Modules Over Color Hom-Poisson Algebras

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Abstract
In this paper we introduce color Hom-Poisson algebras and show that every color Hom-associative algebra has a non-commutative Hom-Poisson algebra structure in which the Hom-Poisson bracket is the commutator bracket. Then we show that color Poisson algebras (respectively morphism of color Poisson algebras) turn to color Hom-Poisson algebras (respectively morphism of Color Hom-Poisson algebras) by twisting the color Poisson structure. Next we prove that modules over color Hom–associative algebras extend to modules over color Hom-Lie algebras L(A), where L(A) is the color Hom-Lie algebra associated to the color Hom-associative algebra A. Moreover, by twisting a color Hom-Poisson module structure map by a color Hom-Poisson algebra endomorphism, we get another one.

Keywords: Color hom-associative algebras; Color hom-Lie algebras; Homomorphism; Formal deformation; Hom-modules; Modules over color Hom-Lie algebras; Modules over color Hom-Poisson algebras

Introduction
Color Hom-Poisson algebras are generalizations of Hom-Poisson algebras introduced in [1], where they emerged naturally in the study of 1-parameter formal deformations of commutative Hom-associative algebras. Color Hom-Poisson algebras generalize, on the one hand, color Hom-associative [2,3] and color Hom-Lie algebras [2,3] which have been recently investigated by various authors. On the other hand, they generalize Hom-Lie superalgebras [4]. These structures are well-known to physicists and to mathematicians studying differential geometry and homotopy theory. The cohomology theory of Lie superalgebras [5] has been generalized to the cohomology of Hom-Lie superalgebras in [6]. A cohomology theory of color Lie algebras was introduced and investigated in [7], and the representations of color Lie algebras were explicitly described in [8]. Modules over Poisson algebras receive various definitions [9,10] and we will use these introduced in [9]. The aim of this paper is to study color Hom-Poisson algebras and modules over color Hom-Poisson algebras. The paper is organized as follows. In section 4, we recall some basic notions related to color Hom-associative algebras and color Hom-Lie algebras. In section 5, we define color Hom-Poisson algebras and point out that to any color Hom-associative algebra ones can associate a color Hom-Poisson algebra. Next, starting from a color Poisson algebra and Poisson algebra morphism we get another one by twisting the associative product and Lie bracket. In section 6, we introduce modules over color Hom-Poisson algebras and prove that starting from a color Hom-Poisson module we get another one by twisting the module structure map by a color Hom-Poisson algebra endomorphism. All vector spaces considered are supposed to be over fields of characteristics different from 2.

Preliminaries
Let G be an abelian group. A vector space V is said to be a G-graded if there exists a family \( \{V_a\}_{a \in G} \) of vector subspaces of V such that

\[ V = \bigoplus_{a \in G} V_a \]

An element \( x \in V \) is said to be homogeneous of degree \( a \in G \) if \( x \in V_a \). We denote H(V) the set of all homogeneous elements in V.

Let \( V = \bigoplus_{a \in G} V_a \) and \( V' = \bigoplus_{a \in G} V'_a \) be two G-graded vector spaces. A linear mapping \( f: V \rightarrow V' \) is said to be homogeneous of degree b if

\[ f(V_a) \subseteq V'_{a+b}, \forall a \in G \]

If \( f \) is homogeneous of degree zero i.e. \( f(V_a) \subseteq V'_{a} \) holds for any \( a \in G \) then \( f \) is said to be even.

An algebra \((A, \mu)\) is said to be G-graded if its underlying vector space is G-graded i.e. \( A = \bigoplus_{a \in G} A_a \) and if furthermore \( \mu(A_a, A_b) \subseteq A_{a+b} \) for all \( a, b \in G \).

Let \( A' \) be another G-graded algebra. A morphism \( f: A \rightarrow A' \) of G-graded algebras is by definition a graded algebra morphism from A to \( A' \) which is, in addition an even mapping.

