Modeling Agricultural Drainage Hydraulic Nets

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Abstract

A review on mathematical models available in the literature to design and evaluate agricultural drainage hydraulic nets is presented, including open ditch and buried pipe alternatives, for steady state and no steady state soil water flows, homogeneous and stratified soil profiles and smooth and corrugated drainage pipes.

Effective drainage pipe radius effects on drainage performance is considered, based on its perforation density and distribution, as it affects pipe weight bearing strength and pipe deformation intensity. Also, quantitative considerations on perforation density upon water flow resistance into the drain pipe are analyzed.

Keywords: Agriculture; Hydraulic nets; Pipe; Soil profile

Introduction

Agricultural land drainage consists of a set of technical strategies and hydraulic structures allowing the removal of excessive water and/or salts present in the soil volume occupied by root crops, to provide an adequately oxygenated environment, suitable for root normal development, keeping adequate water and air relative proportions according to crop physiological needs, to enable soil sustainability for crop productive conditions [1-4].

There are two drainage systems for controlling underground waters: open ditches (Figure 1a) and subsurface perforated piping (Figure 1b). Open ditches systems consist of excavations in the soil that collect the water stored at existing phreatic layers; it also can be used to remove surface run-off; it can account for significant land farming losses, smaller soil units for farm machinery operation and interference with irrigation systems, making agricultural tasks more expensive [5-7].

Subsurface pipe drainage systems consists on plastic tubes, either smooth or corrugated, with perforations, placed at specified distances and depths, buried within the soil; this system is used mainly to lower the water table in unconfined aquifers [8-10]. These drainage systems in most cases consist on a main drain pipe, a collector drain pipe and a network of field drains pipes (Figure 2); the position of the main drain depends on the field slope and the location of the lowest field level, through which the collected water is removed from the drained area. The collector drain and the network of field drains are usually located in parallel to each other; field drains are perforated pipes along their extension (Figure 3), and its function is the phreatic level control by receiving water excesses present in the soil profile and convey this effluent towards the collector drain. Secondary drains and the main drain main conduct water from the drain pipes to the site of water discharge. These conductive drains are either open ditch type or underground pipes, the selected option will depend on costs and dimensions of piping [8-13].

Subsurface drain design corresponds to a set of agronomic, hydraulic and engineering characteristics that a lateral drainage system must fulfill, to eliminate the excessive volume of soil water, enabling soil aeration values required to satisfy crop optimal growth and production [1-3,14,15]. In general terms, design features must define the proper criteria and parameters relevant to spacing among lateral drains, its depth placement inside the soil profile and the hydraulic characteristics of the hydraulic net, required to transport the volume of water to be collected and remove it from the cultivated area. In relation to construction aspects, drainage design must include definitions about drain hydraulic net layout, the materials to be used, the density and kind of perforations, as well as building techniques, network installation and maintenance.

Optimal distances between consecutive lateral drains are closely related to water flow towards the drains. The development of a mathematical model for quantitative description of the sub-surface
flow towards lateral drains is possible only based on mathematics simplifications, deduced from the theory of underground water saturated flow, with pre-established initial and border conditions.

According to Ernst [16], water flow to a subsurface drain (Figure 4) consists of:

a) Descendent vertical flow from the phreatic level down to the drain level,
b) Horizontal flow towards the area nearby the drain,
c) Radial flow towards the drain and,
d) Input water flow into the drain.

Each flux magnitude and direction occurring simultaneously, can be represented by means of vector components, using Darcy’s law, where: \( q \) – the difference of the corresponding hydraulic potentials multiplied by specific resistances [10]. Differences in hydraulic potentials for saturated flow correspond to water hydrostatic pressure differences between the soil and the drain system (Figure 4).

In steady state regimes, water flow total resistance is the sum of the vertical, horizontal, radial and inflow (entrance) resistance. These resistances can be measured by means of piezometers strategically set (Figure 5). A piezometer consists of a small diameter tube without perforations, provided with a short filter in its lower end. Water level in the piezometer represents the hydraulic head in the soil around the filter.

