Modeling and Output Feedback Distributed Control for an Absorption Packed Column

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Abstract

This work consists of modeling, simulating and finally multiple models control of an industrial absorption packed column designated to remove CO₂ from natural gas. The multiple models approach is an elegant way of turning nonlinear problems into linear ones. We used in this paper the output feedback distributed control (ODC) coupled with local linearization of the model of the absorption packed column. We compared the results with those obtained with the traditional PID control and the results are satisfactory.

Keywords: Multiple models; PDC control; Methyl diethanolamine (MDEA) Absorption packed column; LMI; Lyapunov function.

Introduction

The absorption packed column is a physicochemical separation unit largely used in the chemistry industry. It consists of a tube where we send gas mixtures in order to separate one or more compounds from the principal mixture. It is largely used for the separation of acid gases (CO₂, H₂S) from natural gas.

The model presented in this paper is a dynamic model of the absorption packed column and consists of a set of nonlinear partial differential equations; it is elaborated starting from considerations on CO₂ sorption packed column and consists of a set of nonlinear partial differential equations. We send gas mixtures in order to separate one or more compounds from the principal mixture. It is largely used for the separation of acid gases (CO₂, H₂S) from natural gas.


Few studies have also been published concerning the modelling and simulation of CO₂ absorption by aqueous solutions of MEA or MDEA on pilot and industrial columns [10-12].

For the model developed in our study, it seemed interesting for us to use the multiple models approach for the command of the absorption packed column because it enables us to obtain good performances for complex dynamics processes. We develop in first stage the PID regulation to compare the performances of the classical techniques with the performances of the Takagi-Sugeno multiple model approach.

Modeling and Open Loop Simulation of the Industrial Absorption Packed Column

The absorption packed column presented here is located at Khrechba and is part of the In Salah Gaz project. It removes CO₂ from natural gas by using an aqueous solution of methyl diethanolamine (MDEA) as a washing liquid. It is a packed type column measuring 8 meter height and 4 meter in diameter with Pall rings to improve the surface of contact between phases. For a better elimination of CO₂ from the natural gas, the liquid flow (water + MDEA) is counter-current with gas flow. The working pressure and temperature is respectively 71.5 bar and 55°C [2].

At contact between liquid and gas phase occurs on the surface of the Pall rings, CO₂ passes from the gas phase to the liquid phase; this diffusion is accelerated by chemical reaction of CO₂ with the MDEA in the liquid phase. The liquid flow (water + MDEA) and the CO₂ concentration in the gas mixture are respectively selected as control variable and output variable (Figure 1).

Figure 1: The absorption packed column of In Salah Gas (ISG).
Model equations

In order to simplify the model, the following assumptions are done [2,13]:
- There is no resistance in gas phase
- The reaction between CO\textsubscript{2} and MDEA is fast (H\textsubscript{a}>5)
- Axial dispersion is negligible in the gas phase and the liquid phase
- The MDEA does not pass in gas phase

The mass balance on an elementary section dz of the column for CO\textsubscript{2} in the gas phase is written [2,13,14]:

\[ \frac{d(CG)}{dz} = \frac{G}{S} \left( \frac{dCG}{dt} + \phi \right) \]

Where \( G \) (m/s) is the volumic gas flow, \( \phi \) the CO\textsubscript{2} flow transferred from the gas phase to the liquid phase, \( S \) the section of the column and \( CG \) (mol/m\textsuperscript{3}) the CO\textsubscript{2} concentration in the gas phase. Given \( U_G=\frac{G}{S} \) the gas velocity, we obtain then:

\[ \frac{dCG}{dt} + \frac{dCG}{dz} + \phi = 0 \]  

The chemical reaction between CO\textsubscript{2} and the MDEA is [10-12]:

\[ \text{CO}_2 + \text{R}_1\text{R}_2\text{NCH}_3 + \text{H}_2\text{O} \rightarrow \text{R}_1\text{R}_2\text{NCH}_4 + \text{HCO}_3^- \]  

The reaction rate \( r_A \) has the following form [14,15]:

\[ r_A = k C_{Al} C_{Bl} \]  

Where \( k \) is the constant for reaction rate [14,15]:

\[ k = 2.96 \times 10^5 \exp(-\frac{5332 R}{T}) \]  

\( C_{Al} \) is the CO\textsubscript{2} concentration in the liquid phase and \( C_{Bl} \) the MEA concentration in the liquid phase. The mass balance for CO\textsubscript{2} in the liquid phase gives finally:

\[ \phi = [k C_{Al} C_{Bl}] \]  

Which means that the totality of CO\textsubscript{2} transferred to the liquid phase reacts with the MDEA.

