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New Exact Solutions for the Maccari System

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Abstract

In this article we apply three mathematical methods for solving the Maccari system, namely, the exp(-$\varphi(\xi)$)-expansion method, the sine-cosine approach and the Riccati-Bernoulli sub-ODE method. These methods are used to construct new and general exact periodic and soliton solutions of the Maccari system. This nonlinear system can be turned into another nonlinear ordinary differential equation by suitable transformation. It is shown that the exp(-$\varphi(\xi)$)-expansion method, the sine-cosine method and the Riccati-Bernoulli sub-ODE method provide a powerful mathematical tool for solving a great many systems of nonlinear partial differential equations in mathematical physics.

Keywords: Exp(-$\varphi(\xi)$)-expansion method; Sine-cosine method; The Riccati-Bernoulli sub-ODE method; Maccari system; Solitary wave solutions

Introduction

Nonlinear wave phenomenon play a major role in many natural sciences including mathematics, biology, and particularly in many branches of physics, such as nonlinear optics, condensed matter physics, plasma physics, chemical physics, solid-state physics, fluid dynamics, [1-7]. Actually, there are so many interesting applications of the previous natural sciences like the schrodinger equation, the Phi-4 equation, the 2D Ginzburg-Landau equation, the generalized-Zakharov system, the model of blood flow in arteries, the diffusive predator-prey system, shallow water equations, etc.

According to the import role of these equations, various papers interested in finding solutions of them even numerically or analytically. These solutions might be essential and important for the exploring some physical phenomena. Therefore investigating new technique for deterministic case to solve more complicated problems. Thus, many new methods have been proposed, like as the homogeneous balance method [8,9], the F-expansion method [10-12], the tanh-sech method [13-15], the Riccati-Bernoulli sub-ODE method [16,17], Jacobi elliptic function method [18,19], sine-cosine method [20-22], extended tanh-method [23-25], $(\frac{G'}{G})$ -expansion method [26,27], and exp-function method [28-30].

The objective of this paper is to apply three mathematical methods, specially, the exp(-$\varphi(\xi)$)-expansion method, the sine-cosine method and the Riccati-Bernoulli sub-ODE method, for finding various new exact solutions of the Maccari system. This system is extremely presented to define the motion of the isolated waves, localized in a very small part of space, in many fields such as hydrodynamics, optics, quantum field theory. Moreover it used in plasma physics to describe the behavior of the sonic Langmuir solitons. We give new solutions and compare it with other methods and show that this method is efficient robust and adequate for solving other type of NPDEs. Moreover the Riccati-Bernoulli sub-ODE method gives infinite sequence of solutions. Actually all presented solutions have so important contribution for the explanation of some practical physical problems.

The rest of this work is given as follows: In section 2 we prescribe the exp(-$\varphi(\xi)$)-expansion method, the sine-cosine method and the Riccati-Bernoulli sub-ODE method. In Section 3 these methods are applied to solve the Maccari system. In Section 4, some three-dimensional graphs of some solutions is provided. Finally, in Section 5 we give the summary of our contribution.

Analysis of the Methods

Any nonlinear evolution equation can be presented in following form:

$$P(u,u_t,u_x,u_{tt},u_{xx},....)=0, \tag{1}$$

where P is a polynomial in u(x,t) and its partial derivatives in which the highest order derivatives. The basic steps for this method are

Using the wave transformation for a positive constant c,

$$u(x,t)=u(\xi), \qquad \xi=x-ct, \tag{2}$$

Eqn. (1) will be transformed to the following ODE:

$$D(u,u',u'',u''',.....)=0, \tag{3}$$

The exp(-$\varphi(\xi)$)-expansion method

Assume that the solution of eqn. (3) can be written in a polynomial form of exp(-$\varphi(\xi)$)as follows [29,30]:

$$u(\xi)=\sum_{m=0}^{n}a_m\left(exp(-\varphi(\xi))\right)^m,\ a_n\neq 0, \tag{4}$$

where $\varphi(\xi)$ satisfies the following OD

$$\varphi'(\xi)=exp(-\varphi(\xi))+\mu exp(\varphi(\xi))+\lambda. \tag{5}$$

Eqn. (5) has the following solutions:

1. At $\lambda^2-4\mu>0,\mu\neq 0,$

$$\varphi(\xi)=ln\left(\frac{-\sqrt{\lambda^2-4\mu}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+k)\right)-\lambda}{2\mu}\right), \tag{6}$$

Where k is an arbitrary constant.

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2. At $\lambda^2-4\mu<0,\mu\neq 0,$

$$\varphi(\xi)=ln\left(\frac{\sqrt{4\mu-\lambda^2}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+k)\right)-\lambda}{2\mu}\right), \tag{7}$$

3. At $\lambda^2-4\mu>0,\mu=0,\lambda\neq 0$

$$\varphi(\xi)=-ln\left(\frac{\lambda}{exp(\lambda(\xi+k))-1}\right), \tag{8}$$

4. At $\lambda^2-4\mu=0,\mu\neq 0,\lambda\neq 0,$

$$\varphi(\xi)=ln\left(-\frac{2(\lambda(\xi+k)+2)}{\lambda^2(\xi+k)}\right), \tag{9}$$

5. At $\lambda^2-4\mu=0,\mu=0,\lambda=0,$

$$\varphi(\xi)=ln(\xi+k). \tag{10}$$

Finally, superseding eqn. (4) with eqn. (5) into eqn. (3) and aggregating all terms of the same power

exp(*m*-$\varphi(\xi)$), m=0,1,2,3,4…Then equating each term to zero to get algebraic equations which can be solved analytically or by using Mathematica or Maple to obtain the values of a_i Hence, we get the solutions (4), which give the exact solutions of eqn. (3).