Four head losses can be identified in Figure 5 [10]:

- Vertical head loss \( (h_v) \) is the difference in water levels between piezometers 1 and 2, located at the midpoints (half distance) between two consecutive drains, with its filters situated in the proximity of the phreatic level and at the depth where the drain is installed, respectively.
- Horizontal head loss \( (h_h) \), mainly due to the horizontal flow towards the drain, corresponds to the difference in the water level between piezometers 2 and 3; with the filters situated at the level of the drain, one is at the midpoint (half distance) between two drains, and the other is in the close proximity to the drain.
- The radial head loss \( (h_r) \) is given by the difference in the water levels between piezometers 3 and 4, with the filters located at the depth where the drain is installed: one besides the drain and the other at some specific distance.
- The inflow head loss \( (h_e) \) is the difference in water levels between piezometer 4 and a piezometer situated over the drain.

Total head loss \( (h_t) \) is the sum of all those differences, as indicated in Figure 6. Head losses are measurements of the resistances to the corresponding flows; the relation between head loss and the corresponding resistance is:

\[
    h_i = q \cdot L \cdot W_i
\]

where

\[
    i = 1, 2, 3, 4
\]

\[
    L/2 = \text{average distance between two consecutive drainage pipes.}
\]

\[
    r_0 = \text{tile effective radius}
\]

\[
    K_1 \text{ and } K_2 = \text{Saturated hydraulic conductivity for 2 soil layers}
\]

\[
    r_p = \text{perforations}
\]

\[
    b_1, b_2, b_3, b_4 = \text{specific distances}
\]

\[
    h_i = \text{head losses}
\]

\[
    q = \text{flux magnitude and direction}
\]

\[
    L = \text{average distance between two consecutive drainage pipes.}
\]

\[
    W_i = \text{corresponding resistance}
\]


\[ h = \text{head loss (m)}, \quad L = \text{distance between drains (m)}, \quad q = \text{specific flow (m}^2\text{d}^{-1}) \]

\[ W = \text{Resistance (d/m}^{-1}) \]

\[ v = \text{subscripts; } v (\text{vertical}), \quad r (\text{radial}), \quad e (\text{entrance}), \quad t (\text{total}) \]

Total head loss is:

\[ h_t = h_v + h_r + h_e + h_t \tag{2} \]

Sometimes, in mathematical models describing saturated water flow towards drains, resistances \( W \) are substituted by dimensionless coefficients \( \alpha \), which are independent of soil hydraulic conductivity:

\[ \alpha = K^*W \quad \text{or} \quad W = \alpha / K \tag{3} \]

where:

\[ K^* = \text{saturated hydraulic conductivity (m}^2\text{d}^{-1}), \quad \alpha = \text{dimensionless geometric factor} \]

Therefore, total head can be expressed as:

\[ H_t = q L (W_v + W_r + W_e) = q L \left( \frac{\alpha_v}{K_v^*} + \frac{\alpha_r}{K_r^*} + \frac{\alpha_e}{K_e^*} + \frac{\alpha_t}{K_t^*} \right) \tag{4} \]

Mathematical models describing drainage systems are used for the calculation of drain spacing; these models are based in a set of assumptions related to drain hydraulic characteristics and to soil physical properties. One of these assumptions is that the drain is an ideal drain pipe, without inflow or entrance resistance, and flow is considered as an equipotential line (Figure 6). In these models it is assumed that the environment of the drain (surrounding materials and the soil altered by the trench excavated for perforated pipe installation) present a saturated hydraulic conductivity (\( K^* \)) much greater than the \( K \) of the natural unaltered soil, thus disregarding inflow resistance for the envelope material. However, practical experiences have shown that this condition is not always an adequate assumption [7,9].