The mass balance for the MDEA in the liquid phase gives:

\[ (LC_{Bl}) = (LC_{Bl})_{z=0} - [k C_{Al} C_{Bl}] S dz - \frac{dC_{Bl}}{dt} dz \]  

Where \( L \) is the volume liquid flow. By taking account of (5) and noting by \( U_L=\frac{L}{S} \) (m/s) the mean liquid flow velocity, we obtain:

\[ U_L \frac{dC_{Bl}}{dz} - \phi = -\frac{dC_{Bl}}{dt} \]  

Our absorption packed column is finally described by the following set of partial derivative equations:

\[ \begin{cases} 
\frac{dCG}{dt} + \phi = -\frac{dCG}{dz} \\
\frac{dC_{Bl}}{dt} - \phi = -\frac{dC_{Bl}}{dz} 
\end{cases} \]  

The procedure to compute flow \( \phi \) is given in [2] according to [14-16].

We have finally to consider the boundary conditions which for gas phase are the CO\textsubscript{2} concentration at the column bottom or input concentration \( C_{At} \), and for liquid phase the MEA concentration at the column top or input concentration \( C_{Bl} \):

\[ \begin{cases} 
C_{At} = C_{At}^0 \\
C_{Bl} = C_{Bl}^0 
\end{cases} \]

Chemical reactions within the industrial column induces a strong heat emission and the appearance of a temperature gradient throughout the column; the temperature variation is approximately 5°C between the input and the output of the column, which leads us to establish an energy balance in order to describe the temperature changes which affects the various concentrations along the column [16]:

\[ \begin{cases} 
U_G \frac{\partial T}{\partial z} + a h_f (T_l - T_g) = \frac{\partial T_l}{\partial t} \\
\sum \Delta H_r = \frac{\partial T_g}{\partial t} 
\end{cases} \]

With:

\( C_g \): concentration in gas phase at the interface (mol/m\textsuperscript{3})

\( C_l \): concentration in liquid phase at the interface (mol/m\textsuperscript{3})

\( h_f \): coefficient of heat transfer (convection) (J/m\textsuperscript{2}.K.s)

\( T_l \): Liquid temperature (K)

\( T_g \): Gas temperature (K)

\( \Delta H_r \): enthalpy of the reaction (J/mol)

\( c_p \): specific heat in the liquid phase at the interface (J/mol.K)

We finally takes into account the boundary conditions for the temperature which are the temperatures for gas and the liquid at the column input.

\[ \begin{cases} 
T_l = T_{l_0} = T_{l_0}^0 & \frac{\partial T_l}{\partial z} \big|_{z=0} = 0 \\
T_g = T_{g_0} = T_{g_0}^0 & \frac{\partial T_g}{\partial z} \big|_{z=0} = 0 
\end{cases} \]

Model validation

A test was carried out on our industrial absorption column to compare the output CO\textsubscript{2} concentration given by the model with the real one and this for a step input variation of 10 t/h. The data were collected on a horizon of 6800 seconds. The results are grouped in Figure 2 where we represent respectively, the flows of MDEA and gas at the output and then the concentrations of CO\textsubscript{2} at the column output either experimental or given by the model [2]. We note that the model dynamics of the CO\textsubscript{2} concentration at the column output agree with the experimental results.

Open loop Simulation of the industrial column

By considering the equations (2) and (8), the dynamic model of the absorption column is that of a nonlinear, distributed parameters system. The results from open loop simulations are presented in Figures 3 and 4.

