Mathematical Integration of the EVA and RI Approach

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Abstract
This paper aims to show the connection between the added value and residual value approaches in management accounting and corporate finance. The research project was carried out at the Copenhagen Business School. The research is located in the field of management and finance research. The outcome will be to see whether and to which extent the Lucke-theory can be applied on the residual and the added value metrics. Furthermore it aims to answer whether these metrics give decision makers the same decision direction.

Keywords: Finance; Lucke-theory; EVA; RI; Management; Accounting

Introduction
The concepts of cost and investment calculation developed in the younger past more and more similarities [1]. This makes it necessary to ask the question why the cost accounting focusses less on cash flows throughout modern IT systems in organizations it is possible to perform high complex calculations within a realistic amount of time and costs [2]. The question regarding the combinability of the profit, capital value and added value calculations and models creates the possibility to use these systems in an integrated model. Those integration aspects can be already found in current research [3]. As a base for this paper, the theory of Gabriel Preinreich is used [4]. This theory was further developed by Wolfgang Lucke [3]. The theory represents an approach that can be seen in comparison to the investment calculations [4].

The target of this paper is to show the relationship between the capital models and the added value models. The core of the paper answers whether and with which limitations can the Preinreich-Lucke theory applied on the added value models [4]. Here for uses the paper beside the residual income the Economic value added (EVA) to show this relationship in theoretical mathematical equilibrium. Possible limitations of the equilibrium will be stated in the paper [5].

Beside the study of current research the paper will use mathematical equilibrium to show the relation between the residual income model and the EVA model. The analysis may show that both models are inter exchangeable with each other when some facts are fulfilled.

Theoretical Concepts
Preinreich lucke theory
Investments of a corporation are related to all parts of this organization and can affect multiple parts of it. The related tasks can therefore be seen as a core function in an organization and can be seen as highly relevant for the strategic development of a company [6]. In order to make suitable investment decisions a plan for this is necessary. A central part of this is the investment calculation which is a way for a rational decision support. Beside the decision support and other important function of the investment calculation is the integration function. The separation of some of the accounting actions in a company raised the integration function of the investment calculation in order to show the overall connection to the other accounting parts in an organization [1]. This integration function leads to an integration of the different parts of the accounting with that the company in its whole can be described from a financial perspective. One solution to achieve this integration function is when the investment calculation will be used as basis for the accounting in an organization [1,7]. In his papers Lucke differentiates between direct and indirect integration. According to Lucke is it possible to explain the connections between Investment, Financial, Cost and Cash-Flow calculation with direct integration. For tax calculations and special KPIs the indirect integration is suitable. But this last way of explanation will not be covered by this paper.

The integration gets possible throughout the concept of discounting. It is crucial to mention that the discounting needs to be made over the whole time a company exists. Based on this calculation it is possible to calculate all balance sheets of the different years with the equilibrium for each period.

This follows the approach that cash-flows will be sorted according to their influence and balance sheet positions. This development of plan balance sheets represents on the passive side the interest rate and on the active side a correction in the cash positions [7-9]. Because of this every year fulfills this equilibrium:

\[ Ct \text{ at the active side is the same as } Ct \text{ of the passive side with:} \]

\[ C_t = C_{t-1} + i \times C_{t-1} \]  
(1)

\[ C_t = \sum_{i=1}^{t} CF_i(t+i)^{-j}[(t+i)^j] \]  
(2)

\[ CF_t = E_t - A_t \]  
(3)

\[ Et \text{ describes the cash inflow of the period } t \text{ and } At \text{ die cash outflow of the period } t. \]  
\[ i \text{ is the interest rate. } Ct \text{ stands for the capital value in the period } t. \]  
\[ T \text{ describes in this case the end of the organization or the investment. When it comes to the cash in and outflows it can happen that they count for a different period because of a time lack. This will be taken into account in the concept by using positive or negative interest rates which lead to a cost reduction or cost increase [10].} \]

As a result out of this, capital models for an investment project lead to the same results by using different calculation bases like Cash-Flow.

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Profit-Loss, and Income-Cost. The theory shows furthermore that an investment calculation can be performed with all of these measures [10]. The Preinreich-Lucke-Theory shows that there is a connection between these measures and that they can be used inter exchangeable for one another [3,7].

A requirement for this theory is that after a specific time frame, the cashflows and the profit loss accounts accumulate to the same amount. Another requirement is that all values are based on future prices for the sale and purchase of goods. It is furthermore not suitable to use in the calculation interest, repayment and dividend payouts [11].

The capital commitment in the period -1, G-1, and at the end of the timeframe in T, GT, should be both 0.