Actual drains, being only permeable through its perforations, can be considered as continuously permeable drains, with an effective radius of drainage that is significantly lower than its physical radius; this fact is due to pipe mechanical resistance losses, due to a specific perforation density (distance between consecutive perforations). As the effective radius directly depends of the inflow resistance, it can be taken as an alternative to the entrance resistance; the smaller the inflow resistance, the larger will be the effective radius. Therefore, it is necessary to take into account the inflow (entrance) resistances in the equations allowing to define the optimal spacing between consecutive drain-pipes, and also to introduce the concept of effective radius in the outflow calculations, instead of using the physical radius of the lateral drain. If the physical radius is taken into account, calculated spacing between consecutive drains is larger than the spacing needed to optimize water outflow. Also, if the drain pipe physical radius is considered in these models, the actual phreatic layer depth after drainage is higher that the model results; under these conditions, optimal drain pipe layout criteria will not represent an optimal water extraction hydraulic drain net performance [8,9].

The effective radius not only depends both on the physical radius of the drain, as well as on its perforation density (Figure 7), but also, it is dependent on the inflow resistance; being this resistance smaller for larger drainpipe effective radius. Evaluation of the drainage effective radius is not only useful for the determination of the spacing between consecutive drain pipes, but it can be used to compare different materials’ drainage efficiency [8,10,17]. Both from theoretical and experimental points of view, research on inflow resistances to drains is needed, since it can significantly affect calculations of the optimal distances between consecutive drains.

\[ \alpha = \frac{1}{\pi m} \left( \frac{1}{\delta_p} - \frac{1}{2 \lambda} \left( 3.91 - 2 \frac{v}{\lambda} \right) \right) \tag{7} \]

Inflow resistance for a flat border (\( \alpha_{ea} \)) is:

Drain pipe external wall shape can be smooth or corrugated, affecting water inflow resistance. Similarly, soil particle sedimentation around the drain pipe perforations has highly significant effects on inflow resistance; if the corrugations are full with soil particles, the geometric limit between the soil and the perforation is important for the determination of the effective radius; also, if the corrugations are kept free of soil sediments, the limit of the interface soil-drain tile, when no filtering material is used as an envelope, significantly reduces inflow resistance. Corrugations shapes, (waves or blocks), have only a minor influence over water inflow resistance. For certain shapes and distribution of the perforations on a smooth wall drain pipe, inflow resistance may be determined for curved or flat borders. Dierickx [18] has made an extensive review on these analytic solutions and experimentally tested its accuracy.

Drains with circular perforations

a) Inflow resistance for a curved border (\( \alpha_{ae} \)):

\[ \alpha_{ae} = \frac{1}{\pi m} \left[ \sum_{n=1}^{\infty} \left( \frac{\alpha_n \delta_p}{\lambda_p} \right) + 2 \sum_{n=1}^{\infty} \sum_{i=1}^{N_p} \left( \frac{4 \alpha_n \delta_p}{\lambda_p} \sin \frac{\theta_i}{2} \right) + \ln \frac{2R_o}{\delta_p} \right] \tag{5} \]

b) Inflow resistance for a flat border (\( \alpha_{af} \)):

\[ \alpha_{af} = \frac{1}{\pi m} \left[ \sum_{n=1}^{\infty} \left( \frac{\alpha_n \delta_p}{\lambda_p} \right) + 2 \sum_{n=1}^{\infty} \sum_{i=1}^{N_p} \left( \frac{4 \alpha_n \delta_p}{\lambda_p} \sin \frac{\theta_i}{2} \right) + b \frac{2R_o}{\delta_p} \right] \tag{6} \]

Where:

\[ N = \text{number of perforations rows}, \quad R_o = \text{external radius of drainpipe}, \quad K = \text{Bessel function of the second kind, of zero order, } \delta_p \text{ permeation diameter, } \lambda_p \text{ perforation spacing between rows; } n \text{ and } i \text{ integer numbers, } \theta_i \text{ angle relative to the } i^{th} \text{ row, measured from the baseline (ordinate) explained in Figure 8}. \]

The Bessel function is based on solutions to Laplace’s and Helmholtz equations, by using the variable separation method in cylindrical or spherical coordinates [19].

