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Matrix Representation for Seven-Dimensional Nilpotent Lie Algebras

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Abstract

This paper is concerned with finding linear representations for seven-dimensional real, indecomposable nilpotent Lie algebras. We consider the first 39 algebras presented in Gong's classification which was based on the upper central series dimensions. For each algebra, we give a corresponding matrix Lie group, a representation of the Lie algebra in terms of left-invariant vector field and left-invariant one forms.

Keywords: Lie algebra; Lie group; Representation, Nilpotent

Introduction

Given a real Lie algebra g of dimension n a well known theorem due to Ado [1,2] asserts that g has a faithful representation as a subalgebra of $gl(p,R)$ for some p. The theorem does not give much information about the value of p but leads one to believe that p may be very large in relation to the size of n and consequently it seems to be of limited practical value. In previous work we have found linear representation for all indecompsable real Lie algebras in dimesion six or less [3-5] In this paper we are concerned with finding linear representations for seven-dimensional nilpotent Lie algebras. Let g be a nilpotent Lie algebra, we shall assume throughout that g is indecomposable in the sense that g is not isomorphic to a direct sum of two proper nilpotent ideals. If g is decomposable and we have representations for both factors then we can easily find a representation for g. It is well known that a nilpotent Lie algebra has a non-trivial center and so finding representations is difficult because the adjoint representation is not faithful. In this article, we consider the list of seven-dimensional nilpoten Lie algebras over R classified by Gong [6], his list contained 147 indecomposable Lie algebras that are classifed according to the upper cenral series. We consider the first 39 cases for which the dimensions the upper centeral series are (27) (37) (247) (257) and (357). For each algebra g we give a corresponding Lie group that is a subgroup of $GL(7,R)$. The representation for the Lie algebra is then easily obtained by differentiating and evaluating at the identity. The structure of the paper is as follows: in Section 2, we give a brief history and background on the classification of the seven-dimensional nilpotent Lie algebras, In section 3, we give a matrix Lie gorup that corresponds to each Lie algebra. We also give a representation of the Lie algebra in terms of left-invariant one-forms and left-invariant vectros fields. Throught out the paper we will use (p,q,r,x,y,z,w) as local coordinates on Lie groups.

Classifying Nilpotent Lie Algebras in Dimension Seven

Classification of solvable Lie algebras is not an easy problem, this is due to the fact the solvable Lie algebras are unlike semisimple Lie algebras. Semisimple Lie algebras are considered very beautiful since over the complex numbers we have the killing form, Dynkin diagrams, root space decompositions, the Serre representation, the theory of of highest weight represenation, the Weyl character formula and much more [7-10]. On the other hand, we have Lie , Engel and Ado's theorms for solvable Lie algebras. There has been several attempts to classify the seven-dimensional nilpotent Lie algebras, we will mention the most recent ones that we are aware of. In 1993, Seeley [10] gave a classification of over the filed of comples numbers. His classification was based on the upper central series of the Lie algebras and knowledge of all lower dimensional nilpotent Lie algebras. His list contained 161 Lie algebras; 130 indecomposable and 31 indecomposable. In 1998, Gong [8], presented a new list of seven-dimensional nilpotent Lie algebras. Gong's classification was based on the Skjelbred-Sund method [11]. Gong provided a classifcation of the seven-dimensional indecomposable algebras over Algebriacally closed fileds ($\chi \neq 2$) and another classification over the field of real numbers. Once again, he used the same labeling as Seeley; the dimensions of the upper central series.

Representations

In this section, we present a matrix Lie group representation for the first 39 seven-dimensional Lie algebras with upper central series (27) (37) (247) (257) and (357)

Algebras with upper central series dimensions (27)

1. $(27A): [x_1,x_2]=x_6, [x_1,x_4]=x_7, [x_3,x_5]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix} 1 & x & y & r & p & q \\ 0 & 1 & 0 & 0 & w & 0 \\ 0 & 0 & 1 & 0 & -z & 0 \\ 0 & 0 & 0 & 1 & 0 & x \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Left invariant differential forms are given by:

$$F_1 = dp - wdx + zdy, \quad F_2 = -xdr + dq, \quad F_3 = dr, \quad F_4 = dx,$$
$$F_5 = dy, \quad F_6 = dz, \quad F_7 = dw$$

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The vector field representation is given by:

$E_1 = -wD_p - D_x,\quad E_2 = -xD_q - D_r,\quad E_3 = D_z,\quad E_4 = D_w,$
$E_5 = zD_p - D_y,\quad E_6 = D_q,\quad E_7 = D_p$

2. $(27B):[x_1,x_4]=x_6,[x_2,x_5]=x_7,[x_3,x_4]=x_6+x_7,[x_3,x_5]=x_6+x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1 & y & x & r & p & q\\ 0 & 1 & 0 & 0 & w+z & z\\ 0 & 0 & 1 & 0 & z & z\\ 0 & 0 & 0 & 1 & 0 & x\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp - zdx + (-w-z)dy,\quad F_2 = -xdr + dq - zdx - zdy,\quad F_3 = dr,\quad F_4 = dx$
$F_5 = dy,\quad F_6 = dz,\quad F_7 = dz \quad dw$

Vector field representation is given by:

$E_1 = D_w,\quad E_2 = -xD_q - D_r,\quad E_3 = D_z,\quad E_4 = (w+z)D_p + zD_q + D_y$
$E_5 = zD_p + zD_q + D_x,\quad E_6 = D_p,\quad E_7 = D_q$

Algebras with upper central series dimensions (37)