By using the theory of Hellauer to interest rates [12], it is possible to build the following equilibrium [13],

\[ \sum_{t=0}^{T} \bar{U}_t = \sum_{t=0}^{T} (\text{Result}_t) \quad (4) \]

\[ \sum_{t=0}^{T} CF_t (1 + i)^{-t} = \sum_{t=0}^{T} (\bar{U}_t - i * G_{t-1})(1 + i)^{-t} \quad (5) \]

\[ \sum_{t=0}^{T} \bar{U}_t - \sum_{t=0}^{T} CF_t = G_{t-1, fort} = 0, 1, ..., T + 1 \quad (6) \]

\[ \sum_{t=0}^{T} \bar{U}_t - \sum_{t=0}^{T} CF_t = G_{t, fort} = 0, 1, ..., T + 1 \quad (7) \]

Us is here the surplus out of the second control variable for example the cash flow (see for that formula 3) or profit/Losses, to the time t. Gt is the capital that is committed in the investment at the time t. This is based on the assumption that the sum of all cashflows CFt is the same as the sum of all profits Ut in this case G-1 = 0 and GT = 0 applies. Formula (6) shows the definition for the capital commitment of the first period and formula (7) the definition for the capital commitment of the actual period. Formula (5) represents the equilibrium between the cash flow and the adjusted control variable Ut, as for example the residual income.

Residual income

The concept of the residual income is based on equal assumptions like the economic value added model which will be presented in the next part. The opportunity costs of the investor will be integrated into the model which leads to the fact that a decision for investment will be made only under the circumstances that the profit from the investment increases the costs of equity. This concept was developed by Edwards/Bell and Ohlson [14,15].

The concept adjusts the earnings by the interest for the equity which needs to be paid in each period on the invested capital. This can be summarized in the following:

\[ RI = \sum_{t=0}^{T} (\bar{U}_t - i * G_{t-1}) \quad (8) \]

Formula (8) describes the relation between the residual income concept and the Preinreich-Lucke-Theory. For this, look on the second part of the formula (5). RI describes in the formula the residual Income. The rest of the notions are the same as in the formulas above.

In the practical world will Gt-1 be the equity of the company. i is here the interest rate that is used to discount the outcomes of the periods. This interest rate can be calculated by using the Capital-Asset- Pricing-Models (CAPM) or the Arbitrage-Pricing-Theory (ATP) [16,17]. The differences between these calculation methods for the discount factor will be not covered in this paper.

Economic value added

The Economic value Added (EVA) was developed in the 1990’s by the consulting firm Stern & Steward. Mainly responsible for the concept are Stern J and Stewart G [18,19]. Like the residual Income so is also the EVA a member of the family of the residual income methods. This means that the earnings will be reduced by the capital cost which means that the opportunity costs for an investment will be a part of the calculation. According to certain scholars should it be possible to use the EVA concept in multiple areas of a business. One could be the resource allocation or the budgeting [18,19]. The calculation of the EVA is like followed:

\[ EVA = NOPAT - WACC * NOA \quad (9) \]

NOPATt describes the earnings before interest. The tax effect will be not covered in this paper in order to reduce complexity in the equilibriums. Also the adjustments of the NOPAT that are suggested from Stern & Steward will be not applied in this paper [18,19]. The variable Weighted Average Cost of Capital WACC will be used instead of the interest rate i. Since the concept of the EVA is based on the whole capital of a business, it is crucial to cover all costs for resources in the WACC [5]. The Net Operating Assets (NOA) replacing here the position of the capital commitment G. See for this formula (7).

\[ NOA_t = \sum_{s=0}^{T} NOPAT_s - \sum_{s=0}^{T} FCF_s \quad (10) \]

The Free-Cash-Flow (FCF) needs to be used to make the equilibrium work since the whole capital is part of the calculation. Based on formula (9) and (10), the formula for the EVA can be generated.

\[ EVA_t = NOPAT_t - WACC_t * NOA_{t-1} \quad (11) \]

With this the amount that is in addition to the capital costs produces will be described by the EVA measure.

Analysis of Concept Connection

Residual income

The Preinreich-Lucke-Theory shows that the residual value concept is able to be used as a calculation for short and long term focus. In a normal case the focus on cashflows would not lead to the same result as if the discounted profits of the periods would be analyzed. The reason for this is that the profits and the cashflows in each period differs and therefore lead to different results. In order to come with both values to the same result, the connection between the cashflows and the profits will be shown with the residual income (profit) and the free cash flow (Cashflow)

Based on formula (5) which shows the equilibrium of the Lucke-Theory and formula (8) which represents the residual income, it is possible to present the validity of the theory.