Circular perforations

Engelund [20] considered water flow to drains with circular perforations distributed in a rectangular pattern, located over a flat surface (Figure 8b). Inflow resistance for a curved border (\( \alpha_{ae} \)) in a cylindrical surface, having the same perforations pattern, can be described by:

\[ \alpha_{ae} = \frac{1}{\pi m} \left[ \frac{1}{\delta_p} - \frac{1}{2 \lambda} \left( 3.91 - 2 \frac{v}{\lambda} \right) \right] \tag{7} \]
\[
\alpha_{ep} = \frac{1}{2\delta_p} \left( 1 - \frac{1}{2\lambda_p} \left( 3.91 - 2\ln \left( \frac{\lambda_2}{\lambda_1} \right) \right) \right)
\]

Where:

- \(m\) = number of perforations per drain unit length,
- \(\lambda_1\) and \(\lambda_2\) correspond to the spacing between the smaller and larger perforations, respectively. These equations are valid when \(\delta_p << 2\lambda_1\).

For square perforations, \(\lambda_1\) and \(\lambda_2\) in equations 7 and 8 are converted to

\[
\alpha_{ea} = \frac{1}{2\lambda_p} \left( 1 - \frac{1}{2\lambda_p} \left( 3.91 - 2\ln \left( \frac{\lambda_2}{\lambda_1} \right) \right) \right)
\]

and

\[
\alpha_{ep} = \frac{1}{2\lambda_p} \left( 1 - \frac{1}{2\lambda_p} \left( 3.91 - 2\ln \left( \frac{\lambda_2}{\lambda_1} \right) \right) \right)
\]

where \(\alpha_{ea}\) and \(\alpha_{ep}\) are the inflow resistances for a curved and a flat border, respectively.

For square orifices, perforation spacing \(\lambda_p\) can be described as

\[
\lambda_p = \left( \frac{2\pi R_o}{m} \right)^{1/2}
\]

Drains with discontinuous grooving

Inflow resistances for a curved border (\(\alpha_{ea}\)) and for a flat border (\(\alpha_{ep}\)) are, respectively:

\[
\chi_{ea} = \frac{c}{2\pi^2 R_o} \left( \ln \frac{2c}{\lambda_p \beta} - \frac{\pi \lambda_p}{2 \beta} + \frac{4\beta}{\beta} + 0.577 \right)
\]

and

\[
\chi_{ep} = \frac{c}{2\pi^2 R_o} \left( \ln \frac{2c}{\lambda_p \beta} - \frac{\pi \lambda_p}{2 \beta} + \frac{8\beta}{\beta} + 0.577 \right)
\]

where:

- \(C\) = distance between rows,
- \(R_o\) = external radius of drainpipe,
- \(\beta\) = the slit width,
- \(\lambda_p\) = spacing between perforations over the drain circumference. Explained in Figure 9 [18].

Corrugated drains

Equations developed to calculate inflow resistances for smooth drain pipes, can be applied only to corrugated drains having orifices on the top of the corrugations; however, corrugated drain pipes usually have its perforations on the corrugations valleys. For corrugated drains with a square wave profile, with an external major radius \(R_o\) and with its external minor radius \(R'_{o}\), provided with a circular opening \(\beta_v\) with the same width that the valley \(\beta_v\), (Figure 10), the inflow resistance for the flat border (\(\alpha_{ep}\)) condition is: [18]

\[
\chi_{ep} = \frac{c}{R_o \pi^2} \left( \ln \frac{2c}{\lambda_p \beta} - \frac{c}{4\pi R_o} \right) + \frac{1}{2\pi R_o} \left( \ln \frac{2\pi \beta}{2\beta} - \frac{1}{\pi} \right)
\]

For circular diameters smaller than the valley widths, inflow resistances result from the convergent flow lines towards drain perforations (Figure 11) [18].