The sine-cosine method

The solutions of the reduced ODE eqn. (3) can be expressed in the form [31,32]

$$u(x,t)=\begin{cases}\lambda sin^r(\mu\xi), & |\xi|\le\frac{\pi}{\mu},\\ 0, & otherwise,\end{cases} \tag{11}$$

or in the form

$$u(x,t)=\begin{cases}\lambda cos^r(\mu\xi), & |\xi|\le\frac{\pi}{2\mu},\\ 0, & otherwise,\end{cases} \tag{12}$$

where λ, μ and $r^1$0, are parameters, which will be determined. From eqn. (11) we have

$$\begin{aligned}&u(\xi)=\lambda sin^r(\mu\xi),\\ &u^n(\xi)=\lambda^n sin^{nr}(\mu\xi),\\ &(u^n)_{\xi}=n\mu r\lambda^n cos(\mu\xi)sin^{nr-1}(\mu\xi),\\ &(u^n)_{\xi\xi}=-n^2\mu^2r^2\lambda^n sin^{nr}(\mu\xi)+n\mu^2\lambda^n r(nr-1)sin^{nr-2}(\mu\xi),\end{aligned} \tag{11a}$$

and from (12) we have

$$\begin{aligned}&u(\xi)=\lambda cos^r(\mu\xi),\\ &u^n(\xi)=\lambda^n cos^{nr}(\mu\xi),\\ &(u^n)'=-n\mu r\lambda^n sin(\mu\xi)cos^{nr-1}(\mu\xi),\\ &(u^n)''=-n^2\mu^2r^2\lambda^n cos^{nr}(\mu\xi)+n\mu^2\lambda^n r(nr-1)cos^{nr-2}(\mu\xi),\end{aligned} \tag{12a}$$

and so on for other derivatives.

Finally, we substitute the above equations into the reduced equation obtained above in eqn. (3), balance the terms of the cosine functions when (11a) is used, or balance the terms of the sine functions when (12a) is used, and solving the resulting system of algebraic equations by using the computerized symbolic calculations. We next collect all terms with the same power in $cos^k(\mu\xi)$ or $sin^k(\mu\xi)$ and set to zero their coefficient to get a system of algebraic equations in the unknowns λ, μ and r We obtained all possible values of the parameters λ, μ and r

Riccati-Bernoulli sub-ODE method

Due to the Riccati-Bernoulli Sub-ODE technique [16,17,33], we assume that eqn. (3) has a solution given by:

$$u'=au^{2-n}+bu+cu^n, \tag{13}$$

where *a*,*b*,*c* and *n* are constants determined later. From eqn. (13), we get

$$u''=ab(3-n)u^{2-n}+a^2(2-n)u^{3-2n}+nc^2u^{2n-1}+bc(n+1)u^n+(2ac+b^2)u, \tag{14}$$

$$\begin{aligned}u'''=&(ab(3-n)(2-n)u^{1-n}+a^2(2-n)(3-2n)u^{2-2n}\\&+n(2n-1)c^2u^{2n-2}+bcn(n+1)u^{n-1}+(2ac+b^2))u'.\end{aligned} \tag{15}$$

The solitary solutions $u_i(\xi)$ of eqn. (13) are given as follow:

1. For *n*=1

$$u(\xi)=\mu e^{(a+b+c)\xi}. \tag{16}$$

Where μ is an arbitrary constant.

2. For $n\neq 1$ *b*=0 and c=0

$$u(\xi)=\left(a(n-1)(\xi+\mu)\right)^{\frac{1}{n-1}}. \tag{17}$$

3. For $n\neq 1$, $b\neq 0$ and $c=0$

$$u(\xi)=\left(\frac{-a}{b}+\mu e^{b(n-1)\xi}\right)^{\frac{1}{n-1}}. \tag{18}$$

4. For $n\neq 1$, $a\neq 0$ and b^2-4ac<0

$$u(\xi)=\left(\frac{-b}{2a}+\frac{\sqrt{4ac-b^2}}{2a}\tan\left(\frac{(1-n)\sqrt{4ac-b^2}}{2}(\xi+\mu)\right)\right)^{\frac{1}{1-n}} \tag{19}$$

and

$$u(\xi)=\left(\frac{-b}{2a}-\frac{\sqrt{4ac-b^2}}{2a}\cot\left(\frac{(1-n)\sqrt{4ac-b^2}}{2}(\xi+\mu)\right)\right)^{\frac{1}{1-n}}. \tag{20}$$

5. For $n\neq 1$, $a\neq 0$ and b^2-4ac>0

$$u(\xi)=\left(\frac{-b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}\coth\left(\frac{(1-n)\sqrt{b^2-4ac}}{2}(\xi+\mu)\right)\right)^{\frac{1}{1-n}} \tag{21}$$

and

$$u(\xi)=\left(\frac{-b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}\tanh\left(\frac{(1-n)\sqrt{b^2-4ac}}{2}(\xi+\mu)\right)\right)^{\frac{1}{1-n}}. \tag{22}$$

6. For $n\neq 1$, $a\neq 0$ and b^2-4ac=0

$$u(\xi)=\left(\frac{1}{a(n-1)(\xi+\mu)}-\frac{b}{2a}\right)^{\frac{1}{1-n}}. \tag{23}$$