The formulas (6) and (7) can be set in relation to each other which results in formula (12) as shown here

\[ G_t - G_{t-1} = \left( \sum_{s=0}^{t} U_s - \sum_{s=0}^{t} CF_s \right) - \left( \sum_{s=0}^{t-1} U_s - \sum_{s=0}^{t-1} CF_s \right) \quad (12) \]

Formula (12) describes the difference between the capital commitment in the current period and the period before. It is possible to change this formula to calculate the profit Ut.

\[ \bar{U}_t - CF_t = G_t - G_{t-1} \quad (13) \]

\[ \bar{U}_t = CF_t + G_t - G_{t-1} \quad (14) \]
Now it is possible to see that the profits in t, Ut, by using the cashflow and the difference in the capital commitment in t and t-1. In the next step it is possible to insert the formula (14) into formula (8) and simplify the formula.

\[ \sum_{i=0}^{T} [U_i - i \times G_{i,1}] (1+i)^{-t} = \sum_{i=0}^{T} (CF_i + G_{i,1} - i \times G_{i,1}) (1+i)^{-t} \]  

(15)

It is possible to show the equilibrium (16) in a different way and split the last sum. Throughout this change it will be clear that the discounted cashflows and the discounted residual values are the same.

\[ \sum_{i=0}^{T} \frac{U_i - i \times G_{i,1}}{1+i} = \sum_{i=0}^{T} \frac{CF_i}{1+i} + \sum_{i=0}^{T} \frac{G_{i,1}}{1+i} - \sum_{i=0}^{T} \frac{i \times G_{i,1}}{1+i} \]  

(17)

\[ \sum_{i=0}^{T} \frac{U_i - i \times G_{i,1}}{1+i} = \sum_{i=0}^{T} \frac{CF_i}{1+i} + \sum_{i=0}^{T} \frac{G_{i,1}}{1+i} - \sum_{i=0}^{T} \frac{i \times G_{i,1}}{1+i} \]  

(18)

Since the capital commitment G-1 is zero, so is the capital commitment at the end of the life of the investment/company GT zero. This leads to the make the last two parts of the formula (19) zero.

\[ \sum_{i=0}^{T} \frac{U_i - i \times G_{i,1}}{1+i} = \sum_{i=0}^{T} \frac{CF_i}{1+i} \]  

(20)

The equilibrium (20) is a version of formula (5). This shows the validity for the theory when it comes to the residual income. It needs to be mentioned that the equity value is already contained in the formula since the sum was calculated starting at t=0. See for that the following formula:

\[ \sum_{i=0}^{T} \frac{U_i - i \times G_{i,1}}{1+i} = \sum_{i=0}^{T} \frac{CF_i}{1+i} + EQ_0 \]  

(21)

EQ0 is the value of the equity in t=0.

Economic value added

According to Steward is the cumulated discounted EVA the same as the result of the discounted cash flow method (DCF) [19] Because of this it applies:

\[ \sum_{i=0}^{T} \frac{EVA_i}{1+WACC} = \sum_{i=0}^{T} \frac{FCF_i}{1+WACC} \]  

(22)

Already in this stage it can be seen a parallel to the relationship between the cash flow and the residual income. Steward wants to show with this statement that the company value can be calculated by using the EVA measure and therefore can be used inter exchangeable to the DCF [19]. It shows that the relationship needs to be equal to the one that Lucke described in his work [3]. In order to prove this, it should count the same assumptions as in the previous calculation. First the difference between the capital commitments of two following periods will be shown.

\[ NOPAT_t - FCF_t = NOA_t - NOA_{t-1} \]  

(23)

\[ NOPAT_t = FCF_t + NOA_t - NOA_{t-1} \]  

(24)

The formula is pretty equal to formula (13) and (14). The difference is here that it will be calculated with values of the total capital of the company. In the next step formula (11) will be placed in these which leads to the next formula (25).

\[ \sum_{i=0}^{T} \frac{U_i - i \times G_{i,1}}{1+i} \geq 0 \]  

(25)

\[ \sum_{i=0}^{T} \frac{EVA_i}{1+WACC} \geq 0 \]  

(26)

It is possible to split the equilibrium (27) in sums.

\[ \sum_{i=0}^{T} \frac{FCF_i}{1+WACC} + \sum_{i=0}^{T} \frac{NOA_i}{1+WACC} - \sum_{i=0}^{T} \frac{NOA_{i-1}}{1+WACC} \]  

(27)

Like listed in the assumptions, G-1 and GT are both zero. This leads to the effect that the last parts of the formula (30) are zero.

\[ \sum_{i=0}^{T} \frac{EVA_i}{1+WACC} \geq \sum_{i=0}^{T} \frac{FCF_i}{1+WACC} \]  

(31)

The formula (31) is the same as (22) and this shows that the Preinreich-Lucke-Theory counts also when it comes to the EVA and that the EVA can be used instead of the cashflow calculation.