The inflow resistance for a flat border (\(\alpha_{ep}\)) is:

\[
\chi_{ep} = \frac{c}{R_o \pi^2} \left( \ln \frac{2c}{\lambda_p \beta} - \frac{c}{4\pi R_o} \right) + \frac{c}{4\pi R_o} \ln \frac{2\pi \beta}{\sin^2 \frac{2\pi \beta}{2\beta}} - \frac{1}{2\pi R_o}
\]

Research performed on saturated water flow towards drain pipes, based on mathematical models, have demonstrated that for circular orifices, the inflow resistance depends mainly on the distance between the orifices, as well as to the outer pipe diameter [18]. Efficient inflow resistance reduction into drainage pipes with circular orifices, can be achieved by increasing the number and diameter of perforations per drain pipe length unit, as compared to increments on discontinuous slot length.

Models for Agricultural Drainage

Darcy H and Dupuit J were the pioneer researchers formulating the basic equations for subsurface water flow across saturated porous
media and applying these models to describe water flow towards wells employed these same equations describing subsurface water flow to drains, thus describing the first drainage formulae reported [21-23]. Hooghoudt [24], was one of the first researchers on develop a rational analysis for the drainage problem, studying it in the context of the water - soil - plant system. Since then, scientists from all over the world, such as [25] from England, [26-28] in the United States, [29,30] in Holland, have contributed to the improvement of the rational analysis method first proposed by Hooghoudt [24], in 1940. These models are used for quantitative design of drainage systems, taking into consideration the correlation among some design characteristics (spacing and depth) with certain crops features, as well as to precipitation intensity and to soil saturated hydraulic conductivity (Ks) for each strata present in the soil profile within the area to be drained. Drainable pore space, the optimum depth of the phreatic layer with respect to the effective depth of crop roots, phreatic layer rate of descent and the inflow resulting from either rainfall, irrigation or another water origin [1,31,32], are also relevant parameters to be considered in drainage network designs. Equations describing soil drainage can be defined into two major categories: steady state and non- steady state water flow regimes. **Steady state regime** Equations describing situations of steady state flow regime assume that both the water recharge over an area, and the output of water through the drainage system are constants, meanwhile the level of phreatic layer stays in a steady state condition, thus it neither ascends nor descends. This condition properly describes the situation in wet zones, where rainfall is almost constant during a long period of time and its intensity fluctuations are not significant [32,33]. For the calculation of spacing between consecutive drains under a steady state condition, it is necessary to define (Figure 12):

- The saturated hydraulic conductivity of the different soil profile strata (Ks) [m/d];
- The thickness of the flow region (over and underneath of drains);
- The phreatic layer distance from the surface, at the midpoint (half distance) between two consecutive lateral drains (Pe) [m];
- The depths of drains in the soil profile (Pd) [m];
- The hydraulic head (Ah) [m];
- The depth from drain basis down to the impermeable stratus (D) [m]and
  - The recharge (R) [m].

Equations developed to validate drainage design for this type of steady state regime have been published [16,24,26,34,35].

**Non-steady state regime** It assumes that the water recharge (R) over an area and the discharge of water (Q) by the drainage systems are not constants; for a condition characterized by a discharge smaller than the recharge, a phreatic level raise is produced during the recharge; afterwards, the phreatic level starts descending and subsequently it begins to increase again, when the following event of irrigation or rain starts. This non-steady state condition is found in zones with periodical irrigation or high intensities of rainfall followed by significant dry spells [12,13,35-37]. Equations describing this non-steady sate condition assume that soil hydraulic characteristics are homogeneous throughout the soil profile and that the depth from the surface down to the phreatic layer is such that the thickness of the flow region may be considered constant. Since these conditions are fulfilled on rare occasions in Nature and also, soil parameters like hydraulic conductivity, aquifer thickness and drainable porosity are difficult to measure with certainty, the drain spacing calculated with this kind of equations must be contrasted with spacing calculated using other procedures; such as the Hooghoudt equation [24] for a steady state regime, before making a definitive decision about the optimum burying depth and spacing between parallel drain pipes.