Bäcklund transformation: When $u_{m-1}(\xi)$ and $u_m(\xi)(u_m(\xi)=u_m(u_{m-1}(\xi)))$ are the solutions of eqn. (13), we have

$$\frac{du_m(\xi)}{d\xi} = \frac{du_m(\xi)}{du_{m-1}(\xi)}\frac{du_{m-1}(\xi)}{d\xi} = \frac{du_m(\xi)}{du_{m-1}(\xi)}(au_{m-1}^{2-n} + bu_{m-1} + cu_{m-1}^n),$$

namely

$$\frac{du_m(\xi)}{au_m^{2-n} + bu_m + cu_m^n} = \frac{du_{m-1}(\xi)}{au_{m-1}^{2-n} + bu_{m-1} + cu_{m-1}^n}. \quad (24)$$

Integrating eqn. (24) once with respect to ξ, we get a Bäcklund transformation of eqn. (13) as follows:

$$u_m(\xi) = \left(\frac{-cK_1 + aK_2\left(u_{m-1}(\xi)\right)^{1-n}}{bK_1 + aK_2 + aK_1\left(u_{m-1}(\xi)\right)^{1-n}}\right)^{\frac{1}{1-n}}. \quad (25)$$

where $K_1$and K_2 are arbitrary constants. We use eqn. (25) to obtain infinite sequence of solutions of eqn. (13), as well of eqn. (1)

The Maccari System

Maccari [34] derived a new integrable (2+1) dimensional nonlinear system. This system is obtained by an asymptotically exact reduction method based on Fourier expansion and spatio-temporal rescaling, from the Kadomtsevâ€“Petviashvili equation. The Maccari system is frequently introduced to define the motion of the isolated waves, localized in a very small part of space, in many fields such as hydrodynamics, quantum field theory, nonlinear optics. Indeed, it is plays an important role in plasma physics to describe the behavior of the sonic Langmuir solitons. In this section we are concerned with the Maccari system [35-37], given as follows:

$$\begin{aligned} &i\phi_t + \phi_{xx} + \phi\psi = 0, \\ &\psi_t + \psi_y + (|\phi|)^2 = 0, \end{aligned} \quad (26)$$

Where $\varphi=\varphi(x,y,t)$ and $\psi=\psi(x,y,t)$, represent, respectively, the complex scalar field and the real scalar field. Here x,y are the independent spatial variables and t>0 represents the temporal variable. In this section we apply the expansion exp(-(ξ)) method, the sine-cosine method and the Riccati-Bernoulli sub-ODE method, to construct exact solutions for the Maccari system.

Consider the wave transformation

$$\phi(x,y,t) = e^{i\varepsilon}u(\xi),\ \ \psi(x,y,t) = \psi(\xi),\ \ \xi = c_1x + c_2y + c_3t,\ \ \varepsilon = k_1x + k_2y + k_3t, \quad (27)$$

where c_1,c_2,c_3, k_1 k_2 and k_3 are arbitrary constants. Then, eqn. (26) are transformed into the following equations

$$\begin{aligned} &ic_3u' - k_3u - k_1^2u + 2ik_1c_1u' + c_1^2u'' + u\psi = 0, \\ &(c_2 + c_3)\psi' + c_1(u^2)' = 0,, \end{aligned} \quad (28)$$

Integrating second equation of eqn. (28) and taking integration constant zero for the sake of simplicity, yields

$$\psi = \left(\frac{-c_1}{c_2 + c_3}\right)u^2. \quad (29)$$

Now by separating real and imaginary parts in the first equation of (28), we have

$$i(c_3 + 2k_1c_1)u' = -0. \quad (30)$$

$$-(k_3 + k_1^2)u + c_1^2u'' + u\psi = 0. \quad (31)$$

Substituting eqn. (29) into (31), we get the following NODE:

$$l_1u'' + l_2u^3 + l_3u = 0, \quad (32)$$

where $l_1 = c_1^2, l_2 = \left(\frac{-c_1}{c_2 + c_3}\right), l_3 = -(k_1^2 + k_3)$.

On solving eqn. (26) using the exp (−φ(ξ))-expansion method

Balancing the highest order derivative u'' and non-linear term u^3 in eqn. (32), we obtain n=1. Consequently, eqn. (32) has the following solution

$$U = A_0 + A_1exp(-\varphi), \quad (33)$$

where A_0 and A_1 are constants to be determined, such that $A_1 \neq 0$ It is easy to see that

$$U'' = A_1\left(2\,exp(-3\varphi) + 3\lambda\,exp(-2\varphi) + (2\mu + \lambda^2)\,exp(-\varphi) + \lambda\mu\right), \quad (34)$$

$$U^3 = A_1^3\,exp(-3\varphi) + 3A_0A_1^2\,exp(-2\varphi) + 3A_0^2A_1\,exp(-\varphi) + A_0^3. \quad (35)$$

Superseding u, u'', u^3 into eqn. (32) and then equating the coefficients of exp φ) to zero, we obtain

$$l_1A_1\lambda\mu + l_2A_0^3 + l_3A_0 = 0, \quad (36)$$

$$A_1l_1(\lambda^2 + 2\mu) + 3l_2A_0^2A_1 + l_3A_1 = 0, \quad (37)$$

$$3l_1A_1\lambda + 3l_2A_0A_1^2 = 0, \quad (38)$$

$$2l_1A_1 + l_2A_1^3 = 0. \quad (39)$$

Solving eqns. (36)-(39), we get

$$l_1 = \frac{2l_3}{\lambda^2 - 4\mu}, A_0 = \pm\lambda\sqrt{\frac{l_3}{l_2(4\mu - \lambda^2)}}, A_1 = \pm2\sqrt{\frac{l_3}{l_2(4\mu - \lambda^2)}}.$$