Simulations show that the system is stable. It presents a dead time
PID Regulation of the Industrial Absorption Packed Column

We apply a PID regulation to the dynamic model of our absorption packed column. We chose a sampled control with a sampling period of 10 seconds. The reference for the input CO₂ concentration is 0.25 mole%, which corresponds to a concentration of 7.05 mole CO₂/m³.

The parameters of the PID regulator were optimized using trial and error.

The simulation results are satisfactory; the PID regulator cancels the permanent error and ensures a quick response due to the derivative action. The regulation shows a net asymmetrical behaviour between responses to positive and negative step input disturbances due to the strong non linearity of the relationship between the input and the output.

Multiple Models Control of the Industrial Absorption Packed Column

Introduction

The multiple models are a nonlinear modeling technique which allows achieving a good compromise between precision and model complexity. In the light of the numerous work related to it in recent years [17-20], it arises a great interest, especially in applications dealing with simulation and control. It can also be seen as a particular fuzzy modeling technique [21,22], corresponding to a Takagi Sugeno (TS) approach [23]. A TS model is a composed of a finite number of linear models in response to a step input disturbance on the CO₂ concentration due to the gas propagation along the absorption column.

Figure 2: Output CO₂ Concentrations from the system and the model.

Figure 3: Output CO₂ Concentration for an input step disturbance of ± 5% on gas concentration.

Figure 4: Output CO₂ Concentration for an input step disturbance of ± 10% on gas flow.
connected with nonlinear functions called membership functions and verifying the convex mapping property (i.e., they are non-negative and their sum is equal to 1). It allows us to solve various problems of control, observation and diagnosis for nonlinear systems with linear techniques.

The approach associated with multiple models in control is known as the Parallel Distributed Compensation (PDC) [24]. This method is based on a set of linear controllers designed for each linear model, and stability of the overall closed loop is guaranteed via a Lyapunov function common to all the linear models.

In this paper, we identify the absorption packed column as a multiple model of the TS type and proposes a control based on State feedback Distributed Control (PDC) (Figures 5 and 6).

Problem formulation

The Multiple model approach: The multiple models have three basic structures: Coupled states (TS), uncoupled states [25] and hierarchical structure. The coupled states Structure (TS) is the most popular in the analysis and synthesis of the multiple models. It is written in the following form:

$$\begin{align*}
    \dot{x}(t) &= \sum_{i=1}^{M} \mu_i(z(t))(A_i x(t) + B_i u(t)) \\
y(t) &= \sum_{i=1}^{M} \mu_i(z(t)) C_i x(t)
\end{align*}$$

(13)

Figure 5: Output CO₂ Concentration and washing liquid flow for an input step disturbance of ± 5% on gas concentration.

Figure 6: Output CO₂ Concentration and washing liquid flow for an input step disturbance of ± 10% on gas flow

The activation function or membership function $\mu_i(z(t))$ determines the degree of activation of the $i$th local model. It allows a progressive passage from this model to the other close local models. These functions can depend of the measurable variables of the system (the input and output signals) or of the non-measurable variables of the system (the states). They can be of triangular or Gaussian form and satisfy the properties of convex mapping:

$$\begin{align*}
    \sum_{i=1}^{M} \mu_i(z(t)) &= 1, \forall t \\
    0 \leq \mu_i(z(t)) \leq 1, \forall i = 1, \ldots, M, \forall t
\end{align*}$$

(14)

The multiple models can be viewed as universal approximations since any nonlinear system can be approximated by a multiple models representation with sufficient accuracy and this simply by increasing the number of sub-models. In practice, a reduced number of sub-models can be sufficient to obtain a satisfactory approximation, and we can use the tools of linear systems analysis to achieve this goal.

There are three approaches largely used in the literature allowing us to obtain a TS model: Identification, transformation by nonlinear sectors [26], or linearization. This last one is used in this work (Figure 7).

Using convex analysis for regulator synthesis, the multiple models allows us to obtain control laws by the simultaneous resolution of a finite number of Linear Matrix Inequalities (LMI). In this case, the number of LMI inequalities is polynomial with respect to the number

Figure 7: PDC control for TS multiple models.
of local models. Thus, it is advisable to minimize the number of local models to limit the conservatism of the method.