Definition
Let G be an abelian group. A map \( \varepsilon: G \times G \rightarrow \mathbb{K}^* \) is called a skew-symmetric bicharacter on G if the following identities hold,

1. \( \varepsilon(a,b) = \varepsilon(b,a) = 1 \)
2. \( \varepsilon(a,b+c) = \varepsilon(a,b) \varepsilon(a,c) \)
3. \( \varepsilon(a+b,c) = \varepsilon(a,c) \varepsilon(b,c) \)

\( a, b, c \in G \)

Remark that \( \varepsilon(0,0) = 1 \) and \( \varepsilon(a,0) = \varepsilon(0,a) = \pm 1 \) for all \( a \in G \).

Where, 0 is the identity of G. If x and y are two homogeneous elements of degree a and b respectively and \( \varepsilon \) is a skew-symmetric bicharacter, then we shorten the notation by writing \( \varepsilon(x, y) \) instead of \( \varepsilon(a, b) \).

Definition
A color Hom-associative algebra is a quadruple \( (A, \mu, \varepsilon, \alpha) \) consisting of a G-graded vector space A, an even bilinear map \( \mu: A \times A \rightarrow A \) and an even linear map such \( \alpha: A \rightarrow A \) that

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Received August 12, 2013; Accepted September 30, 2014; Published October 06, 2014

Citation: Bakayoko I (2014) Modules Over Color Hom-Poisson Algebras. J Generalized Lie Theory Appl 8: 212. doi: 10.4172/1736-4337.1000212

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\[ \mu(a(x), a(y)) = a(\mu(x, y)) \]  
(1)

\[ \mu(a(x), \mu(y, z)) = \mu(\mu(x, y), a(z)) \]  
(2)

If in addition \( \mu(x, y) = \varepsilon(x, y) \mu(y, x) \) the color Hom-associative algebra \((A, \mu, \varepsilon, a)\) is said to be a \( \varepsilon \)-commutative color Hom-associative algebra.

**Remark**

When \( a = Id \) we recover the classical associative color algebra.

Recall that the Hom-associator, \( aS_{\alpha} \) of a Hom-algebra A is defined as:

\[ aS_{\alpha} = A \otimes A \otimes A \rightarrow A \]

Observe that \( aS_{\alpha} = 0 \) when A is a color-Hom-associative algebra.

**Definition**

Let \((A, \mu, \varepsilon, a)\) and \((A', \mu', \varepsilon', a')\) be two color Hom-associative algebras. An even linear map \( f: A \rightarrow A' \) is said to be a morphism of color Hom-associative algebras if

\[ f(\mu(x, y)) = \mu'(f(x), f(y)) \]

for all \( x, y \in A \).

**Lemma**

((17)) Let \((A, \mu, \varepsilon)\) be a color associative algebra and \(\alpha\) be an even algebra endomorphism. Then \((A, \mu, \alpha)\) is another color associative algebra and \(\alpha': A' \rightarrow A'\) be an even algebra endomorphism such that \( f \circ \alpha = \alpha' \circ f \) and

\[ f(\mu(x, y)) = \mu'(f(x), f(y)) \]

for all \( x, y \in A \).

**Theorem**

((17)) A color Hom-Lie algebra is a quadruple \((A, \{.,.\}, \varepsilon, \alpha)\) consisting of a \(G\)-graded vector space \(A\), an even bilinear map

\[ \{.,.\}: A \otimes A \rightarrow A \]

(i.e \(A_{a} \cdot A_{b} \subseteq A_{a+b}\) for all \( a, b \in G \)) a bicharacter, and an even linear map \(\alpha': A \rightarrow A'\) such that for any \( x, y \in H(A) \) we have

\[ \{x, y\} = -\varepsilon(x, y)\{y, x\} \]  
(3)

\[ \alpha' = \{.,.\} = \varepsilon \alpha \]  
(4)

\[ \varepsilon(\alpha(x), \{y, z\}) = 0 \]  
(5)

Where \( \varepsilon \) means cyclic summation.

By the \(\varepsilon\) skew symmetry 3 of the color Hom-Lie bracket \(\{.,.\}\), the color Hom-Jacobi identity 5 is equivalent to

\[ \{\{x, y\}, \alpha(z)\} = \{\{x, \alpha(y)\}, y, z\} - \varepsilon(x, y)\{\alpha(y), \{x, z\}\} \]  
(6)

Remark that a color Lie algebra \((A, \{.,.\}, \varepsilon)\) is a color Hom-Lie algebra with \(\alpha = Id\).