Actually we will consider only case from the above cases, the other cases follow in the same way. Now substituting the values of A_0,A_1 into eqn. (33) yields

$$U(\xi) = \pm\sqrt{\frac{l_3}{l_2(4\mu - \lambda^2)}}\left(\lambda + 2exp(-\varphi(\xi))\right). \quad (40)$$

Now substituting eqns. (6)-(7) into eqn. (40) respectively, we have the following solutions:

Case 1. At $\lambda^2 - 4\mu > 0, \mu \neq 0,$

$$u_{1,2}(x,y,t) = \pm\sqrt{\frac{l_3}{l_2(4\mu - \lambda^2)}}\left(\lambda - \frac{4\mu}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(c_1x + c_2y + c_3t + k)\right) + \lambda}\right), \quad (41)$$

where $l_2, l_3, c_1, c_2, c_3, \lambda, \mu$ and k are arbitrary constants. Using eqns. (41), (27) and (29) the solutions of eqn. (26) take the forms:

$$\phi_{1,2}(x,y,t) = \pm e^{i(k_1x + k_2y + k_3t)}\sqrt{\frac{l_3}{l_2(4\mu - \lambda^2)}}\left(\lambda - \frac{4\mu}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(c_1x + c_2y + c_3t + k)\right) + \lambda}\right) \quad (42)$$

and

$$\psi_{1,2}(x,y,t) = \left(\frac{-c_1}{c_2 + c_3}\right)\left(\sqrt{\frac{l_3}{l_2(4\mu - \lambda^2)}}\left(\lambda - \frac{4\mu}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(c_1x + c_2y + c_3t + k)\right) + \lambda}\right)\right)^2. \quad (43)$$

Case 2. At $\lambda^2 - 4\mu < 0, \mu \neq 0,$

$$u_{3,4}(x,y,t) = \pm\sqrt{\frac{l_3}{l_2(4\mu - \lambda^2)}}\left(\lambda + \frac{4\mu}{\sqrt{4\mu - \lambda^2}\tanh\left(\frac{\sqrt{4\mu - \lambda^2}}{2}(c_1x + c_2y + c_3t + k)\right) - \lambda}\right). \quad (44)$$

where $l_2, l_3, c_1, c_2, c_3, \lambda, \mu$ and k are arbitrary constants.

Using eqns. (44), (27) and (29) the solutions of eqn. (26) take the forms:

$$\phi_{3,4}(x,y,t) = \pm e^{i(k_1x+k_2y+k_3t)}\sqrt{\frac{l_3}{l_2(4\mu-\lambda^2)}}\left(\lambda + \frac{4\mu}{\sqrt{4\mu-\lambda^2}\tanh\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(c_1x+c_2y+c_3t+k)\right)-\lambda}\right) \quad (45)$$

and

$$\psi_{3,4}(x,y,t) = \left(\frac{-c_1}{c_2+c_3}\right)\left(\sqrt{\frac{l_3}{l_2(4\mu-\lambda^2)}}\left(\lambda + \frac{4\mu}{\sqrt{4\mu-\lambda^2}\tanh\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(c_1x+c_2y+c_3t+k)\right)-\lambda}\right)\right)^2. \quad (46)$$

Case 3. At $\lambda^2 - 4\mu > 0, \mu = 0, \lambda \neq 0$

$$u_{5,6}(x,y,t) = \pm\sqrt{\frac{l_3}{-l_2\lambda^2}}\left(\lambda + \frac{2\lambda}{exp\left(\lambda\left(c_1x+c_2y+c_3t+k\right)\right)-1}\right). \quad (47)$$

Using eqns. (47), (27) and (29) the solutions of eqn. (26) take the forms:

$$\phi_{5,6}(x,y,t) = \pm e^{i(k_1x+k_2y+k_3t)}\sqrt{\frac{l_3}{-l_2\lambda^2}}\left(\lambda + \frac{2\lambda}{exp\left(\lambda\left(c_1x+c_2y+c_3t+k\right)\right)-1}\right) \quad (48)$$

and

$$\psi_{3,4}(x,y,t) = \left(\frac{-c_1}{c_2+c_3}\right)\left(\sqrt{\frac{l_3}{-l_2\lambda^2}}\left(\lambda + \frac{2\lambda}{exp\left(\lambda\left(c_1x+c_2y+c_3t+k\right)\right)-1}\right)\right)^2, \quad (49)$$

where $l_2, l_3, c_1, c_2, c_3, \lambda, \mu$ and k are arbitrary constants.