1. $(37A):[x_1,x_2]=x_5,[x_2,x_3]=x_6,[x_2,x_4]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1 & 0 & 0 & w & 0 & 0 & q\\ 0 & 1 & 0 & 0 & w & 0 & -x\\ 0 & 0 & 1 & 0 & 0 & w & y\\ 0 & 0 & 0 & 1 & 0 & 0 & z\\ 0 & 0 & 0 & 0 & 1 & 0 & r\\ 0 & 0 & 0 & 0 & 0 & 1 & p\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp,\quad F_2 = dq - zdw\quad F_3 = dr\quad F_4 = -dx - rdw$
$F_5 = dy - pdw,\quad F_6 = dz,\quad F_7 = dw$

Vector field representation is given by:

$E_1 = D_p,\quad E_2 = zD_q - rD_x + pD_y + D_w,\quad E_3 = D_r,\quad E_4 = D_z$
$E_5 = D_y,\quad E_6 = D_x,\quad E_7 = -D_q$

2. $(37B):[x_1,x_2]=x_5,[x_2,x_3]=x_6,[x_3,x_4]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1 & 0 & 0 & w & 0 & -z & q\\ 0 & 1 & 0 & 0 & w & 0 & x\\ 0 & 0 & 1 & 0 & 0 & w & y\\ 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & -r\\ 0 & 0 & 0 & 0 & 0 & 1 & p\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp,\quad F_2 = dq + pdz,\quad F_3 = dr,\quad F_4 = dx + rdw$
$F_5 = dy - pdw,\quad F_6 = dz,\quad F_7 = dw$

Vector field representation is given by:

$E_1 = -D_r,\quad E_2 = D_w - rD_x + pD_y,\quad E_3 = -D_p,\quad E_4 = D_z - pD_q,$
$E_5 = D_x,\quad E_6 = D_y,\quad E_7 = D_q$

3. $(37C):[x_1,x_2]=x_5,[x_2,x_3]=x_6,[x_2,x_4]=x_7,[x_3,x_4]=x_5;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1 & 0 & 0 & w & 0 & r & -q\\ 0 & 1 & 0 & 0 & w & 0 & x\\ 0 & 0 & 1 & 0 & 0 & w & y\\ 0 & 0 & 0 & 1 & 0 & 0 & z\\ 0 & 0 & 0 & 0 & 1 & 0 & r\\ 0 & 0 & 0 & 0 & 0 & 1 & p\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp,\quad F_2 = dr,\quad F_3 = dz,\quad F_4 = dw$
$F_5 = -pdr - dq - zdw,\quad F_6 = dx - rdw,\quad F_7 = dy - pdw$

Vector field representation is given by:

$E_1 = D_p,\quad E_2 = zD_q - rD_x + pD_y + D_w,\quad E_3 = D_r,\quad E_4 = D_z,$
$E_5 = D_y,\quad E_6 = D_x,\quad E_7 = -D_q$

4. $(37D):[x_1,x_2]=x_5,[x_1,x_3]=x_6,[x_2,x_4]=x_7,[x_3,x_4]=x_5;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1 & 0 & 0 & w & -z & 0 & q\\ 0 & 1 & 0 & 0 & w & -z & x\\ 0 & 0 & 1 & 0 & 0 & w & y\\ 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & r\\ 0 & 0 & 0 & 0 & 0 & 1 & p\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp,\quad F_2 = dq + rdz,\quad F_3 = dr,\quad F_4 = dx + pdz - rdw$
$F_5 = dy - pdw,\quad F_6 = dz,\quad F_7 = dw$

Vector field representation is given by:

$E_1 = D_p,\quad E_2 = -rD_q - pD_x + D_z,\quad E_3 = rD_x + pD_y + D_w,\quad E_4 = D_r,$
$E_5 = -D_x,\quad E_6 = D_y,\quad E_7 = D_q$

Algebras with upper central series dimensions (247)

1. $(247A):[x_1,x_i]=x_{i+2}, i=2,3,4,5;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1 & 0 & w & 0 & 1/2w^2 & 0 & q\\ 0 & 1 & 0 & w & 0 & 1/2w^2 & x\\ 0 & 0 & 1 & 0 & w & 0 & y\\ 0 & 0 & 0 & 1 & 0 & w & z\\ 0 & 0 & 0 & 0 & 1 & 0 & r\\ 0 & 0 & 0 & 0 & 0 & 1 & p\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp,\quad F_2 = dq - ydw, F_3 = dr,\quad F_4 = dx - zdw$
$F_5 = dy - rdw,\quad F_6 = dz - pdw,\quad F_7 = dw$

Vector field representation is given by:

$E_1 = -yD_q - zD_x - rD_y - pD_z - D_w,\quad E_2 = D_p,\quad E_3 = D_r,\quad E_4 = D_z$
$E_5 = D_y,\quad E_6 = D_x, E_7 = D_q$

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2. $(247B):[x_1,x_i]=x_{i+2}, i=2,3,4,[x_3,x_5]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix} 1 & 0 & 0 & p & 0 & -z+pw & q \\ 0 & 1 & w & 0 & 1/2w^2 & 0 & x \\ 0 & 0 & 1 & 0 & w & 0 & y \\ 0 & 0 & 0 & 1 & 0 & w & z \\ 0 & 0 & 0 & 0 & 1 & 0 & r \\ 0 & 0 & 0 & 0 & 0 & 1 & p \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Left invariant differential forms are given by:

$F_1=dp,\quad F_2=-zdp+dq+pdz-p^2dw,\quad F_3=dr,\quad F_4=dx-ydw$
$F_5=dy-rdw,\quad F_6=dz-pdw,\quad F_7=dw$

Vector field representation is given by:

$E_1=D_w+yD_x+rD_y+pD_z,\quad E_2=D_r,\quad E_3=D_p+zD_q,\quad E_4=-D_y$
$E_5=pD_q-D_z,\quad E_6=D_x,\quad E_7=2*D_q$