Morphism of color Hom-Lie algebras are defined similarly to the Definition 4.3, where the color Hom-associative product is replaced by the color Hom-Lie bracket. Examples of color Hom-Lie algebras are provided in [2,3].

The following lemma connects color Hom-associative algebras to color Hom-Lie algebras.

**Lemma**

((17)) Let \((A, \mu, \varepsilon, a)\) be a color Hom-associative algebra. Then \((A, \{.,.\} = \mu - \varepsilon(\mu)\mu', \varepsilon, a)\) is a color Hom-Lie algebra, denoted by \(L(A)\).

**Color Hom-Poisson algebras**

**Definition**

A color Hom-Poisson algebra consists of a \(G\)-graded vector space \(A\), a multiplication \(\mu: A \otimes A \rightarrow A\), an even bilinear bracket \(\{.,.\}: A \otimes A \rightarrow A\) and an even linear map \(\alpha: A \rightarrow A\) such that

1. \((A, \mu, \varepsilon, a)\) is a color Hom-associative algebra,
2. \((A, \{.,.\}, \varepsilon)\) is a color Hom-Lie algebra,
3. the color Hom-Leibniz identity is satisfied i.e.

\[ \{\alpha(x), \mu(y, z)\} = \mu([x, y], \alpha(z)) + \varepsilon(x, y)\mu(\alpha(y), \{x, z\}) \]

(7)

For \(x, y, z \in H(A)\) any

If in addition \(\mu\) is \(\varepsilon\) commutative, the color Hom-Poisson algebra \((A, \{.,.\}, \varepsilon, a)\) is said to be a \(\varepsilon\) commutative color Hom-Poisson algebra.

The condition 7 expresses the compatibility between the color Hom-associative product \(\mu\) and the color Hom-Lie bracket \(\{.,.\}\) it can be written equivalently

\[ \{\mu(x, y), \alpha(z)\} = \mu(\{x, \alpha(y)\}, \{y, z\}) + \varepsilon(x, y)\mu(\alpha(y), \{x, z\}) \]

(8)

**Remark**

We recover Poisson algebras ([6, 5]) when \(\alpha = Id\) and \(\varepsilon = 1\).

We need the following lemma in Proposition 6.1.

**Lemma**

If \((A, \mu, \{.,.\}, \varepsilon, a)\) is a \(\varepsilon\) commutative color Hom-Poisson algebra, then \((A, -\mu - \{.,.\}, \varepsilon, a)\) is also a \(\varepsilon\) commutative color Hom-Poisson algebra.

The following theorem is the color version of ([11], Proposition 4.6).

**Theorem**

Let \((A, \mu, \{.,.\}, \varepsilon, a)\) be a color Hom-associative algebra. Then \((A, \mu - \{.,.\}, \varepsilon, a)\) is a color Hom-Poisson algebra.

**Proof**

According to Lemma 4.2, it remains to prove the color Hom-Leibniz identity 7. For any \( x, y, z \in H(A)\)

\[ \{\alpha(x), \mu(y, z)\} - \mu([x, y], \alpha(z)) - \varepsilon(x, y)\mu(\alpha(y), \{x, z\}) = \]

\[ = \varepsilon(x, y)\mu(y, z) - \varepsilon(x, y)\mu(\alpha(y), \{x, z\}) \]

(9)

Remark that a color Lie algebra \((A, \{.,.\}, \varepsilon)\) is a color Hom-Lie algebra with \(\alpha = Id\).

Morphism of color Hom-Lie algebras are defined similarly to the Definition 4.3, where the color Hom-associative product is replaced by the color Hom-Lie bracket. Examples of color Hom-Lie algebras are provided in [2,3].

The following lemma connects color Hom-associative algebras to color Hom-Lie algebras.

**Lemma**

((17)) Let \((A, \mu, \varepsilon, a)\) be a color Hom-associative algebra. Then \((A, \{.,.\} = \mu - \varepsilon(\mu)\mu', \varepsilon, a)\) is a color Hom-Lie algebra, denoted by \(L(A)\).

