^{ip}\in\mathcal{N}^{ip}$, and $A^{qj}\in\mathcal{N}^{qj}$, we have $i'<i<p<q<j$, $[A^{i'i},[A^{ip},A^{qj}]]=0$, so that:

$$0=f([A^{i'i},[A^{ip},A^{pj}]])_{i'j}=[f(A^{i'i}),[A^{ip},A^{pj}]]_{i'j}+[A^{i'i},[f(A^{ip}),A^{pj}]]_{i'j}$$

$$+[A^{i'i},[A^{ip},f(A^{pj})]]_{i'j}=(A^{i'i})_{i'i}f(A^{ip})_{iq}(A^{qj})_{qj}+(A^{i,i})_{i'i}(A^{ip})_{ip}f(A^{qj})_{qj}$$

$$=-(A^{i'i})_{i'i}(A^{ip})_{ip}Y_{pq}(A^{qj})_{qj}+(A^{i,i})_{i'i}(A^{ip})_{ip}X_{pq}(A^{qj})_{qj}.$$

Therefore, $X_{pq}=Y_{pq}$.

(b) Similarly, for any $A^{ip}\in\mathcal{N}^{ip}$, $A^{qj}\in\mathcal{N}^{qj}$, and $A^{jj'}\in\mathcal{N}^{jj'}$, we have $i<p<q<j<j'$, $[A^{ip},[A^{qj},A^{jj'}]]=0$, and $X_{pq}=Y_{pq}$.

Finally, the equalities eqns (61) and (62) are proved.

Proof of Theorem 3.1. By Lemma 3.9, for $(p,q)\in\Omega'$ we can find a matrix $X_{pq}\in\mathcal{N}_{pq}$ that satisfies eqns (61) and (62). Let $X^{pq}\in\mathcal{N}^{pq}$ be the matrix such that the (p,q) block is X_{pq} and 0's elsewhere, and let:

$$X_0:=\sum_{(p,q)\in\Omega'}X^{pq}\in\mathcal{N},\qquad f_0:=f-ad\,X_0. \quad (68)$$

Then f_0 is a Lie triple derivation. The equalities eqns (61) and (62) imply that:

$$f_0(\mathcal{N}^{ip})_{iq}=0\quad\text{for all }\mathcal{N}^{ip}\subseteq\mathcal{N},\quad f_0(\mathcal{N}^{qj})_{pj}=0\quad\text{for all }\mathcal{N}^{qj}\subseteq\mathcal{N}. \quad (69)$$

By Lemmas 3.2, 3.3, 3.4, 3.5, and 3.8 when char$(\mathbb{F})\neq 2$, for any $\mathcal{N}^{ij}\subseteq\mathcal{N}$, the only possibly nonzero blocks of $f_0(^{ij})$ are the (i,j) block and the following:

- The $(1,t),(2,t),(1,t-1)$ blocks when $j\in\{i+1,i+2\}$.

Recall that $\Omega=\{(i,j)\in[t]\times[t]$

$j\in\{i+1,i+2\}\}$. Define $\phi^{\Omega}_{2t},\phi^{\Omega}_{1,t-1}\in End(\mathcal{N})$ such that for $A\in\mathcal{N}$,

$$\phi^{\Omega}_{2t}(A):=\sum_{(i,j)\in\Omega}f_0(A^{ij})^{2t}=\sum_{(i,j)\in\Omega}f(A^{ij})^{2t},$$

$$\phi^{\Omega}_{1,t-1}(A):=\sum_{(i,j)\in\Omega}f_0(A^{ij})^{1,t-1}=\sum_{(i,j)\in\Omega}f(A^{ij})^{1,t-1}.$$

Then Lemmas 3.6, and 3.7 show that $\phi^{\Omega}_{2t},\phi^{\Omega}_{1,t-1}$ are Lie triple derivation of $\mathcal{N}$. Now we get a Lie triple derivation:

$$f_1:=f_0-\phi^{\Omega}_{2t}-\phi^{\Omega}_{1,t-1}=f-ad\,X_0-\phi^{\Omega}_{2t}-\phi^{\Omega}_{1,t-1}.$$