3. $(247C):[x_1,x_i]=x_{i+2}, i=2,3,4,[x_1,x_5]=x_7,[x_3,x_5]=x_6;$

A matrix Lie group is given by:

$$S=\begin{bmatrix} 1 & 0 & w & p & 1/2w^2 & -z+pw & q \\ 0 & 1 & 0 & w & 0 & 1/2w^2 & x \\ 0 & 0 & 1 & 0 & w & 0 & y \\ 0 & 0 & 0 & 1 & 0 & w & z \\ 0 & 0 & 0 & 0 & 1 & 0 & r \\ 0 & 0 & 0 & 0 & 0 & 1 & p \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Left invariant differential forms are given by:

$F_1=dp,\quad F_2=-zdp+dq+pdz+(-p^2-y)dw,\quad F_3=dr,\quad F_4=dx-zdw$
$F_5=dy-rdw,\quad F_6=dz-pdw,\quad F_7=dw$

Vector field representation is given by:

$E_1=D_w+yD_q+zD_x+rD_y+pD_z,\quad E_2=2D_r,\quad E_3=D_p+zD_q,\quad E_4=-2D_y$
$E_5=pD_q-D_z,\quad E_6=2D_q,\quad E_7=D_x$

4. $(247D):[x_1,x_i]=x_{i+2}, i=2,3,[x_1,x_4]=x_6,[x_2,x_5]=x_7,[x_3,x_4]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix} 1 & 0 & 0 & r & 0 & rw-y & q \\ 0 & 1 & 0 & w & 0 & 1/2w^2 & x \\ 0 & 0 & 1 & 0 & w & 0 & y \\ 0 & 0 & 0 & 1 & 0 & w & z \\ 0 & 0 & 0 & 0 & 1 & 0 & r \\ 0 & 0 & 0 & 0 & 0 & 1 & p \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Left invariant differential forms are given by:

$F_1=dp,\quad F_2=-zdr+dq+pdy-prdw,\quad F_3=dr,\quad F_4=dx-zdw,$
$F_5=dy-rdw,\quad F_6=dz-pdw,\quad F_7=dw$

Vector field representation is given by:

$E_1=zD_x+rD_y+pD_z+D_w,\quad E_2=D_p,\quad E_3=zD_q+D_r,\quad E_4=-D_z,$
$E_5=pD_q-D_y,\quad E_6=D_x,\quad E_7=D_q$

5. $(247E):[x_1,x_i]=x_{i+2}, i=2,3,4,[x_1,x_5]=x_6,[x_2,x_5]=x_7,[x_3,x_4]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix} 1 & 0 & 0 & r & 0 & rw-y & q \\ 0 & 1 & w & w & 1/2w^2 & 1/2w^2 & x \\ 0 & 0 & 1 & 0 & w & 0 & y \\ 0 & 0 & 0 & 1 & 0 & w & z+pw \\ 0 & 0 & 0 & 0 & 1 & 0 & r \\ 0 & 0 & 0 & 0 & 0 & 1 & p \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Left invariant differential forms are given by:

$F_1=dp,\quad F_2=(-pw-z)dr+dq+pdy-prdw,\quad F_3=dr,\quad F_4=dx+(-pw-z-y)dw$
$F_5=dy-rdw,\quad F_6=wdp+dz,\quad F_7=dw$

Vector field representation is given by:

$E_1=(pw+z+y)D_x+rD_y+D_w,\quad E_2=(z+pw)D_q+D_r,\quad E_3=D_p-wD_z,\quad E_4=pD_q-D_y$
$E_5=-D_z,\quad E_6=D_x,\quad E_7=D_q$

6. $(247F):[x_1,x_i]=x_{i+2}, i=2,3,[x_2,x_4]=x_6,[x_2,x_5]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix} 1 & 0 & 0 & w & 0 & r+1/2w^2 & q \\ 0 & 1 & r & 0 & -y+rw & 0 & x \\ 0 & 0 & 1 & 0 & w & 0 & y \\ 0 & 0 & 0 & 1 & 0 & w & z \\ 0 & 0 & 0 & 0 & 1 & 0 & r \\ 0 & 0 & 0 & 0 & 0 & 1 & p \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Left invariant differential forms are given by:

$F_1=dp,\quad F_2=-pdr+dq-zdw,\quad F_3=dr,\quad F_4=-ydr+dx+rdy-r^2dw$
$F_5=dy-rdw,\quad F_6=dz-pdw,\quad F_7=dw$

Vector field representation is given by:

$E_1=rD_y+pD_z+D_w,\quad E_2=D_r+yD_x,\quad E_3=-D_p-zD_q,\quad E_4=rD_x-D_y$
$E_5=-pD_q+D_z,\quad E_6=2D_x,\quad E_7=2D_q$

7. $(247G):[x_1,x_i]=x_{i+2}, i=2,3,[x_1,x_4]=x_7,[x_2,x_4]=x_6,[x_3,x_5]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix} 1 & 0 & w & p & 1/2w^2 & -z+pw & q \\ 0 & 1 & r & 0 & -y+rw & 0 & x \\ 0 & 0 & 1 & 0 & w & 0 & y \\ 0 & 0 & 0 & 1 & 0 & w & z \\ 0 & 0 & 0 & 0 & 1 & 0 & r \\ 0 & 0 & 0 & 0 & 0 & 1 & p \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Left invariant differential forms are given by:

$F_1=dp,\quad F_2=-zdp+dq+pdz+(-p^2-y)dw,\quad F_3=dr,\quad F_4=-ydr+dx+rdy-r^2dw$
$F_5=dy-rdw,\quad F_6=dz-pdw,\quad F_7=dw$

Vector field representation is given by:

$E_1=yD_q+rD_y+pD_z+D_w,\quad E_2=2D_r+2yD_x,\quad E_3=-D_p-zD_q,\quad E_4=2rD_x-2D_y$
$E_5=-pD_q+D_z,\quad E_6=8D_x,\quad E_7=2D_q$