Relationship RI and EVA

Now the paper has shown the connections between both the residual income and the cashflow and the EVA and the free cash flow. This part will focus on the relationship between the EVA and the residual income as well as the cashflow and the free cash flow. The previous parts of this paper have shown that both the EVA and the residual income lead under the assumptions of the Pre-inreich-Lucke-Theory to the same results as the linked measures cashflow and free cash flow.

In this part it should be present that also the EVA and the RI lead to the same result and with that give the decision maker the same base. The first situation in which both are the same is when the company is financed with 100% equity since then the interest is 0 and both formulas get the same result. It is then WACC=i und NOPAT=U and NOA=G. Since the input values are the same, the result will also not differ. However this holds only true when there are not changes when calculating the NOPAT [18,19].

Furthermore it will be shown that the discounted EVA has the same result like the discounted RI. This would lead to the same investment and divestment decisions. If the equilibrium (32) holds true, then also formula (33) needs to hold true.

\[ \sum_{i=0}^{T} \frac{U_i - i \times G_{i,1}}{1+i} \geq 0 \]  

(32)

\[ \sum_{i=0}^{T} \frac{EVA_i}{1+WACC} \geq 0 \]  

(33)

For this calculation the NOA will be defined as the whole capital of the project and the capital commitment the same as the equity of a period. The author wants to raise the hypothesis that the graphs of both values over changing situations have a different increase but they meet...
at the point of zero. This means that both values give in all situations the same decision advice when they get discounted. In the following graphic the values of RI and EVA will be plotted by changing interest rates (Figure 1) and in the second graph over changing earnings or NOPAT (Figure 2).

In Figure 1 and 2 it is able to see that the increase of the function of the discounted EVA is higher as the one of the discounted RI. This leads to the interpretation that there is also a combination out of financing and interest rate where both values are the same. However it is much more important that both values cut the x-achses at the same point. This means that by changing variables both measures give the same decision support.

Now it needs to be examined whether EVA and RI can also have the same value. The WACC needs to be calculated on the base of market values to make this work.

\[
WACC = \left( FK* i^{EK} + EK* i^{EK} \right) / GK^{m}
\]  
(34)

\[
EK^{m} = EK + \sum_{r=1}^{T} RI_{t} / \prod_{t=1}^{r} (1+i_t)
\]

(35)

The market equity value \(EK^{m}\) will be calculated based on the balanced equity \(EK\) at \(t\) and the cumulated residual income which will be calculated using the roll back method until the time \(t\) [20].

When the discounted residual income \(RI\) will be over the lifetime of the company with the perpetuity method be cumulated, then it generates the same result like the EVA if this measure will be calculated based on the market value of equity. Based on these assumptions it needs to hold true that the left sides of the equilibriums (17) and (22) needs to be the same. But to make this happen it is important to use the retrograde version while discounting [20,21].

\[
\sum_{r=1}^{T-1} EVA_{r} / WACC_{r} \times WACC_{r} + EVA_{T} / WACC_{T} \times WACC_{T}
\]

\[
= \sum_{r=1}^{T-1} RI_{r} / (1+i_{t}) + RI_{T} / (1+i_{t})
\]

(36)

In this formula \(RI_{t}\) is the residual income in \(t\). \(WACC\) is the based on market equity value calculated.

\(WACC\). The interest rate of the is \(i\). Since a 3rd mathematical equilibrium would be too much for this paper, this relation will be shown in a practical example (Table 1). That the roll back method leads to an exact solution by doing business valuation, can be seen in the work of Casey [21]. The company value will be calculated starting with the entity value and to the time of the valuation [22].

Result

This work showed the assumptions connected to the Preinreich-Lucke-Theory. Furthermore it presented the relationship between the capital and profit accounting concepts. The core of the paper focused on the question in which cases the Preinreich-Lucke-Theory can be used also for the added value concepts. This showed the paper with the residual income and economic value added measure.

It came to the result that the residual income as well as the EVA can lead to the same result as their related cash flow measure (CF and FCF). Furthermore the paper elaborated more the relationship between the both concepts RI and EVA which are based on the one hand on the equity and on the other hand on the entity approach. The single discounted values of the RI and the EVA will give the user to all times the same decision direction (either invest or devest). The reason for this is that both measures change at the same time from negative to positive. The only difference it the increase of their curves. If the WACC in the EVA measure is calculated with market values and if

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Figure 1: Comparison of RI and EVA in changing the borrowing interest rate.

Figure 2: Comparison of RI and EVA at the change in earnings.
both measures are calculated by using the perpetuity method, then the result of both concepts is identical.

**Future Research**

Since this paper answers the last question about the relationship between the EVA and the RI only with an example and not a mathematical equilibrium, future research could look on this and tries to solve this problem in a mathematical way.

**References**