Define a linear map $\varphi_{1t}\in End(\mathcal{N})$ such that for $A\in\mathcal{N}$,

$$\varphi_{1t}(A):=\sum_{(i,j)\in\Omega}f_1(A^{ij})^{1t}=\sum_{(i,j)\in\Omega}f(A^{ij})^{1t}.$$

Then $Im\varphi_{1t}\subseteq Z(\mathcal{N})=^{1t}$ and $Ker\varphi_{1t}\supseteq[\mathcal{N},[\mathcal{N},\mathcal{N}]]$, so that φ_{1t} is a central Lie triple derivation of $\mathcal{N}$. We get a new derivation:

$$f_2:=f_1-\varphi_{1t}=f-ad\,X_0-\phi^{\Omega}_{2t}-\phi^{\Omega}_{1,t-1}-\varphi_{1t}$$

where $f_2(\mathcal{N}^{ij})\subseteq\mathcal{N}^{ij}$.

To get eqn (44), it suffices to prove the following claim regarding f_2: there exist $X^{ii}\in\mathcal{B}^{ii}$ for $i\in[t]$ such that for each $k\in[t-4]$, the Lie triple derivation:

$$f_2^{(k)}:=f_2-\sum_{i=1}^{k+4}ad\,X^{ii}$$

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Satisfies that $f_2^{(k)}(\mathcal{N}^{pq})=0$ for $1\le p<q\le k+4$. The proof is done by induction on k:

1. k=1: We proceed the case k=1 by the following steps:

- For any $A^{12}\in\mathcal{N}^{12}$, $A^{23}\in\mathcal{N}^{23}$, and $A^{34}\in\mathcal{N}^{34}$, we have:

$$\begin{aligned} & f_2(A^{12}A^{23}A^{34})_{14}=f_2([A^{12},[A^{23},A^{34}]])_{14} \\ & =[f_2(A^{12}),[A^{23},A^{34}]]_{14}+[A^{12},[f_2(A^{23}),A^{34}]]_{14}+[A^{12},[(A^{23}),f(A^{34})]]_{14} \\ & =f_2(A^{12})_{12}(A^{23})_{23}(A^{34})_{34}+(A^{12})_{12}f_2(A^{23})_{23}(A^{34})_{34}+(A^{12})_{12}(A^{23})_{23}f(A^{34})_{34}. \end{aligned} \tag{70}$$

Applying Lemma 2.6 in eqn (70), we find $X^{11}\in\mathcal{B}^{11}$, $X^{22}\in\mathcal{B}^{22}$, $-X^{33}\in\mathcal{B}^{33}$ and $-X^{44}\in\mathcal{B}^{44}$ such that

$$f_2(A^{12})_{12}=(X^{11}A^{12}-A^{12}X^{22})_{12},\ f_2(A^{23})_{23}=(X^{22}A^{23}-A^{23}X^{33})_{23},$$

and $f_2(A^{34})_{34}=(X^{33}A^{44}-A^{34}X^{34})_{34}$.

Let $f_2^0:=f_2-ad\,X^{11}-ad\,X^{22}-ad\,X^{33}-ad\,X^{44}$.

Then: $f_2^0(\mathcal{N}^{12})=0, f_2^0(\mathcal{N}^{23})=0, f_2^0(\mathcal{N}^{34})=0$ and $f_2^0(\mathcal{N}^{14})=0$.

- For any $A^{12}\in\mathcal{N}^{12}$, $A^{24}\in\mathcal{N}^{24}$, and $A^{45}\in\mathcal{N}^{45}$, by a similar computation as eqn (70),

$$f_2^0(A^{12}A^{24}A^{45})_{45}=(A^{12})_{12}(f_2^0(A^{24})_{24}(A^{45})_{45}+(A^{24})_{24}f_2^0(A^{45})_{45}). \tag{71}$$