8. $(247H)$:

$[x_1,x_i]=x_{i+2}, i=2,3,[x_1,x_4]=x_7,[x_1,x_5]=x_6,[x_2,x_4]=x_6,[x_3,x_5]=x_7;$

A matrix Lie group is given by:

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$$S=\begin{bmatrix}1&0&w&p&1/2w^2&pw-z&q\\0&1&r&w&rw-y&1/2w^2&x\\0&0&1&0&w&0&y\\0&0&0&1&0&w&z\\0&0&0&0&1&0&r\\0&0&0&0&0&1&p\\0&0&0&0&0&0&1\end{bmatrix}$$

Left invariant differential forms are given by:

$$F_1=dp,\quad F_2=-zdp+dq+pdz+(-p^2-y)dw,\quad F_3=dr,\quad F_4=-ydr+dx+rdy+(-r^2-z)dw$$
$$F_5=dy-rdw,\quad F_6=dz-pdw,\quad F_7=dw$$

Vector field representation is given by:

$$E_1=2yD_q+2zD_x+2rD_y+2pD_z+2D_w,\quad E_2=D_r+yD_x,\quad E_3=D_p+zD_q,\quad E_4=2rD_x-2D_y$$
$$E_5=2pD_q-2D_z,\quad E_6=4D_x,\quad E_7=4D_q$$

9. $(247I):[x_1,x_i]=x_{i+2},i=2,3,[x_2,x_5]=x_6,[x_3,x_4]=x_6,[x_3,x_5]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1&0&0&r&0&rw-y&q\\0&1&0&p&0&-z&x\\0&0&1&0&w&0&y\\0&0&0&1&0&w&z+pw\\0&0&0&0&1&0&r\\0&0&0&0&0&1&p\\0&0&0&0&0&0&1\end{bmatrix}$$

Left invariant differential forms are given by:

$$F_1=dp,\quad F_2=(-z-pw)dr+dq+pdy-prdw,\quad F_3=dr,\quad F_4=zdp+dx+pdz$$
$$F_5=dy-rdw,\quad F_6=wdp+dz,\quad F_7=dw$$

Vector field representation is given by:

$$E_1=rD_y+D_w,\quad E_2=(2z+2pw)D_q+2D_r,\quad E_3=-D_p+(-z-pw)D_x+wD_z,\quad E_4=2pD_q-2D_y$$
$$E_5=-pD_x+D_z,\quad E_6=-2D_q,\quad E_7=2D_x$$

10. $(247J):[x_1,x_i]=x_{i+2},i=2,3,4[x_2,x_5]=x_6,[x_3,x_4]=x_6,[x_3,x_5]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1&0&0&p&0&pw-z&q\\0&1&w&r&1/2w^2&rw-y&x\\0&0&1&0&w&0&y\\0&0&0&1&0&w&z\\0&0&0&0&1&0&r\\0&0&0&0&0&1&p\\0&0&0&0&0&0&1\end{bmatrix}$$

Left invariant differential forms are given by:

$$F_1=dp,\quad F_2=-zdp+dq+pdz-p^2dw,\quad F_3=dr,\quad F_4=-zdr+dx+pdy+(-pr-y)dw$$
$$F_5=dy-rdw,\quad F_6=dz-pdw,\quad F_7=dw$$

Vector field representation is given by:

$$E_1=yD_x+rD_y+pD_z+D_w,\quad E_2=D_r+zD_x,\quad E_3=D_p+zD_q,\quad E_4=pD_x-D_y$$
$$E_5=pD_q-D_z,\quad E_6=D_x,\quad E_7=2D_q$$

11. $(247K):[x_1,x_i]=x_{i+2},i=2,3,4,[x_2,x_5]=x_7,[x_3,x_4]=x_7,[x_3,x_5]=x_6;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1&0&0&r&0&rw-y&q\\0&1&w&p&1/2w^2&-z&x\\0&0&1&0&w&0&y\\0&0&0&1&0&w&z+pw\\0&0&0&0&1&0&r\\0&0&0&0&0&1&p\\0&0&0&0&0&0&1\end{bmatrix}$$

Left invariant differential forms are given by:

$$F_1=dp,\quad F_2=(-z-pw)dr+dq+pdy-prdw,\quad F_3=dr,\quad F_4=-zdp+dx+pdz-ydw$$
$$F_5=dy-rdw,\quad F_6=wdp+dz,\quad F_7=dw$$

Vector field representation is given by:

$$E_1=yD_x+rD_y+D_w,\quad E_2=(2z+2pw)D_q+2D_r,\quad E_3=D_p+(z+pw)D_x-wD_z,\quad E_4=2pD_q-2D_y$$
$$E_5=pD_x-D_z,\quad E_6=2D_x,\quad E_7=2D_q$$

12. $(247L):[x_1,x_i]=x_{i+2},i=2,3,4,5,[x_2,x_3]=x_6$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1&0&w&0&1/2w^2&r&q\\0&1&0&w&0&1/2w^2&x\\0&0&1&0&w&0&y\\0&0&0&1&0&w&z\\0&0&0&0&1&0&r\\0&0&0&0&0&1&p\\0&0&0&0&0&0&1\end{bmatrix}$$

Left invariant differential forms are given by:

$$F_1=dp,\quad F_2=-pdr+dq-ydw,\quad F_3=dr,\quad F_4=dx-zdw$$
$$F_5=dy-rdw,\quad F_6=dz-pdw,\quad F_7=dw$$

Vector field representation is given by:

$$E_1=yD_q+zD_x+rD_y+pD_z+D_w,\quad E_2=-pD_q-D_r,\quad E_3=-D_p,\quad E_4=D_y$$
$$E_5=D_z,\quad E_6=-D_q,\quad E_7=-D_x$$