On solving eqn. (26) using the sine-cosine method

Now we apply the sine-cosine method for solving eqn. (32). Subtitling (31) into (32), gives

$$l_1\left(-\mu^2r^2\lambda sin^r(\mu\xi) + \mu^2\lambda r(r-1)sin^{r-2}(\mu\xi)\right) + l_2\lambda^3 sin^{3r}(\mu\xi) + l_3\lambda sin^r(\mu\xi) = 0. \quad (50)$$

Equating the exponents and the coefficient of the sine functions gives the following system of algebraic equations:

$$r-1 \neq 0, \quad r-2 = 3r,$$

$$l_1\mu^2\lambda r(r-1) + l_2\lambda^3 = 0, \quad (51)$$

$$-l_1\mu^2r^2\lambda + l_3\lambda = 0.$$

Solving this system gives

$$r = -1, \quad \lambda = \pm i\sqrt{\frac{2l_3}{l_2}}, \quad \mu = \pm\sqrt{\frac{l_3}{l_1}}, \quad (52)$$

for $\frac{l_3}{l_1} > 0$ and $\frac{l_3}{l_2} > 0$ with $l_1, l_2 \neq 0$. The result (52) can be easily obtained if we also use the cosine method (12). Consequently, we get the :following periodic solutions

$$\tilde{u}_1(x,y,t) = \pm\sqrt{\frac{2l_3}{l_2}}sec\left(\sqrt{\frac{l_3}{l_1}}(c_1x+c_2y+c_3t)\right), \quad |\sqrt{\frac{l_3}{l_1}}(c_1x+c_2y+c_3t)| < \frac{\pi}{2} \quad (53)$$

and

$$\tilde{u}_2(x,y,t) = \pm\sqrt{\frac{2l_3}{l_2}}csc\left(\sqrt{\frac{l_3}{l_1}}(c_1x+c_2y+c_3t)\right), \quad 0 < \sqrt{\frac{l_3}{l_1}}(c_1x+c_2y+c_3t) < \pi. \quad (54)$$

Using eqns. (53), (54), (27) and (29) the solutions of eqn. (26) take the forms:

$$\tilde{\phi}_1(x,y,t) = \pm e^{i(k_1x+k_2y+k_3t)}\sqrt{\frac{2l_3}{l_2}}sec\left(\sqrt{\frac{l_3}{l_1}}(c_1x+c_2y+c_3t)\right), \quad |\sqrt{\frac{l_3}{l_1}}(c_1x+c_2y+c_3t)| < \frac{\pi}{2}, \quad (55)$$

$$\tilde{\phi}_2(x,y,t) = \pm e^{i(k_1x+k_2y+k_3t)}\sqrt{\frac{2l_3}{l_2}}csc\left(\sqrt{\frac{l_3}{l_1}}(c_1x+c_2y+c_3t)\right), \quad 0 < \sqrt{\frac{l_3}{l_1}}(c_1x+c_2y+c_3t) < \pi, \quad (56)$$

$$\tilde{\psi}_1(x,y,t) = \left(\frac{-c_1}{c_2+c_3}\right)\left(\sqrt{\frac{2l_3}{l_2}}sec\left(\sqrt{\frac{l_3}{l_1}}(c_1x+c_2y+c_3t)\right)\right)^2, \quad |\sqrt{\frac{l_3}{l_1}}(c_1x+c_2y+c_3t)| < \frac{\pi}{2} \quad (57)$$

and

$$\tilde{\psi}_2(x,y,t) = \left(\frac{-c_1}{c_2+c_3}\right)\left(\sqrt{\frac{2l_3}{l_2}}csc\left(\sqrt{\frac{l_3}{l_1}}(c_1x+c_2y+c_3t)\right)\right)^2, \quad 0 < \sqrt{\frac{l_3}{l_1}}(c_1x+c_2y+c_3t) < \pi. \quad (58)$$

However, for $\frac{l_3}{l_2} < 0$ and $l_2 \neq 0$, we obtain the soliton and complex solutions

$$\tilde{u}_3(x,y,t) = \pm\sqrt{\frac{2l_3}{l_2}}sech\left(\sqrt{-\frac{l_3}{l_1}}(c_1x+c_2y+c_3t)\right) \quad (59)$$

and

$$\tilde{u}_4(x,y,t) = \pm\sqrt{\frac{2l_3}{l_2}}csch\left(\sqrt{\frac{l_3}{l_1}}(c_1x+c_2y+c_3t)\right). \quad (60)$$

Using eqns. (59), (60), (27) and (29), we get the solutions of eqn. (26) in the same way as in eqns. (55)-(58).

On solving eqn. (26) using the Riccati-Bernoulli Sub-ODE method

Now we apply the Riccati-Bernoulli Sub-ODE method for solving eqn. (32). Substituting eqn. (14) into eqn. (32), we obtain

$$l_1\left(ab(3-n)u^{2-n} + a^2(2-n)u^{3-2n} + nc^2u^{2n-1} + bc(n+1)u^n + (2ac+b^2)u\right) + l_2u^3 + l_3u = 0. \quad (61)$$

Setting n=0, eqn. (61) is reduced to

$$l_1(3abu^2 + 2a^2u^3 + bc + (2ac+b^2)u) + l_2u^3 + l_3u = 0. \quad (62)$$

Putting each coefficient o $u^i(i = 0,1,2,3)$ to zero, we get

$$l_1bc = 0, \quad (63)$$

$$l_1(2ac+b^2) + l_3 = 0, \quad (64)$$

$$3l_1ab = 0, \quad (65)$$

$$2l_1a^2 + l_2 = 0. \quad (66)$$

Solving eqns. (63)-(66), we get

$$b=0 \quad (67)$$

$$c = \mp\frac{l_3}{\sqrt{-2l_1l_2}}, \quad (68)$$

$$a = \pm\sqrt{\frac{-l_2}{2l_1}}. \quad (69)$$

Hence, we give the cases of solutions for the eqns. (32) and (26), respectively

1. When b=0 and c=0the solution of eqn. (32) is

$$\hat{u}_1(x,y,t) = \left(-a\left(c_1x+c_2y+c_3t+\mu\right)\right)^{-1}. \quad (70)$$

Using eqns. (70), (27) and (29) the solutions of eqn. (26) take the forms:

$$\hat{\phi}_1(x,y,t) = e^{i(k_1x+k_2y+k_3t)}\left(-a\left(c_1x+c_2y+c_3t+\mu\right)\right)^{-1}, \quad (71)$$

$$\hat{\psi}_1(x,t)=\left(\frac{-c_1}{c_2+c_3}\right)\left(\left(-a\left(c_1x+c_2y+c_3t+\mu\right)\right)^{-1}\right)^2, \tag{72}$$

where $k_1,k_2,k_3,c_1,c_2,c_3,\mu$ are arbitrary constants.