In the case of TS multiple models, this technique of regulator synthesis corresponds to the PDC method. It supposes that all the linear sub-models are at least stabilizable. Subsequently, they will also be supposed commendable.

Given the TS model given by equation (13), a control law resulting from PDC synthesis will thus be the combination of linear control laws for each sub-model, given by:

\[ u(t) = - \sum_{i=1}^{M} \mu_i(z(t)) F_i x(t) \]  

(15)

Applying this control law to the TS multiple model, we obtain in closed loop:

\[
\begin{align*}
\dot{x}(t) &= (A_i - B_i F_i) x(t) \\
y(t) &= C_i x(t)
\end{align*}
\]

(16)

Or, in a more explicit way:

\[
\dot{x}(t) = \sum_{i=1}^{M} \sum_{j=1}^{M} \mu_i(z(t)) \mu_j(z(t)) (A_i - B_i F_j) x(t)
\]

(17)

The stability conditions for the closed loop system amounts to control a gain \( F \) such that the derivative of the candidate Lyapunov function associated with the system is negative. Stabilizing the system thus amounts to solve the following problem:

Find a positive definite matrix \( P \) and \( F \) matrices, \( i=1, \ldots, M \) such that:

\[
(A_i - B_i F_i)^T P + P (A_i - B_i F_i) < 0
\]

(18)

We notice that this inequality is nonlinear with respect to \( P \) and \( F \). Therefore, we have to solve the following problem:

\[
\sum_{i=1}^{M} \sum_{j=1}^{M} \mu_i(z(t)) \mu_j(z(t)) (A_i - B_i F_j) x(t) < 0
\]

(22)

And we can express the following result:

**Theorem 4.1** [24] Given a continuous TS model, the PDC control law (14) and the \( Y_i \), if it exists a positive definite matrix \( X \) and \( M_i \) matrices, such that (21) is satisfied for all \( i=1, \ldots, M \), then the closed loop is overall asymptotically stable. Moreover, if the problem has a solution, the gains of the PDC control are given by:

\[
F_i = XM_i^{-1}
\]

(24)

And the PDC control is:

\[
u(t) = - \sum_{i=1}^{M} \mu_i(z(t)) F_i x(t)\]

(25)

The closed loop multiple systems model of (16) is rewritten without the crossing terms \( B_i F_j \):

\[
\dot{x}(t) = \sum_{i=1}^{M} \mu_i(z(t)) (A_i - B_i F_i) x(t)
\]

(27)

The stability conditions of theorem 4.1 reduce then to the stability of the dominant models: \( P > 0 \),

\[
(A_i - B_i F_i) P + P (A_i - B_i F_i) < 0, \quad \forall (i=1, \ldots, M)
\]

(28)

Substituting \( B_i \) by \( B \), the control law leads to similar conditions.

**Multiple models identification**

The structural identification of a multiple models representation consists in the determination of the local models structures and the operation zones (or validity zones) for each local model [25]. The local models can be of various structures but in general we use simple structures, such as linear models.

The identification leads to a family of functions parameterized by the parameters vector \( \theta \) defining the structure of the \( i \)-th local model, and the parameters vector \( \beta \) characterizing the zone of validity of this local model. The parametric estimate consists in determining for each local model \( i \) the parameters vector: \( \theta = [\theta^T \beta^T]^T \)

The parametric estimation (also called training) is done on the basis of minimization of a functional binding the inputs and outputs system to the characteristics parameters of the model.

\[
J_{tr} = \frac{1}{2} \sum_{i=1}^{N} \sum_{t=0}^{T} (y_i(t) - \hat{y}(t))^2 = \frac{1}{2} \sum_{i=1}^{N} \sum_{t=0}^{T} \sum_{j=1}^{M_i} \omega_j \beta_j \phi_j(y(t), \theta_i, t, j)
\]

(29)

In order to simplify the model, we chose linear sub-models of ARX type (auto regressive with exogenous inputs) with 3 inputs (system MISO):

- Liquid Flow \( U_l \) [ton/h]
- Gaz flow \( U_g \) [ton/h]
- Input CO2 concentration \( C_{P_0} \) [mol%]

