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New Near Open Set In Topological Space

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Abstract

The aim of this paper is to introduce new class of near open sets namely, b^*-open set. And studs same of their properties, also we study the relation between this class among this classes. Also, we introduce some topological properties and we shall study same of their properties.

Keywords: b^*-open set, b^*-interior, b^*-closure, b^*-boundary, b^*-neighbourhood.

Introduction

Topological ideas are present not only in almost all areas of today mathematics for example biochemistry [1] information systems [2] and others for more fields of topology applications see ref. [3] and its related links. The subject of topology itself consist of several different branches such as point set topology, algebraic topology and differential topology which have relatively little in common this richness of applications and difference between branches of topology. implied a difficulty to give an accurate definition for topology. In 1937 [4] M.H. Stone introduced the concept of regular open sets. In 1963 [5] Levine introduced the concept of semi open sets. In 1965 [6] Najasted introduced the concept of α-open sets. In 1982 [7,8] Mashhour, Abd El-Monsef and El-Deeb introduced the concept of pre-open sets. In 1983 [9] Abd El-Monsef and et al. introduced the concept of β-open sets. In 1996 [10] Andrijevic introduced the concept of b-open sets, In 2013 [11] Hariwan Z Ibrahim introduced the concept of Bc-Open Set.

Definition 1.1: *A subset A of topological space(X,τ) is called:$A\subseteq int(cl(int(A)))$*

(1) *α-open* if $A\subseteq int(cl(int(A)))$ [6]

(2) preopen if $A\subseteq int(cl(A))$ [8]

(3) semi open if $A\subseteq cl(int(A))$ [5]

(4) Regular open if $A=int(cl(A))$ β [4]

(5) *β-open* (or semi pre open) if, $A\subseteq (cl(int(cl(A)))$ [9-15]

(6) *b-open*. $A\subseteq cl(int(A)\,)$ Ç $int(cl(A)\,)$ [10]

(7) *A* subset *A* of a space *X* is called *Bc-open* if for each $x \in A \in bO(X)$, there exists a closed set *F* such that $x \in F \subset A$ [11]

Remark 1.1: *The complement of a α-open (resp. preopen, semi open, Regular open, β-open and b-open) sets is called α-closed (resp. pre closed, semi closed, Regular closed, β-closed and b-closed) sets. The intersection of all α-closed (resp. pre closed, semi closed, Regular closed, β-closed and b-closed) sets containing A is called the α-closure (resp. pre-closure, semi-closure, Regular closure, β-closure and b-closure) of A and is denoted by αcl(A) (resp. pcl (A), scl(A), Rcl(A), βcl(A) or spcl (A), and bcl(A)).*

The union of all a *α-open* (resp. preopen, semi open, Regular open, *β-open*, and *b-open*) sets contained in *A* is called *α-intrior* (resp. *pre-intrior*, *semi-intrior*, Regular intrior, *β-intrior* and *b-intrior*) of *A* and is denoted by $\alpha int(A)$ (resp. pint (A), sint (A), Rint(A), $\beta int(A)$ or spint (A), and bint (A)). The family of all a *α-open* (resp. *α-closed*, preopen, pre closed, semi open, semi closed, Regular open, Regular closed, *β-open*, *β-closed* and *b-open*, *b-closed*) sets is denoted by $\alpha O(X)$ (resp $\alpha C(X)$, PO(X), PC(X), SO(X), SC(X), RO(A),C(A), $\beta O(A)$, $\beta C(A)$, bO(A) and bC(A)).

Proposition 1.1: *For sub set A,B a spase(X,τ), the following statmant hold :*

(1) $pcl(A)=A\cup cl(int(A)), pint(A)=A\cap int(cl(A))$ [10].

(2) $spcl(A)=A\cup int(cl(int(A))), spint(A)=A\cap cl(int(cl(A)))$ [10].

(3) $pcl(A\cup B)\subseteq pcl(A)\cup pcl(B)$, $spcl(A\cup B)\subseteq spcl(A)\cup spcl(B)$ [12,13].