2. When $\frac{l_3}{l_1}<0$, substituting eqns. (67)-(69), (27) and (29) into eqns. (19) and (20), we obtain the exact solutions of eqn. (26) as follows

$$\hat{u}_{2,3}(x,y,t)=\pm\sqrt{\frac{l_3}{l_2}}\,tan\left(\sqrt{\frac{-l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right) \tag{73}$$

and

$$\hat{u}_{4,5}(x,y,t)=\pm\sqrt{\frac{l_3}{l_2}}\,cot\left(\sqrt{\frac{-l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right). \tag{74}$$

Using eqns. (73), (74), (27) and (29) the solutions of eqn. (26) take the forms:

$$\hat{\phi}_{2,3}(x,y,t)=\pm\sqrt{\frac{l_3}{l_2}}e^{i(lx+kt)}\,tan\left(\sqrt{\frac{-l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right) \tag{75}$$

and

$$\hat{\phi}_{4,5}(x,y,t)=\pm\sqrt{\frac{l_3}{l_2}}e^{i(k_1x+k_2y+k_3t)}\,cot\left(\sqrt{\frac{-l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right), \tag{76}$$

$$\hat{\psi}_{2,3}(x,y,t)=\left(\frac{-c_1}{c_2+c_3}\right)\left(\sqrt{\frac{l_3}{l_2}}\,tan\left(\sqrt{\frac{-l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right)\right)^2, \tag{77}$$

$$\hat{\psi}_{4,5}(x,y,t)=\left(\frac{-c_1}{c_2+c_3}\right)\left(\sqrt{\frac{l_3}{l_2}}\,cot\left(\sqrt{\frac{-l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right)\right)^2, \tag{78}$$

Where $k_1,k_2,k_3,c_1,c_2,c_3,l_i,l_2,l_3,\mu$ are arbitrary constants.

3. When $\frac{l_3}{l_1}>0$, substituting eqns. (67)-(69) and (27) into eqns. (21) and (22), we get exact solutions of eqn. (26),

$$\hat{u}_{6,7}(x,y,t)=\pm\sqrt{\frac{-l_3}{l_2}}\,tanh\left(\sqrt{\frac{l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right) \tag{79}$$

and

$$\hat{u}_{8,9}(x,y,t)=\pm\sqrt{\frac{-l_3}{l_2}}\,coth\left(\sqrt{\frac{l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right). \tag{80}$$

Using eqns. (79), (80), (27) and (29) the solutions of eqn. (26) take the forms:

$$\hat{\phi}_{6,7}(x,y,t)=\pm\sqrt{\frac{-l_3}{l_2}}e^{i(k_1x+k_2y+k_3t)}\,tanh\left(\sqrt{\frac{l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right) \tag{81}$$

and

$$\hat{\phi}_{8,9}(x,y,t)=\pm\sqrt{\frac{-l_3}{l_2}}e^{i(k_1x+k_2y+k_3t)}\,tanh\left(\sqrt{\frac{l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right), \tag{82}$$

$$\hat{\psi}_{6,7}(x,y,t)=\left(\frac{-c_1}{c_2+c_3}\right)\left(\sqrt{\frac{-l_3}{l_2}}\,tanh\left(\sqrt{\frac{l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right)\right)^2, \tag{83}$$

$$\hat{\psi}_{8,9}(x,y,t)=\left(\frac{-c_1}{c_2+c_3}\right)\left(\sqrt{\frac{-l_3}{l_2}}\,coth\left(\sqrt{\frac{l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right)\right)^2, \tag{84}$$

Where $k_1,k_2,k_3,c_1,c_2,c_3,l_i,l_2,l_3,\mu$ are arbitrary constants.

Remark 3.1

Applying eqn. (25) to $u_i(x,y,t)$ i=1,2,...,9, we obtain an infinite sequence of solutions of eqn. (32). Consequently, we get an infinite sequence of solutions of eqn. (26). For illustration, by applying eqn. (25) to $u_i(x,y,t)$ i=1,2,...,9, once, we have new solutions of eqn. (32)

$$u_1^*(x,y,t)=\frac{B_3}{-aB_3\left(c_1x+c_2y+c_3t+\mu\right)\pm1}, \tag{85}$$

$$u_{2,3}^*(x,y,t)=\frac{\pm\frac{l_3}{\sqrt{-2l_1l_2}}\pm B_3\sqrt{\frac{l_3}{l_2}}\,tan\left(\sqrt{\frac{-l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right)}{B_3\pm\sqrt{\frac{l_3}{l_2}}\,tan\left(\sqrt{\frac{-l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right)}, \tag{86}$$

$$u_{4,5}^*(x,y,t)=\frac{\pm\frac{l_3}{\sqrt{-2l_1l_2}}\pm B_3\sqrt{\frac{l_3}{l_2}}\,cot\left(\sqrt{\frac{-l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right)}{B_3\pm\sqrt{\frac{l_3}{l_2}}\,cot\left(\sqrt{\frac{-l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right)}, \tag{87}$$