13. $(247M):[x_1,x_i]=x_{i+2},i=2,3,4,[x_2,x_3]=x_6,[x_3,x_5]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1&0&0&w&0&r+1/2w^2&q\\0&1&r&0&-y+rw&0&x\\0&0&1&0&w&0&y\\0&0&0&1&0&w&z\\0&0&0&0&1&0&r\\0&0&0&0&0&1&p\\0&0&0&0&0&0&1\end{bmatrix}$$

Left invariant differential forms are given by:

$$F_1=dp,\quad F_2=-pdr+dq-zdw,\quad F_3=dr,\quad F_4=-ydr+dx+rdy-r^2dw$$
$$F_5=dy-rdw,\quad F_6=dz-pdw,\quad F_7=dw$$

Vector field representation is given by:

$$E_1=-zD_q-rD_y-pD_z-D_w,\quad E_2=D_p,\quad E_3=pD_q+D_r+yD_x,\quad E_4=D_z$$
$$E_5=-rD_x+D_y,\quad E_6=D_q,\quad E_7=-2D_x$$

14. $(247N):[x_1,x_i]=x_{i+2},i=2,3,[x_1,x_5]=x_6,[x_2,x_3]=x_7,[x_2,x_4]=x_6;$

A matrix Lie group is given by:

Citation: Ghanam R, Basim Mustafa B, Mustafa MT, Thompson G (2016) Matrix Representation for Seven-Dimensional Nilpotent Lie Algebras. J Phys Math 7: 155. doi:10.4172/2090-0902.1000155

$$S=\begin{bmatrix}1&0&0&0&0&r&q\\0&1&r&w&rw-y&1/2w^2&x\\0&0&1&0&w&0&y\\0&0&0&1&0&w&z\\0&0&0&0&1&0&r\\0&0&0&0&0&1&p\\0&0&0&0&0&0&1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1=dp,\quad F_2=-pdr+dq,\quad F_3=dr,\quad F_4=-ydr+dx+rdy+(-r^2-z)dw$
$F_5=dy-rdw,\quad F_6=dz-pdw,\quad F_7=dw$

Vector field representation is given by:

$E_1=zD_x+rD_y+pD_z+D_w,\quad E_2=pD_q+D_r+yD_x,\quad E_3=2D_p,\quad E_4=rD_x-D_y$
$E_5=-2D_z,\quad E_6=2D_x,\quad E_7=-2D_q$

15. $(247O):[x_1,x_i]=x_{i+2},i=2,3,4,[x_1,x_5]=x_7,[x_2,x_3]=x_7,[x_3,x_5]=x_6;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1&0&w&0&1/2w^2&r&q\\0&1&r&w&-y+rw&1/2w^2&x\\0&0&1&0&w&0&y\\0&0&0&1&0&w&z\\0&0&0&0&1&0&r\\0&0&0&0&0&1&p\\0&0&0&0&0&0&1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1=dp,\quad F_2=-pdr+dq-ydw,\quad F_3=dr,\quad F_4=-ydr+dx+rdy+(-r^2-z)*w$
$F_5=dy-rdw,\quad F_6=dz-pdw,\quad F_7=dw$

Vector field representation is given by:

$E_1=2yD_q+2zD_x+2rD_y+2pD_z+2D_w,\quad E_2=4D_p,\quad E_3=2pD_q+2D_r+2yD_x,\quad E_4=-8D_z$
$E_5=4rD_x-4D_y,\quad E_6=16D_x,\quad E_7=8D_q$

16. $(247P):[x_1,x_i]=x_{i+2},i=2,3,[x_2,x_3]=x_6,[x_2,x_5]=x_7,[x_3,x_4]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1&0&0&0&p&0&q\\0&1&0&r&0&rw-y&x\\0&0&1&0&w&0&y\\0&0&0&1&0&w&z\\0&0&0&0&1&0&r\\0&0&0&0&0&1&p\\0&0&0&0&0&0&1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1=dp,\quad F_2=-rdp+dq,\quad F_3=dr,\quad F_4=-zdr+dx+pdy-prdw$
$F_5=dy-rdw,\quad F_6=dz-pdw,\quad F_7=dw$

Vector field representation is given by:

$E_1=rD_y+pD_z+D_w,\quad E_2=-D_r-zD_x,\quad E_3=-D_p-rD_q,\quad E_4=-pD_x+D_y$
$E_5=D_z,\quad E_6=D_q,\quad E_7=D_x$

17. $(247Q):[x_1,x_i]=x_{i+2},i=2,3,4,[x_2,x_3]=x_6,[x_2,x_5]=x_7,[x_3,x_4]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1&0&0&w&0&r+1/2w^2&q\\0&1&p&r&pw-z&-y+rw&x\\0&0&1&0&w&0&y\\0&0&0&1&0&w&z\\0&0&0&0&1&0&r\\0&0&0&0&0&1&p\\0&0&0&0&0&0&1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1=dp,\quad F_2=-pdr+dq-zdw,\quad F_3=dr,\quad F_4=-zdr-ydp+dx+pdy+rdz-2prdw$
$F_5=dy-rdw,\quad F_6=dz-pdw,\quad F_7=dw$

Vector field representation is given by:

$E_1=2zD_q+2rD_y+2pD_z+2D_w,\quad E_2=-2D_p-2yD_x,\quad E_3=4pD_q+4D_r+4zD_x,\quad E_4=-4rD_x+4D_z$
$E_5=8pD_x-8D_y,\quad E_6=-8D_q,\quad E_7=-32D_x$