(4) $pint(A\cap B)\subseteq pint(A)\cap pint(B)$, $pint(A\cup B)\supseteq pint(A)\cup pint(B)$ [14].

(5) $X/(int(A))=cl(X/(A))$, $int(X/A)=X/cl(A)$.

2 b^*-*Open sets*

Definition 2.1: *let (X,τ) be topological space. Then a subset A of Xis said to be*

1. a b^*-*Open set* if A $\subseteq cl(int(cl(A)))\cup int(cl(A))$.

2. a b^*-*closed set* if A $\supseteq int(cl(int(A)))\cap cl(inl(A))$.

The family of all b^*-*Open set* (resp. b^*-*closed set*)subsets of a spase (X,τ) will be as always denoted by $bO^*(X)$ (resp. $bC^*(X)$)

Example 2.1: *Let X={a,b,c,d}with topology τ={X,ϕ,{a},{a,b},{a,c,d}}. Then the classes of b^*-open set and b^*-closed set*

$b^*O(X)=\{X,\Phi,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$, and

$$b^*C(X)=\{X,\Phi,\{b\},\{c\},\{d\},\{c,d\},\{b,d\},\{b,c\},\{b,c,d\}\}$$

Proposition 2.1: *Let A be a sub set of a space (X,τ). Then* (1) Every preopen (resp. *Bc-open*) set is b^*-*open*

Remark 2.1: *The converse of the above propsition is not necessarily true as shown by the following example.*

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Example 2.2: *Let X={a,b,c} with topology τ={X,Φ {a},{b},{a,b}}. Then*

(1) A subset {a,c} of X is b^*-*open* but not preopen.

(2) A subset {a} of X is b^*-*open* but not *Bc-open*.

Remark 2.2: *According to Definition (2.1) and Proposition (2.1), the following diagram holds for a subset A of a space X :*

Lemma 2.1: *Let* (*X*,*τ*) *be topological space. Then the following statements are hold*

(1) The union of b^*-*Open sets* is b^*-*open*

(2) The intersection of b^*-*closed sets* is b^*-*closed*

Proof: (1) let $\{A_i, i\in I\}$ be a family of b^*-*Opensets*. Then $A_i \subseteq cl(int(cl(A_i))) \cup int(cl(A_i))$, hence $U_i A_i \subseteq U_i(cl(int(cl(A_i))) \cup int(cl(A_i))) \subset cl(int(cl(U_i A_i))) \cup int(cl(U_i A_i))$, for all *iI*. Thus $U_i A_i$ is b^*-*Open*

(2) let $\{A_i, i \in I\}$ be a family of b^*-*closed sets*.Then $A_i \supseteq int(cl(int(A_i))) \cap cl(inl(A_i))$, hence $\bigcap_i A_i \supseteq \bigcap_i (int(cl(int(A_i))) \cap cl(inl(A_i))) \supset int(cl(int(\bigcap_i A_i))) \cap cl(inl(\bigcap_i A_i))$. for all *i*∈*I*. Thus $\bigcap A_i$ is b^**closed*

Remark 2.3: *The intersection of any two b^*-open sets is not b^*-open. Let X={a,b,c,d},*

τ={X,Φ {a},{c,d},{a,c,d}}. Then *A*={*a*,*b*} and *B*={*b*, *c*} are b^*-*open* sets, but *A* ∩ *B*={*b*} is not b^*-*open*.

Definition 2.2: *Let* (*X*,*τ*) *be topological space. Then:*

(1) The union of all b^*-*open* sets of *X* contained in *A* is called the b^*-*interior* of *A* and is denoted by b^*-*int*(*A*).

(2) The intersection of all b^*-*closed* sets of *X* contained in *A* is called the b^*-*closure* of *A* and is denoted by b^*-*Cl*(*A*).