**Corollary**

Let \((A, \mu, \{.,.\}, \varepsilon)\) be a color associative algebra and \(\alpha\) an even color algebra endomorphism. Then \((A, \mu - \{.,.\}, \varepsilon, a)\) where
\( \mu = a \circ \mu \cdot (\gamma) = a - e(\gamma) p^n \) is a color Hom-Poisson algebra.

**Proof**

The proof follows from Lemma 4.1 and Theorem 3.1.

**Lemma**

Let \((A, \mu, \epsilon, \alpha)\) be a color Poisson algebra and \(\alpha\) be an even color Poisson algebra endomorphism. Then \((A, \mu_a, \epsilon_a, \alpha)\) is a color Hom-Poisson algebra.

**Proof**

By Lemma 4.1 and [(3) Example1.2], we only need to prove the color Hom-Leibniz identity. For any \(x, y, z \in H(A)\),

\[
[a(x), \mu(y,z)]_a = \mu_a([x,y]_a, \alpha(z)) - \epsilon(x) \mu_a(\alpha(y), [x,z]_a) =
\]

\[
= \alpha^2(x, \alpha^2(y,z)) - \mu(\alpha^2(x,y), \alpha^2(z)) - \epsilon(x) \mu(\alpha^2(y), \alpha^2(z)) =
\]

\[
= \alpha(\alpha(x, y, z) - \mu(x, y, z) - \epsilon(x) \mu(y, x, z)) = 0.
\]

This completes the proof.

**Theorem**

Let \((A, \mu, \epsilon, \alpha)\) be a color Poisson algebra and \(\beta : A \rightarrow A\) an even color Poisson algebra endomorphism. Then \((A, \mu_{\beta}, \epsilon_{\beta}, \alpha)\) is a color Hom-Poisson algebra.

Moreover, suppose that \((A', \mu', \epsilon', \alpha')\) is a color Poisson algebra and \((A, \mu, \epsilon, \alpha)\) is an even color Poisson algebra endomorphism. If \(f : A \rightarrow A'\) is a color Poisson algebra morphism that satisfies \(f \circ \beta = \alpha \circ f\), then

\[
f : (A, \mu_{\beta}, \epsilon, \alpha) \rightarrow (A', \mu_{\beta'}, \epsilon_{\beta'}, \alpha')
\]

is a color Hom-Poisson algebra homomorphism.

**Proposition 5.1**

Let \((A, \mu, \epsilon, \alpha)\) be a color Poisson algebra and \(\alpha_k\) an even color Poisson algebra endomorphism of the form \(\alpha_k = \alpha_0 + \sum_{i=1}^{k} \alpha_i\), where \(\alpha_i\) are endomorphism of \(A\) (as color Poisson algebra). Then \(\alpha_k \circ \alpha_l = \sum_{i=0}^{k+l} \alpha_i\) is a color Hom-Poisson algebra which is a deformation of the color Poisson algebra \((A, \mu, \epsilon, \alpha)\) viewed as a color Hom-Poisson algebra \((A, \mu, \epsilon, \alpha_k \circ \alpha_l)\).

**Proof**

The proof follows from Theorem 2.

As in the case of Poisson algebras ([10,12,13]), the cohomology of color Hom-Poisson algebras is described by the cohomology of the underlying color Hom-Lie algebras ([3]).

**Modules Over Color Hom-Poisson Algebras**

**Definition 6.1**

Let \(G\) be an abelian group. A Hom-module is a pair \((M, \alpha_M)\) in which \(M\) is a \(G\)-graded vector space and \(\alpha_M : M \rightarrow M\) is an even linear map.

**Definition 6.2**

Let \((A, \mu, \epsilon, \alpha)\) be a color Hom-associative algebra. An A-module is a Hom-module \((M, \alpha_M)\) together with a bilinear map \(\mu_M : A \otimes M \rightarrow M\) called structure map, such that

\[
\mu_M(A_{\alpha}, A_{\beta}) \subseteq M_{\alpha \beta}
\]

\[
\alpha_M \circ \mu_M = \mu_M \circ (\alpha_A \otimes \alpha_M),
\]

\[
\mu_M \circ (\alpha_M \otimes \mu_M) = \mu_M \circ (\mu_M \otimes \alpha_M)
\]

Twisting a module structure map by an algebra endomorphism, we get another one as stated in the following Lemma.