$$u_{6,7}^*(x,y,t)=\frac{\pm\frac{l_3}{\sqrt{-2l_1l_2}}\pm B_3\sqrt{\frac{-l_3}{l_2}}\,tanh\left(\sqrt{\frac{l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right)}{B_3\pm\sqrt{\frac{-l_3}{l_2}}\,tanh\left(\sqrt{\frac{l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right)}, \tag{88}$$

$$u_{8,9}^*(x,y,t)=\frac{\pm\frac{l_3}{\sqrt{-2l_1l_2}}\pm B_3\sqrt{\frac{-l_3}{l_2}}\,coth\left(\sqrt{\frac{l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right)}{B_3\pm\sqrt{\frac{-l_3}{l_2}}\,coth\left(\sqrt{\frac{l_3}{l_1}}\left(c_1x+c_2y+c_3t+\mu\right)\right)}, \tag{89}$$

where B_3, $c_1,c_2,c_3,l_i,l_2,l_3,\mu$ are arbitrary constants.

Remark 3.2

1. Comparing our results concerning eqn. (26) with the results [35-37], one can see that our results are new and most extensive. Moreover it can be seen that by choosing suitable values for the parameters similar solutions can be verified

2. One very important characteristic, that the Riccati-Bernoulli Sub-ODE method admits infinite sequence of solutions of equation, which has never given for any another methods.

3. Consequently, the method is efficacious robust and adequate for solving such types PDEs.

Graphs for the Solutions

In this section 3D graphics of some solutions have been plotted, namely Figures 1-7.

Conclusion

In this work we have introduced the exp(-$\varphi(\xi)$)-expansion, the sine-cosine methods and the Riccati-Bernoulli sub-ODE method in order to obtain the exact travelling wave solutions for the Maccari system. Many new and more general traveling wave solutions are obtained during the analytical treatment. The calculations demonstrate that this method has a very important role to obtain analytical solutions in a unified and more general form. It is shown that these three methods provide a very effective and powerful mathematical tool for solving nonlinear evolution equations in mathematical physics, such as the schrodinger

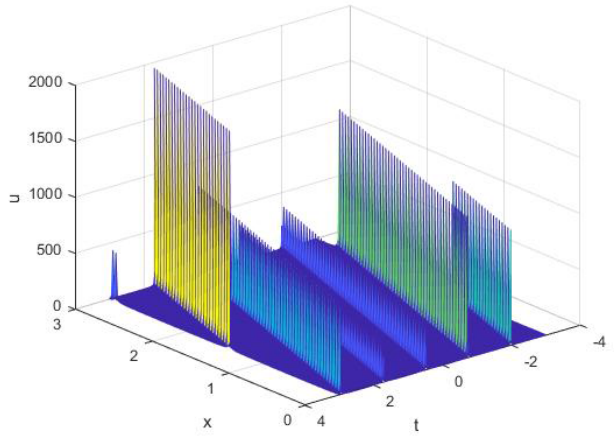

Figure 1: The solution $u=u_1$ in eqn. (41) with l_2=1.5, l_3=2, *mu*=2.2, λ=3.5, l_1=1.1594, c_1=1, c_3=1.6, *k*=4 and $-2 \le x \le 2$, $0 \le t \le 2$.

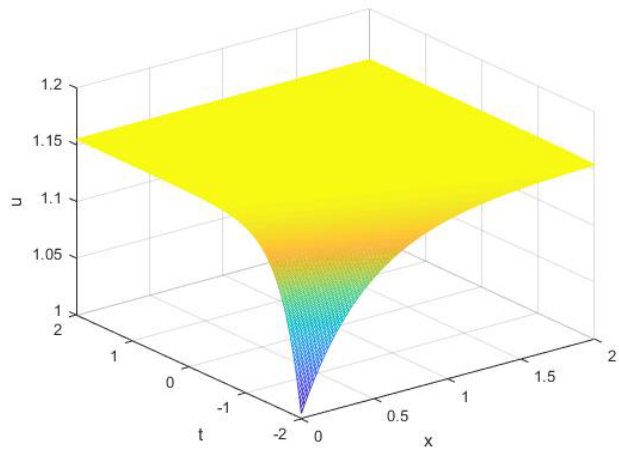

Figure 2: The stochastic solution $u=u_3$ in eqn. (44) with l_2=1.5, l_3=2, *mu*=3, λ=2, l_1= -0.5, c_1=1.2, c_3=1.8, *k*=4 and $-3 \le x \le 3$, $0 \le t \le 3$.

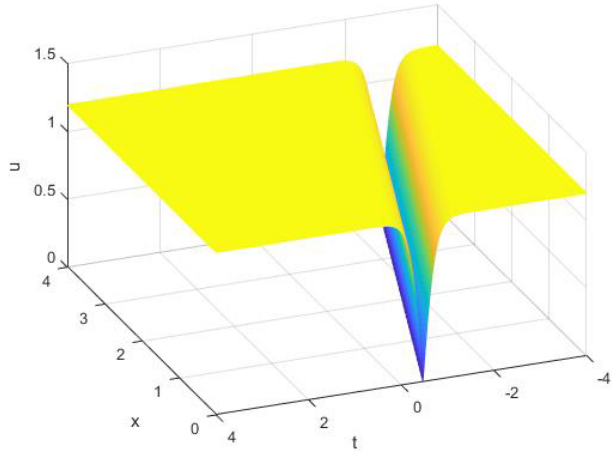

Figure 3: The solution $u=u_9$ in eqn. (47) with l_2=2.5, l_3=2.4, *mu*=1.6, λ=4, l_1=0.5, c_1=1.2, c_3=1.7, *k*=2 and $-4 \le x \le 4$, $0 \le t \le 4$.