Example 2.3: *Let X={a,b,c,d} with topology τ={X,Φ {a},{c},{a,c}}. and A={a,b}, B={a, c} are b^*open then*

$b^* - int(A) = \{a,b\}, b^* - int(B) = \{a,c\}$ and $b^* - cl(A) = \{a,b\}, b^* - cl(B) = X$

Theorem 2.1: *Let* (*X*,*τ*) *be topological space and A ⊂ X, then the following statement are equivalent:*

(1) *A* is a b^*-*open* set,

(2) *A*=*spint*(*A*)∪ *pint* (*A*)

Proof: (1)→(2). Let *A* be a b^*-*open* set. Then $A \subseteq cl(int(cl(A))) \cup int(cl(A))$, hence by proposition (1.1).

$spint(A) \cup pint(A) = (A \cap cl(int(cl(A))) \cup (A \cap int(cl(A))) = A \cap (cl(int(cl(A))) \cup int(cl(A))) = A$

(2)→(1). Suppose that *A*=*spint*(*A*)∪ *pint* (*A*). Than by proposition (1.1)

$A = (A \cap cl(int(cl(A))) \cup (A \cap int(cl(A))) \subset cl(int(cl(A))) \cup int(cl(A))$. Therefore, *A* is a b^*-*open*.

Theorem 2.2: *Let* (*X*,*τ*) *be topological space and A ⊂ X, then the following statement are equivalent:*

(1) *A* is a b^*-*closed* set,

(2) *A*=*spcl*(*A*)∩*pcl*(*A*)

Proof: (1)→(2) Let *A* be a b^*-*closed* set. Then $A \supseteq int(cl(int(A))) \cap cl(int(A))$, hence by proposition(1.1).

$spcl(A) \cap pcl(A) = (A \cup int(cl(int(A))) \cap (A \cup cl(int(A))) = A \cup (int(cl(int(A))) \cap cl(int(A))) = A$

(2)→(1). Suppose that *A*=*spcl*(*A*)∩*pcl*(*A*). Than by proposition (1.1)

$A = (A \cup int(cl(int(A))) \cap (A \cup cl(int(A))) \supset int(cl(int(A))) \cap cl(int(A))$. Therefore, *A* is a b^*-*closed*.

Theorem 2.3: *Let A be a supset of a space* (*X*,*τ*). *Then*

(1) $b^* - cl(A) = spcl(A) \cap pcl(A)$,

(2) $b^* - int(A) = spint(A) \cup pint(A)$.

Proof: (1) It is easy to see that $b^* - cl(A) \subseteq spcl(A) \cap pcl(A)$. Also $spcl(A) \cap pcl(A) = (A \cup int(cl(int(A))) \cap (A \cup cl(int(A))) = A \cup (int(cl(int(A))) \cap cl(int(A)))$. But, b^*-*cl*(*A*) is b^*-*closed*, hence *x* $A \cup (int(cl(int(A))) \cap cl(int(A))) \subset A \cup b^* - cl(A) = b^* - cl(A) = b^*$ Thus $A \cup (int(cl(int(A))) \cap cl(int(A))) \subset A \cup b^* - cl(A) = b^* - cl(A) = b^*$ there for, $spcl(A) \cap pcl(A) \subset b^* - cl(A)$. So $b^* - int(A) \subseteq spint(A) \cup pint(A)$.

(2) It is easy to see that $b^* - int(A) \subseteq spint(A) \cup pint(A)$. Also $spint(A) \cup pint(A) = (A \cap cl(int(cl(A))) \cup (A \cap int(cl(A))) = A \cap (cl(int(cl(A))) \cup int(cl(A)))$. But,$b^*$-*int*(*A*)is$b^*$-*open*,hence $b^* - int(A) \subset cl(int(cl(b^* - int(A)))) \cup int(cl(b^* - int(A))) \subset (cl(int(cl(A))) \cup int(cl(A)))$ .Thus $A \cap (cl(int(cl(A))) \cup int(cl(A))) \subset A \cap b^* - int(A) = b^* - int(A)$ there for $spint(A) \cup pint(A) \subset b^* - int(A)$.

So $b - int(A) = spint(A) \cup pint(A)$.