**Lemma 6.1**

Let \((A, \mu, \epsilon, \alpha)\) be a color Hom-associative algebra and \(M\) an A-module with structure map \(\mu_M : A \otimes M \rightarrow M\). Define the map

\[
\tilde{\mu}_M = \mu_M \circ (\alpha_M \otimes \text{Id}_M) : A \otimes M \rightarrow M.
\]

Then \(\tilde{\mu}_M\) is the structure map of another A-module structure on \(M\).
such that be a color Hom-associative algebra as in \(A\), we have \(\mu = \mu_{M} = \mu_{M}\),
\[\mu_{M}(x, y) = \mu_{M}(x, y) = \mu_{M}(x, y) = \mu_{M}(x, y) = \mu_{M}(x, y),\]
for any \(m \in H(M), x, y \in H(L)\).

Remark 6.1
When \(\alpha_{M} = \text{Id}_{M}\) and \(\alpha = \text{Id}_{L}\) we recover the definition of Lie modules ([15-17]).

The following statement is the Lie analogue of Lemma 6.1.

Lemma 6.2
Let \((L, \{.,.,\}, \varepsilon, \alpha)\) be a color Hom-Lie algebra and \(M\) an \(L\)-module with structure map \(\mu_{M} = \mu_{M}\). Define the map
\[\mu_{M} = \mu_{M} \circ (\alpha_{L} \otimes \text{Id}_{M}) : L \otimes M \to M\]
Then \(\mu_{M}\) is the structure map of another \(L\)-module structure on \(M\).

Proof
Equalities 19 and 20 are proved as in Lemma 6.1. Now, we prove 21 for \(\mu_{M}\). For any \(x, y, z \in L, m \in M\)
\[\mu_{M}(x, y, z, m) = \mu_{M}(x, y, z, m) = \mu_{M}(x, y, z, m) = \mu_{M}(x, y, z, m),\]
Hence the conclusion holds.

The following result shows that \(L\)-modules extend to \(L(A)\)-modules with samemodule structure map.

Theorem 6.1
Let \((A, \mu, \varepsilon, \alpha)\) be a color Hom-associative algebra and \((M, \alpha_{M})\) an \(A\)-module with structure map \(\mu_{M}\). Then, \(M\) is an \(L(A)\)-module with structure map \(\mu_{M}\).

Proof
In fact, it suffices to show the relation 21. For any \(x, y \in H(A), m \in H(M)\), we have
\[\mu_{M}(x, y, m) = \mu_{M}(x, y, m) = \mu_{M}(x, y, m) = \mu_{M}(x, y, m),\]
This establishes the Theorem.

The corollaries below give a large class of examples of \(L(A)\)-modules.

Corollary 6.1
Let \(A = (A, \mu_{A}, \varepsilon, \alpha_{A})\) be a color Hom-associative algebra as in Lemma 6.1 and \((M, \alpha_{M})\) an \(A\)-module with structure map \(\mu_{M}\). Then, \(M\) is an \(L(A)\)-module with structure map \(\mu_{M}\).

Proof

We conclude that \(M\) is an \(L(A)\)-module with structure map \(\mu_{M}\).

Corollary 6.2
Let \((A, \mu, \varepsilon, \alpha)\) be a color Hom-associative algebra and \((M, \alpha_{M})\) an \(A\)-module with structure map \(\mu_{M}\). Put
\[\tilde{\mu}_{M} = \mu_{M} \circ (\alpha_{L} \otimes \text{Id}_{M})\]
Then \(M\) is an \(L(A)\)-module with structure map \(\mu_{M}\).

Proof
We know from Lemma 6.1 that \(\tilde{\mu}_{M}\) is an \(A\)-module structure map. And, for \(x, y \in H(A), m \in H(M)\), one has
\[\tilde{\mu}_{M}(x, y, m) = \mu_{M}(x, y, m) = \mu_{M}(x, y, m),\]
This is similar to the relation 21 for \(\tilde{\mu}_{M}\).

Now we define modules for color Hom-Poisson algebras.