The chosen local models are second order ones, they are written in the following form:

\[
C_{P_0}(t+1) = A_{ii} C_{P_0}(t) + A_{i2} C_{P_0}(t-1) + B_1 U_1(t) + C_{ii} U_g(t) + D_1 C_{ge}(t) + P + e_i(t+1)
\]

(30)

With:

\( i=1, \ldots, M \) : local model indices

\( C_{P_0}(t+1) \) : output (concentration of CO2) of local model \( i \)

\( e_i(t+1) \) : Gaussian white noise

\( A_{ii}, A_{i2}, B_1, C_{ii}, D_1, P_i \) : parameters of local model \( i \)

We choose Gaussian membership functions with the decision variable \( \tilde{z}(t) = C_{P_0}(t) \) being the system output at time \( t \).

The expression of the membership function is:

\[
a_i(y, \sigma) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left( \frac{(y - a_i)^2}{2\sigma^2} \right)
\]

(31)

Where \( a_i \) is the mean. And \( \sigma \) the standard deviation for the Gaussian membership function.

The global identification diagram is shown in Figure 8.

**Training data** (Figure 9)
Validating data (Figure 10)

Implementation of the method and simulation results

The decision variable \( x(t) \) is in our case the state vector \( x(t) \) output gas concentration of the industrial column \( C_{gs} \),

\[ x(t) = \begin{bmatrix} C_{gs}(t) & C_{gs}(t-1) \end{bmatrix}^{T}. \]

A multiple models with four sub-models can easily be obtained in the form:

\[ \dot{x}(t) = \sum_{i=1}^{4} \mu_{i}(x(t)) (A_{i} - B_{i}F_{i}) x(t) \tag{31} \]

With the memberships functions \( \mu_{i}(x(t)) \) either triangular or Gaussian. The parameter of the four models is:

\[ A_{i} = \begin{bmatrix} -0.4436 & 0.5744 \\ -0.4772 & -0.2732 \end{bmatrix}, B_{i} = \begin{bmatrix} -1.8937 & 0.3093 \\ -1.2364 & -0.2006 \end{bmatrix}, \]

\[ C_{i} = C_{j} = C_{k} = C_{l} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{32} \]

And the State Feedback Distributed Control (PDC) is:

\[ U_{L}(t) = -\sum_{i=1}^{4} H_{i} (C_{gs}(t) F_{i} x(t)) + U_{in} \tag{33} \]

\( U_{L} \) is the liquid flow velocity, and \( U_{in} \) the mean flow velocity corresponding to the chosen operating point of the column.

The \( F_{i} \) feedback gains are determined by resolution of \( LMI \) in order to ensure good regulation performances, less than a 0.25% variation of \( CO_{2} \) around the operating points.

The selected gain is finally \( F_{i} = [980 985 995 1002] \) t/h

Closed loop simulation:

Case (a): triangular membership functions

The column simulation shows us the output \( CO_{2} \) concentration for step input disturbances on either the gas flow or the \( CO_{2} \) concentration. The evolutions of the output \( CO_{2} \) concentration and the corresponding control are on Figures 11-13

Case (b): Gaussian membership functions

The evolutions of the output \( CO_{2} \) concentration and the corresponding control are on Figures 14-16

Discussions

The oscillations of the output \( CO_{2} \) concentration for the industrial column are mainly due to fuzzy control, but the operating point is quickly reached in less than 50 seconds. We note on all the curves (5, 6 and 11-16) that disturbance is always rejected with both PID and PDC regulation. But comparison of the peak values as well as the oscillations show us that PDC control acts more quickly than PID control.

Conclusion

In this paper, a TS multiple models obtained by linearization was
used for the regulation of an industrial absorption packed column used for gas washing. We take a reduced number of sub models, four, to ease identification. The results obtained with state distributed feedback (PDC) are better than those by classical PID regulation, for a method which is not significantly complicated. Further investigations will be undertaken in the use of multiple models in control, multiple observers and multiple models in systems diagnosis.
back controller for a packed trickle bed column with carbon dioxide absorption”, in Joint Automatic Control Conference, USA.


17. CO2 absorber CO2 control. Data Sheet ISG. Ain Salah Gas.