Theorem 2.4: *Let A be a supset of a space (X,τ). Then*

(1) A is a b^*-*open* set if and only if $A=b^*\text{-}int(A)$

(2) A is a b^*-*closed* set if and only if $A=b^*\text{-}cl\ (A)$

Proof: (1) Let A be a b^*-*open* set. Then by theorem (2.1), $A=spint(A)\cup\cup\ pint\ (A)$ and by theorem (2.3), we have $A=b^*\text{-}int(A)$ Conversely, let $A=b^*\text{-}int(A)$ Then by theorem(2.3), $A=spint(A)\cup\ pint\ (A)$ and by theorem (2.1), A is b^*-*open* $A=spint(A)\cup\ pint\ (A)$

(2) Let A be a b^*-*closed* set. Then by theorem (2.1), $A=spcl(A)\cap pcl(A)$and by theorem (2.3), we have $A=b^*\text{-}cl\ (A)$ Conversely, let $A=b^*\text{-}cl\ (A)$ Then by theorem (2.3), $A=spcl(A)\cap pcl(A)$ and by theorem (2.1), A is b^*-*closed*

Theorem 2.5: *Let A and B be a subsets of a space(X,τ). Then the following are hold*

(1) $b^*-cl(X\setminus A)=X\setminus b^*-int(A)$.

(2) $b^*-int(X\setminus A)=X\setminus b^*-cl(A)$.

(3) If $A\subseteq B$, then $x\in b^*\text{-}cl\ (A)$

(4) $x\in b^*\text{-}cl\ (A)$if and only if there exists a b^*-*open* set U and $x\in U$ such that $U\cap A\neq\phi$.

(5) $x\in b^*\text{-}int(A)$ if and only if there exists a b^*-*open* set G and $x\in G$ such that $x\in G\subseteq A$

(6) $b^*-cl(b^*-cl(A))=b^*-cl(A)$ and $b^*-int(b^*-int(A))=b^*-int(A)$.

(7) $b^*-cl(A)\cup b^*-cl(B)\subseteq b^*-cl(A\cup B)$ *and* $b^*-int(A)\cup b^*-int(B)\subseteq b^*-int(A\cup B)$

(8) $b^*-int(A\cap B)\subseteq b^*-int(A)\cap b^*-int(B)$, $b^*-cl(A\cap B)\subseteq b^*-cl(A)\cap b^*-cl(B)$

Proof: (1) Since $(X\setminus A)\subseteq X$, by theorem (2.4) $b^*-cl(X\setminus A)=spcl(X\setminus A)\cap pcl(X\setminus A)$ and by proposition (1.1) $b^*-cl(X\setminus A)=(X\setminus spint(A))\cap(X\setminus pint(A))=X\setminus(spint(A)\cup pint(A))$, hence by theorem (2.4), $b^*-cl(X\setminus A)=X\setminus b^*-int(A)$

(2) Since $(X\setminus A)\subseteq X$, by theorem (2.4) $b^*-int(X\setminus A)=spint(X\setminus A)\cup pint(X\setminus A)$ and by proposition (1.1) $b^*-int(X\setminus A)=(X\setminus spcl(A))\cup(X\setminus pcl(A))=X\setminus(spcl(A)\cap pcl(A))$, hence by theorem (2.4), $b^*-int(X\setminus A)=X\setminus b^*-cl(A)$

(3) Since, $b^*-cl(A)=spcl(A)\cap pcl(A)$ and $A\subseteq B$, $b^*-cl(A)=spcl(A)\cap pcl(A)\subseteq spcl(B)\cap pcl(B)=b^*-cl(B)$

(4) Let $x\notin b^*\text{-}cl(A)$then $x\notin\bigcap F$ where F is b^*-*closed* with $A\subset F$, so $x\notin X\setminus\bigcap F$ and $X\setminus\bigcap F$ is a b^*-*open* set containing x and hence $(X\setminus\bigcap F)\cap A\subseteq(X\setminus\bigcap F)\cap(\bigcap F)=\phi$. Converly, suppose that exists a b^*-*open* set containing x with $A\cap U=\phi$.