To calculate the optimal spacing between drains, under a condition of non-steady state regime, it is necessary to define the variables indicated in Figure 13 [35].

- The unsaturated hydraulic conductivity (K=5(soil water content)) [m/d];
- The drainable porosity (µ) [%];
- The time (t) that water needs for descending from an initial position (h,) down to a final position (h,);
- The instantaneous recharge rate (Ri) [m];
- The drain depth (Pd) [m];
- The effective depth (Pe) for crop root development [m]

Equations describing non-steady state regimes have been published [38-41].
Simulation Models in Agricultural Drainage

Since the beginning of digital computing technology, mathematical models have found wide application to different fields in applied natural sciences [42]; a mathematical model is defined as a set of equations and computing programs that can be used to quantify the performance of a natural system, in relation to specific functions [43]. For agricultural drainage, several mathematical models have been proposed, for different applications in basic research and applied engineering (Table 1). Most models are relatively simple and usually consider a single flow process or function, using finite element and finite differences analysis techniques [44], which enable calculating the main parameters needed for drainage design, (optimal depth and spacing between consecutive drains) (Table 1) [45].

Most simulation model results have been evaluated under field conditions, and applied to describe and characterize the phreatic layer behavior, and the agricultural drainage effects on agricultural production [46-48]. Other agricultural drainage models have been developed to predict soil salinity buildup, fertilizer leaching, and sediment and contaminant distributions in the soil drained profile [47-53]. Most commonly used drainage models are:

**Swap**

It is developed based on the agro-hydrological models SWATRE and SWACROP, and some of its later derivations, as SWASALT, for soil salt transportation and FLOCR, developed for the study of contraction and expansion in clay soils [54]. Swap integrates water flow, solutes transportation, and crop growth and development. It includes Richardson numerical solution methods [55], solutes incorporation and the heat transport, soil heterogeneity and the contraction and expansion of clay soils.

**Drain mod**

It is a field model based on poorly drained soil hydrology and the corresponding artificial drainage. The model accounts for water balances at the soil surface and within the soil profile, enabling to calculate drainage rates as a function of soil hydraulic properties, the phreatic layer water level and the design of the drainage network [56].

**Sahys mod**

It is a mathematical model used for simulating and predicting the increment in soil salinity, as related to soil moisture variations, underground water flow, phreatic layer depth and the drainage network discharge, under conditions of agricultural irrigation in soils with different hydro-geological conditions [57].

**Espadren**

It allows the calculation of drains spacing for a steady state regime,
by using a specific set of equations [24,26,29,34]; it accounts also for a non-steady state regimes, by using [37,39] equations, either for open drains or for subsurface drainage pipes. It includes routines valid for homogeneous soil profiles as well as for soils having two layers (strata) with different Ks values [58].

The increasing development and applicability of these simulation models oriented to agricultural applications, has become an important tool for research on the quantification of crop productivity and the impacts of deficient drainage over the soil – root environment. However, most model results do not fully satisfy defining specific technical needs of agricultural drainage: also, accurate measurements of its input variables are difficult and costly and do not take into account soil hydraulic properties in terms of its geo – spatial variability. Therefore, for each specific drainage problem, it is necessary to consider specifics criteria to select and validate one or more of these simulation models.

**Conclusion**

The normative and protocols established for agricultural land drainage in countries having expertise in the subject, have not been validated for the specific conditions of soils and situations of deficient drainage existing in local agricultural conditions in different countries. Thus, no international standards for drainage network design are available. Specifications for drainage pipe resistances are also seldom available. For example, for plastic drains, standard norms specify only the use of limited proportions of recycled plastic raw materials allowances. Additionally, physical and mechanical dimensions for drainage nets, like internal and external diameter, perforation size, location and density has not been universally defined.

For each specific drainage problem, it is necessary to consider specifics criteria to select and validate one or more of these simulation models.

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