Definition 6.4
Let \((A, \mu, \{.,.,\}, \varepsilon, \alpha)\) be a color Hom-Poisson algebra and \((M, \alpha_{M})\) a Hom-module.
A color Hom-Poisson module structure on M consists of two K-bilinear maps $\mu_M : A \otimes M \to M$ and $\lambda_M : A \otimes M \to M$ such that

(i) M is an A-module and an L-module,

(ii) And for any $x, y \in H(A), m \in H(M)$,

\[ \lambda_M(\alpha(x), \mu_M(y, m)) = \mu_M(\{\alpha, y\}, \alpha_M(m)) + \varepsilon(x, y)\mu_M(\alpha(y), \lambda_M(x, m)), \]  

\[ \lambda_M(\mu(x, y), \alpha_M(m)) = \mu_M(\alpha(x), \lambda_M(y, m)) + \varepsilon(x, y)\mu_M(\alpha(y), \lambda_M(x, m)), \]  

Then $M$ is a module over the $\varepsilon$ commutative color Hom-Poisson algebra (resp. $\varepsilon$ commutative color Hom-associative algebra) with the trivial color Hom-Lie algebra (resp. color Hom-Lie algebra) can be seen as a module over a $\varepsilon$ commutative color Hom-Poisson algebra with the trivial color Hom-Lie bracket (resp. trivial color Hom-associative product).

Example 6.1

(i) Any module over a $\varepsilon$ commutative color Hom-associative algebra (resp. color Hom-Lie algebra) can be seen as a module over a $\varepsilon$ commutative color Hom-Poisson algebra with the trivial color Hom-Lie bracket (resp. trivial color Hom-associative product).

(ii) Any $\varepsilon$ commutative color Hom-Poisson module is a module over itself.

Example 6.2

Let $(V, \mu_V, \lambda_V, \alpha_V)$ and $(W, \mu_W, \lambda_W, \alpha_W)$ be two modules over the $\varepsilon$ Commutative color Hom-Poisson algebra $(A, \mu, \{\cdot, \cdot\}, \varepsilon, \alpha)$,

Then the direct product $M = V \times W$ is a module over A with structure maps $\mu_M : A \otimes M \to M, \lambda_M : A \otimes M \to M$ and $\alpha_M : M \to M$.

Defined by

$\mu_M((\mu, v), (\lambda, w)) = (\mu(x, v), \mu(w, y))$, $\lambda_M((\mu, v), (\lambda, w)) = (\lambda(x, v), \lambda(w, y))$ and $\alpha_M((\mu, v), (\lambda, w)) = (\alpha(x, v), \alpha(w, y))$

for any $x \in H(A), v \in H(V)$ and $w \in H(W)$.

Proposition 6.1

If $(M, \mu_M, \lambda_M, \alpha_M)$ is a module over the $\varepsilon$-commutative color Hom-Poisson algebra $(A, \mu, \{\cdot, \cdot\}, \varepsilon, \alpha)$ then $(M, -\mu_M, -\lambda_M, -\alpha_M)$ is also a module over the $\varepsilon$-commutative color Hom-Poisson algebra $(A, -\mu, -\{\cdot, \cdot\}, -\varepsilon, -\alpha)$.

Proof

The proof comes from Definition 6.4 and Lemma 5.1.

Theorem 6.2

Let $(A, \mu, \{\cdot, \cdot\}, \varepsilon, \alpha)$ be a $\varepsilon$ commutative color Hom-Poisson algebra and $(M, \mu_M, \lambda_M, \alpha_M)$ color Hom-Poisson module. Then

$\mu_M = \mu_M \circ (\alpha^2 \otimes Id_M) : A \otimes M \to M$,  

$\lambda_M = \lambda_M \circ (\alpha^2 \otimes Id_M) : A \otimes M \to M$,  

Define another color Hom-Poisson module structure on M.

Proof

We know that $\mu_M$ is a structure of another $A$-module structure map on M (Lemma 6.1) and $\lambda_M$ is a structure of another $L$-module structure map on M (Lemma 6.2). Show relations 23 and 24 for $\mu_M$ and $\lambda_M$. For all $x, y \in H(A)$ and $m \in H(M)$,

$\lambda_M(\alpha(x), \mu_M(y, m)) = \lambda_M(\alpha(x), \mu_M(y, m))$  

$i$ $ii$}

References