18. $(247R):$

$[x_1,x_i]=x_{i+2},i=2,3,4,[x_1,x_5]=x_6,[x_2,x_3]=x_6,[x_2,x_5]=x_7,[x_3,x_4]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1&0&w&w&p+1/2w^2&1/2w^2&q\\0&1&0&r&0&rw-y&x\\0&0&1&0&w&0&y\\0&0&0&1&0&w&z\\0&0&0&0&1&0&r\\0&0&0&0&0&1&p\\0&0&0&0&0&0&1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1=dp,\quad F_2=-rdp+dq+(-z-y)dw,\quad F_3=dr,\quad F_4=-zdr+dx+pdy-rpdw$
$F_5=dy-rdw,\quad F_6=dz-pdw,\quad F_7=dw$

The Vector field representation is given by:

$E_1=(z+y)D_q+rD_y+pD_z+D_w,\quad E_2=D_r+zD_x,\quad E_3=D_p+rD_q,\quad E_4=pD_x-D_y,$
$E_5=-D_z,\quad E_6=D_q,\quad E_7=D_x$

Algebras with upper central series dimensions (257)

1. $(257A):[x_1,x_2]=x_3,[x_1,x_3]=x_6,[x_1,x_5]=x_7,[x_2,x_4]=x_6;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1&x&-y&-r&p&q\\0&1&x&0&w&0\\0&0&1&0&z&0\\0&0&0&1&0&x\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1=dp+(-w+zx)dx+zdy,\quad F_2=xdr+dq,\quad F_3=dr,\quad F_4=dx,$
$F_5=xdx+dy,\quad F_6=dz,\quad F_7=-zdx+dw$

Vector field representation is given by:

$E_1=-D_x-wD_p+xD_y-zD_w,\quad E_2=D_z,\quad E_3=D_w,\quad E_4=-D_y+zD_p,$
$E_5=D_r-xD_q,\quad E_6=D_p,\quad E_7=D_q$

2. $(257B):[x_1,x_2]=x_3,[x_1,x_3]=x_6,[x_1,x_4]=x_7,[x_2,x_5]=x_7;$

A matrix Lie group is given by:

Citation: Ghanam R, Basim Mustafa B, Mustafa MT, Thompson G (2016) Matrix Representation for Seven-Dimensional Nilpotent Lie Algebras. J Phys Math 7: 155. doi:10.4172/2090-0902.1000155

$$S=\begin{bmatrix} 1 & 0 & -w & -w & 0 & 1/2w^2 & q \\ 0 & 1 & 0 & 0 & -w & z & -x \\ 0 & 0 & 1 & 0 & 0 & -w & -y \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & r \\ 0 & 0 & 0 & 0 & 0 & 1 & p \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp,\quad F_2 = dq - ydw,\quad F_3 = dr,\quad F_4 = dx + pdz - rdw,$
$F_5 = dy - pdw,\quad F_6 = dz,\quad F_7 = dw$

Vector field representation is given by:

$E_1 = -yD_q - rD_x - pD_y - D_w,\quad E_2 = D_p,\quad E_3 = D_y,\quad E_4 = D_r,$
$E_5 = pD_x - D_z,\quad E_6 = D_q,\quad E_7 = D_x$

3. $(257C):[x_1,x_2]=x_3,[x_1,x_3]=x_6,[x_2,x_4]=x_6,[x_2,x_5]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix} 1 & x & -y & -r & p & q \\ 0 & 1 & z & 0 & w & 0 \\ 0 & 0 & 1 & 0 & z & 0 \\ 0 & 0 & 0 & 1 & 0 & x \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp + (-w + z^2)dx + zdy,\quad F_2 = xdr + dq,\quad F_3 = dr,\quad F_4 = dx,$
$F_5 = zdx + dy,\quad F_6 = dz,\quad F_7 = -zdz + dw$

Vector field representation is given by:

$E_1 = D_z + zD_w,\quad E_2 = wD_p + D_x - zD_y,\quad E_3 = zD_p - D_y,\quad E_4 = -D_w,$
$E_5 = xD_q - D_r,\quad E_6 = D_p,\quad E_7 = D_q$

4. $(257D):[x_1,x_2]=x_3,[x_1,x_3]=x_6,[x_1,x_4]=x_7,[x_2,x_4]=x_6,[x_2,x_5]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix} 1 & 0 & -w & -w & p & 1/2w^2 & q \\ 0 & 1 & 0 & 0 & -w & z & -x \\ 0 & 0 & 1 & 0 & 0 & -w & -y \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & r \\ 0 & 0 & 0 & 0 & 0 & 1 & p \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp,\quad F_2 = -rdp + dq - ydw,\quad F_3 = dr,\quad F_4 = dx + pdz - rdw,$
$F_5 = dy - pdw,\quad F_6 = dz,\quad F_7 = dw$

Vector field representation is given by:

$E_1 = D_w + yD_q + rD_x + pD_y,\quad E_2 = D_p + rD_q,\quad E_3 = -D_y,\quad E_4 = -D_r,$
$E_5 = -D_z + pD_x,\quad E_6 = D_q,\quad E_7 = D_x$

5. $(257E):[x_1,x_2]=x_3,[x_1,x_3]=x_6,[x_2,x_4]=x_7,[x_4,x_5]=x_6;$

A matrix Lie group is given by:

$$S=\begin{bmatrix} 1 & 0 & -w & -w & z & 1/2w^2 & q \\ 0 & 1 & 0 & 0 & 0 & z & -x \\ 0 & 0 & 1 & 0 & 0 & -w & -y \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & r \\ 0 & 0 & 0 & 0 & 0 & 1 & p \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp,\quad F_2 = dq - rdz - ydw,\quad F_3 = dr,\quad F_4 = dx + pdz,$
$F_5 = dy - pdw,\quad F_6 = dz,\quad F_7 = dw$

Vector field representation is given by:

$E_1 = yD_q + pD_y + D_w,\quad E_2 = D_p,\quad E_3 = -D_y,\quad E_4 = -rD_q + pD_x - D_z,$
$E_5 = D_r,\quad E_6 = D_q,\quad E_7 = D_x$

6. $(257F):\ \lfloor 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \rfloor$

A matrix Lie group is given by:

$$S=\begin{bmatrix} 1 & 0 & -w & -w & z & 1/2w^2 & q \\ 0 & 1 & 0 & 0 & -w & 0 & -x \\ 0 & 0 & 1 & 0 & 0 & -w & -y \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & r \\ 0 & 0 & 0 & 0 & 0 & 1 & p \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp,\quad F_2 = dq - rdz - ydw,\quad F_3 = dr,\quad F_4 = dx - rdw,$
$F_5 = dy - pdw,\quad F_6 = dz,\quad F_7 = dw$

Vector field representation is given by:

$E_1 = -D_p,\quad E_2 = -yD_q - rD_x - pD_y - D_w,\quad E_3 = D_y,\quad E_4 = D_r,$
$E_5 = rD_q + D_z,\quad E_6 = D_q,\quad E_7 = D_x$

7. $(257G):[x_1,x_2]=x_3,[x_1,x_3]=x_6,[x_1,x_5]=x_7,[x_2,x_4]=x_7,[x_4,x_5]=x_6;$

A matrix Lie group is given by:

$$S=\begin{bmatrix} 1 & 0 & -w & -w & z & 1/2w^2 & q \\ 0 & 1 & 0 & 0 & -w & z & -x \\ 0 & 0 & 1 & 0 & 0 & -w & -y \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & r \\ 0 & 0 & 0 & 0 & 0 & 1 & p \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp,\quad F_2 = dq - rdz - ydw,\quad F_3 = dr,\quad F_4 = dx + pdz - rdw,$
$F_5 = dy - pdw,\quad F_6 = dz,\quad F_7 = dw$

Vector field representation is given by:

$E_1 = -yD_q - rD_x - pD_y - D_w,\quad E_2 = D_p,\quad E_3 = D_y,\quad E_4 = -rD_q + pD_x - D_z,$
$E_5 = D_r,\quad E_6 = D_q,\quad E_7 = D_x$

8. $(257H):[x_1,x_2]=x_3,[x_1,x_3]=x_6,[x_2,x_4]=x_6,[x_4,x_5]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1 & x & -y & -r & p & q\\ 0 & 1 & z & 0 & w+1/2z^2 & 0\\ 0 & 0 & 1 & 0 & z & 0\\ 0 & 0 & 0 & 1 & 0 & w\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp + (-w + (1/2)z^2)dx + zdy,\quad F_2 = wdr + dq,\quad F_3 = dr,\quad F_4 = dx,$
$F_5 = zdx + dy,\quad F_6 = dz,\quad F_7 = dw$

Vector field representation is given by:

$E_1 = -D_z,\quad E_2 = D_x + (w + (1/2)z^2)D_p - zD_y,\quad E_3 = -zD_p + D_y,\quad E_4 = -D_w,$
$E_5 = -wD_q + D_r,\quad E_6 = D_p,\quad E_7 = D_q$

9. $(257I): [x_1,x_2]=x_3, [x_1,x_3]=x_6, [x_1,x_4]=x_6, [x_1,x_5]=x_7, [x_2,x_3]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1 & 0 & -w & -w & 0 & 1/2w^2 & q\\ 0 & 1 & p & 0 & -w & y & -x\\ 0 & 0 & 1 & 0 & 0 & -w & -y-pw\\ 0 & 0 & 0 & 1 & 0 & 0 & z\\ 0 & 0 & 0 & 0 & 1 & 0 & r\\ 0 & 0 & 0 & 0 & 0 & 1 & p\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp,\quad F_2 = dq + (-y - pw + z)dw,\quad F_3 = dr,\quad F_4 = ydp - dx - pdy + rdw,$
$F_5 = wdp + dy,\quad F_6 = dz,\quad F_7 = dw$

Vector field representation is given by:

10. $(257J): [x_1,x_2]=x_3, [x_1,x_3]=x_6, [x_1,x_5]=x_7, [x_2,x_3]=x_7, [x_2,x_4]=x_6;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1 & 0 & -w & -w & 0 & 1/2w^2+z & q\\ 0 & 1 & p & 0 & -w & y & -x\\ 0 & 0 & 1 & 0 & 0 & -w & -y-pw\\ 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & r\\ 0 & 0 & 0 & 0 & 0 & 1 & p\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp,\quad F_2 = dq - pdz + (-y - pw)dw,\quad F_3 = dr,\quad F_4 = ydp - dx - pdy + rdw,$
$F_5 = wdp + dy,\quad F_6 = dz,\quad F_7 = dw$

Vector field representation is given by:

$E_1 = (y + pw)D_q + rD_x + D_w,\quad E_2 = D_p + (y + pw)D_x - wD_y,\quad E_3 = pD_x - D_y,\quad E_4 = pD_q + D_z,$
$E_5 = -2D_r,\quad E_6 = D_q,\quad E_7 = 2D_x$

11. $(257K): [x_1,x_2]=x_3, [x_1,x_3]=x_6, [x_2,x_3]=x_7, [x_4,x_5]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1 & 0 & -z & y-wz & r & 0 & q\\ 0 & 1 & w & 1/2w^2 & 0 & 0 & x\\ 0 & 0 & 1 & w & 0 & 0 & 2y\\ 0 & 0 & 0 & 1 & 0 & 0 & 2z\\ 0 & 0 & 0 & 0 & 1 & 0 & p\\ 0 & 0 & 0 & 0 & 0 & 1 & r\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp,\quad F_2 = -pdr + dq - 2zdy + 2ydz + 2z^2dw,\quad F_3 = dr,\quad F_4 = dx - 2ydw,$
$F_5 = dy - zdw,\quad F_6 = dz,\quad F_7 = dw$

