

New Methods for Solving Integral – Differential Transport Equations

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The integral-differential equations describe many phenomena in different field of mechanical and nuclear engineering, chemistry, astronomy, biology and potential theory. The resolution of boundary problems for these equations are the subject of several papers in which the authors have approached in most cases numerical methods: Monte Carlo, truncated series of Chebyshev polynomials, the fictitious domain method, S_N method and the finite element method [1-4]. An exact solution was found only in the particular cases. Generally, these are obtained with the help of the methods of mathematical analysis, abstract functional analysis and the spectral methods [5].

In recent years, the basic ideas of the homotopy, which is a concept of the topology and differential geometry, were used to obtain the approximate solutions for a wide class of differential, integral and integral–differential equations. We mention here the homotopy perturbation method (HPM) proposed by *Ji-Huan He* in 1998 and the homotopy analysis method (HAM) proposed by *Liao* in 1992. The perturbation methods approximate the solution of given problem by a series of small parameters. Unfortunately, the majority of non-linear problems have no small parameters and an unsuitable choice of these parameters can lead to bad effects. The new homotopy perturbation technique (HPM) embeds a parameter p that ranges from zero to one [6,7]. When the embedding parameter is zero, we get a linear equation and if it is equal to one, we get the original transport equation. This embedding parameter that belongs to the interval $(0, 1)$ can be considered as a small parameter.

The homotopy analysis method (HAM) is one of the most effective methods to obtain the exact and approximate series solutions for the integral–differential equations [8]. Being a great freedom in choosing of the initial guess functions and the auxiliary functions, are proposed for these such forms that correspond to the conditions imposed at the ends of the spatial interval and thus the algorithm becomes fast convergent. In the papers [9-11] is shown that the homotopy perturbation method, variational iteration method and Adomian decomposition method are

only special cases of the homotopy analysis method solutions. Today, a new homotopy method is added to the above. With this we can solve the multipoint boundary problem for a stationary transport equation in the non-homogeneous media. Comparative studies between the solutions obtained with the help of mentioned algorithms for a numerical example with the appropriate exact solutions illustrate the accuracy and computational efficiency of these new methods.

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