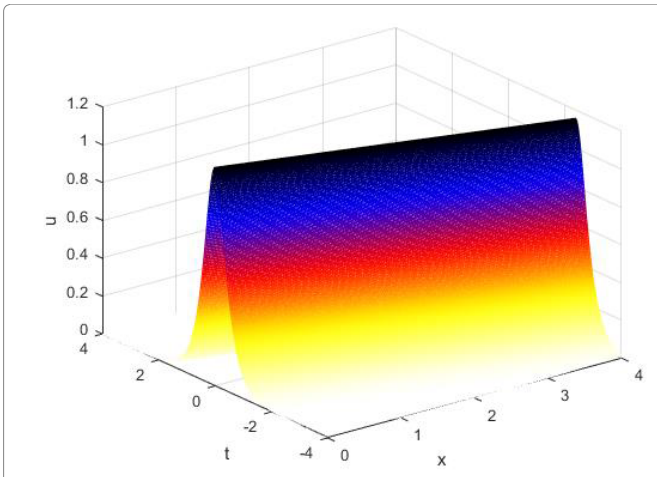

Figure 4: The solution $u=\tilde{u}_1$ in eqn. (53) with l_1=1.1, l_3=2, c_1=1.3, c_3=2.2, and $-4 \le x \le 4$, $0 \le t \le 4$.

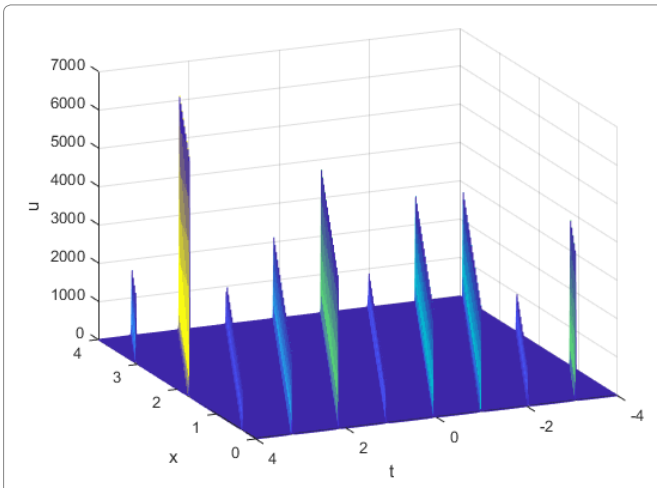

Figure 5: The solution $u=\tilde{u}_1$ in eqn. (59) with l_1=1.1, l_2=3, l_3=-2, c_1=1.3, c_3=2.2, and $-4 \le x \le 4$, $0 \le t \le 4$.

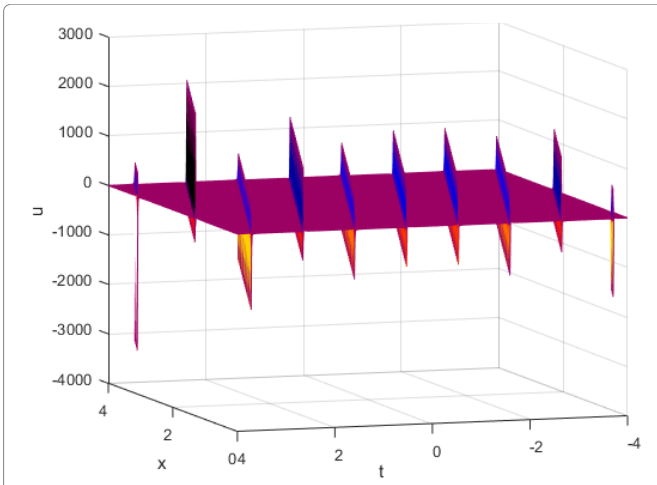

Figure 6: The solution $u=\hat{u}_2$ in eqn. (53) with l_1=1.5, l_2=1.6, l_3=-2, μ=1.2, c_1=1.5, c_3=2.6, and $-4 \le x \le 4$, $0 \le t \le 4$.

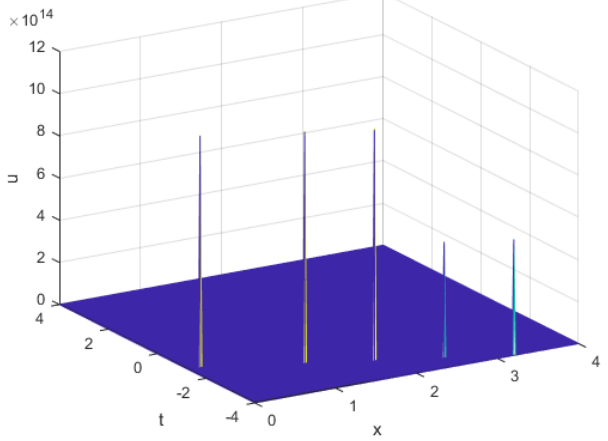

Figure 7: The solution $u = \hat{u}_6$ in eqn. (59) with l_1=1.4, l_2=3.1, l_3=3, μ=1.2, c_1=2.5, c_3=2.6, and $-4 \le x \le 4$, $0 \le t \le 4$.

equation, the Phi-4 equation, the 2D Ginzburg-Landau equation, the generalized-Zakharov system, etc.

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