Vector field representation is given by:

$E_1 = D_w + 2yD_x + zD_y,\quad E_2 = D_z - 2yD_q,\quad E_3 = -D_y - 2zD_q,\quad E_4 = -4D_p,$
$E_5 = D_r + pD_q,\quad E_6 = 2D_x,\quad E_7 = -4D_q$

12. $(257L): [x_1,x_2]=x_3, [x_1,x_3]=x_6, [x_2,x_3]=x_7, [x_2,x_4]=x_6, [x_4,x_5]=x_7;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1 & 0 & -z & y-wz & r & 0 & q\\ 0 & 1 & w & 1/2w^2 & 0 & z & x\\ 0 & 0 & 1 & w & 0 & 0 & 2y\\ 0 & 0 & 0 & 1 & 0 & 0 & 2z\\ 0 & 0 & 0 & 0 & 1 & 0 & p\\ 0 & 0 & 0 & 0 & 0 & 1 & r\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp,\quad F_2 = -pdr + dq - 2zdy + 2ydz + 2z^2dw,\quad F_3 = dr,\quad F_4 = dx - rdz - 2ydw,$
$F_5 = dy - zdw,\quad F_6 = dz,\quad F_7 = dw$

Vector field representation is given by:

$E_1 = D_w + 2yD_x + zD_y,\quad E_2 = D_z - 2yD_q + rD_x,\quad E_3 = -D_y - 2zD_q,\quad E_4 = -2D_r - 2pD_q,$
$E_5 = -2D_p,\quad E_6 = 2D_x,\quad E_7 = -4D_q$

Algebras with upper central series dimensions (357)

1. $(357A): [x_1,x_2]=x_3, [x_1,x_3]=x_5, [x_1,x_4]=x_7, [x_2,x_4]=x_6;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & r & -q\\ 0 & 1 & 0 & w & 0 & 1/2w^2 & x\\ 0 & 0 & 1 & 0 & w & 0 & y\\ 0 & 0 & 0 & 1 & 0 & w & z\\ 0 & 0 & 0 & 0 & 1 & 0 & r\\ 0 & 0 & 0 & 0 & 0 & 1 & p\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp,\quad F_2 = -pdr - dq,\quad F_3 = dr,\quad F_4 = dx - zdw,$
$F_5 = dy - rdw,\quad F_6 = dz - pdw,\quad F_7 = dw$

Vector field representation is given by:

$E_1 = -zD_x + rD_y - pD_z - D_w,\quad E_2 = D_p,\quad E_3 = D_z,\quad E_4 = -pD_q + D_r,$
$E_5 = D_x,\quad E_6 = -D_q,\quad E_7 = -D_y$

2. $(357B): [x_1,x_2]=x_3, [x_1,x_3]=x_5, [x_1,x_4]=x_7, [x_2,x_3]=x_6;$

A matrix Lie group is given by:

$$S=\begin{bmatrix}1 & 0 & 0 & p & 0 & -z+pw & 2q\\ 0 & 1 & 0 & w & 0 & 1/2w^2 & x\\ 0 & 0 & 1 & 0 & w & 0 & y\\ 0 & 0 & 0 & 1 & 0 & w & z\\ 0 & 0 & 0 & 0 & 1 & 0 & r\\ 0 & 0 & 0 & 0 & 0 & 1 & p\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

Left invariant differential forms are given by:

Citation: Ghanam R, Basim Mustafa B, Mustafa MT, Thompson G (2016) Matrix Representation for Seven-Dimensional Nilpotent Lie Algebras. J Phys Math 7: 155. doi:10.4172/2090-0902.1000155

$F_1 = dp, \quad F_2 = -zdp + 2dq + pdz - p^2dw, \quad F_3 = dr, \quad F_4 = dx - zdw,$
$F_5 = dy - rdw, \quad F_6 = dz - pdw, \quad F_7 = dw$

The vector field representation is given by:

$E_1 = zD_x + rD_y + pD_z + D_w, \quad E_2 = D_p + z/2D_q, \quad E_3 = p/2D_q - D_z, \quad E_4 = -D_r,$
$E_5 = D_x, \quad E_6 = D_q, \quad E_7 = D_y$

3. $(357C) : [x_1, x_2] = x_3, [x_1, x_3] = x_5, [x_1, x_4] = x_7, [x_2, x_3] = x_6, [x_2, x_4] = x_5;$

A matrix Lie group is given by:

$$S = \begin{bmatrix} 1 & 0 & 0 & p & 0 & -z + pw & 2q \\ 0 & 1 & 0 & w & p & 1/2w^2 & x \\ 0 & 0 & 1 & 0 & w & 0 & y \\ 0 & 0 & 0 & 1 & 0 & w & z \\ 0 & 0 & 0 & 0 & 1 & 0 & r \\ 0 & 0 & 0 & 0 & 0 & 1 & p \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Left invariant differential forms are given by:

$F_1 = dp, \quad F_2 = -zdp + 2dq + pdz - p^2dw, \quad F_3 = dr, \quad F_4 = -rdp + dx - zdw,$
$F_5 = dy - rdw, \quad F_6 = dz - pdw, \quad F_7 = dw$

Vector field representation is given by:

$E_1 = D_w + zD_x + rD_y + pD_z, \quad E_2 = D_p + ((1/2)z)D_q + rD_x, \quad E_3 = -D_z + ((1/2)p)D_q, \quad E_4 = -D_r,$
$E_5 = D_x, \quad E_6 = D_q, \quad E_7 = D_y$

Conclusion

The Explanation regarding finding linear representations for seven-dimensional real, indecomposable nilpotent Lie algebras is done.

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