Applying Lemma 2.2 in eqn (71), we find $-X^{55}\in\mathcal{B}^{55}$ such that $f_2^0(A^{15})_{15}=(-A^{15}X^{55})_{55}$ and by eqn (71)

$$f_2^0(A^{24})_{24}(A^{45})_{45}=(A^{24})_{24}((-A^{45}X^{55})_{45}-f_2^0(A^{45})_{45}). \tag{72}$$

Applying lemma 2.4 in eqn (74), we find $Y^{44}\in\mathcal{B}^{44}$ such that $f_2^0(A^{24})_{24}=(A^{24}Y^{44})_{24}$. We choose Y^{44}=0, then $f_2^0(A^{24})=0$ and $f_2^0(A^{45})_{45}=(-A^{45}X^{55})_{45}$.

Let $f_2^{(1)}:=f_2^0-ad\,X^{55}=f_2-ad\,X^{11}-ad\,X^{22}-ad\,X^{33}-ad\,X^{44}-ad\,X^{55}$. It is easy to show that $f_2^{(1)}(\mathcal{N}^{pq})=0$ for $1\le p<q\le 5$. So k=1 is done.

2. k=2 : Similar to eqn (70), for any $A^{12}\in\mathcal{N}^{12}$, $A^{25}\in\mathcal{N}^{25}$, and $A^{56}\in\mathcal{N}^{56}$

$$f_2^{(1)}(A^{12}A^{25}A^{56})_{16}=(A^{12})_{12}(A^{23})_{25}f_2^{(1)}(A^{56})_{56}. \tag{73}$$

Applying Lemma 2.2 in eqn (73), we find $-X^{66}\in\mathcal{B}^{66}$ such that $f_2^{(1)}(A^{16})_{16}=(-A^{16}X^{66})_{16}$ and $f_2^{(1)}(A^{56})_{56}=(-A^{56}X^{66})_{66}$. Let $f_2^{(2)}:=f_2^{(1)}-ad\,X^{66}$. It is easy to show that $f_2^{(2)}(\mathcal{N}^{pq})=0$ for $1\le p<q\le 6$. So k=2 is done.

3. k= ℓ>2: Suppose the claim holds for all $k<\ell$ where $\ell>2$ is given. In other words, there exist $X^{ii}\in\mathcal{B}^{ii}$ for all $i\in[\ell]$ such that $f_2^{(\ell-1)}:=f_2-\sum_{i=1}^{\ell+3}ad\,X^{ii}$ satisfies that $f_2^{(\ell-1)}(\mathcal{N}^{pq})=0$ for $1\le p<q\le \ell+3$. Similar to eqn (70), for any $A^{1,\ell+2}\in\mathcal{N}^{1,+2}$, $A^{+2,\ell+3}\in\mathcal{N}^{\ell+2,\ell+3}$, and $A^{\ell+3,\ell+4}\in\mathcal{N}^{\ell+3,+4}$.

$$f_2^{(\ell-1)}(A^{1,\ell+2}A^{\ell+2,\ell+3}A^{\ell+3,\ell+4})_{1,\ell+4}=(A^{1,\ell+2})_{1,\ell+2}(A^{\ell+2,\ell+3})_{\ell+2,\ell+3}f_2^{(\ell-1)}(A^{\ell+3,\ell+4})_{\ell+3,\ell+4}.$$

Applying Lemma 2.2 in the last equation, we find $-X^{+4,\ell+4}\in\mathcal{B}^{\ell+4,+4}$ such that:

$$f_2^{(\ell-1)}(A^{1,\ell+4})_{1,\ell+4}=(-A^{1,\ell+4}X^{\ell+4,\ell+4})_{1,\ell+4}.$$

Let $f_2^{(\ell)}:=f_2^{(\ell-1)}-ad\,X^{\ell+2,\ell+2}$. It is easy to show that $f_2^{(\ell)}(\mathcal{N}^{p,q})=0$ for $1\le p<q\le \ell+4$. So $k=\ell$ is proved.

Overall, the claim is completely proved; in particular, $f_2^{(\ell-4)}(\mathcal{N})=0$. Let $X:=X_0+\sum_{i=1}^{t}X^{ii}$, then we get (43).

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