Then $A\subseteq X/U$ and X/U is a b^*-*closed*. Hence $x\notin b^*\text{-}cl(A)$.

(5) Necessity. Let $x\in b^*\text{-}int(A)$. Then $x\in\cup\{G:G\ is\ b^*-open\ \ G\subseteq A\}$ and hence there exists b^*-*open* set G such that $x\in G\subseteq A$ sufficiency. Let G be a b^*-*open* set such that $x\in G\subseteq A$. Then $A=\cup\{G: x\in G\}$ which is the union of b^*-*open* set. There for, $x\notin b^*\text{-}cl(A)$.

(6) Since $b^*-cl(b^*-cl(A))=spcl(b^*-cl(A))\cap pcl(b^*-cl(A))$. by theorem (2.4).

$spcl(spcl(A)\cap pcl(A))\cap pcl(spcl(A)\cap pcl(A))\subseteq(spcl(A)\cap spcl(pcl(A)))\cap pcl(spcl(A)\cap pcl(A))=spcl(A)\cap pcl(A)=b^*-cl(A)$ hence:

$b^*-cl(b^*-cl(A))\subseteq b^*-cl(A)$. But, $b^*-cl(A)\subseteq b^*-cl(b^*-cl(A))$, there for, $b^*-cl(b^*-cl(A))=b^*-cl(A)$

(7) Since $A\subseteq A\cup B$ and $B\subseteq A\cup B$ we have $b^*\text{-}cl\ (A)\subseteq b^*\text{-}cl(A\cup B)$ and $b^*\text{-}cl\ (B)\subseteq b^*\text{-}cl(A\cup B)$. There for $b^*\text{-}cl\ (A)\cup b^*\text{-}cl\ (B)\subseteq b^*\text{-}cl(A\cup B)$ $A\subseteq(A\cup B)$ and and $B\subseteq(A\cup B)$ we have $b^*\text{-}int(A)\subseteq b^*\text{-}int(A\cup B)$ and $b^*\text{-}int(B)\subseteq b^*\text{-}int(A\cup B)$. There for $b^*\text{-}int(A)\cup b^*\text{-}int(B)\subseteq b^*\text{-}int(A\cup B)$.

(8) Since $A\supseteq A\cap B$ and $B\supseteq A\cap B$ we have $b^*\text{-}cl\ (A)\supseteq b^*\text{-}cl(A\cap B)$ and $b^*\text{-}cl\ (B)\supseteq b^*\text{-}cl(A\cap B)$, $b^*\text{-}cl\ (B)\supseteq b^*\text{-}\ cl\ ((A\cap B)$. There for for $b^*\text{-}cl\ (A)\cap b^*\text{-}cl\ (B)$ $b^*\text{-}cl(A\cap B)$ and $A\ (A\cap B)$ and $B\supseteq(A\cap B)$ we have $b^*\text{-}int(A)\supseteq b^*\text{-}int(A\cap B)$ and $b^*\text{-}int(B)\supseteq b^*\text{-}int(A\cap B)$.There for $b^*\text{-}int(A)\cap b^*\text{-}int(B)\supseteq b^*\text{-}int(A\cap B)$.

Remark 2.4: *The inclusion relation in part (6),(7) of the above theorem cannot be replaced by equality as shown by the following example.*

Example 2.4: *Let X={a,b,c,d} with topologyτ={X,Φ {a},{b},{c}{a,b},{a,c}{b,c}{b,d}{a,b,c}{a,b,d}{b,c,d}}.*

Then $(A\cup B)=\{a,b,d\}$

(1) If A={a,b},B={d} and $(A\cup B)=\{a,b,d\}$, then $b^*\text{-}int(A)=A$ $b^*\text{-}int(B)=\phi$ and $b^*\text{-}int(A\cup B)=\{a,b,c\}$ So, $b^*\text{-}int(A\cup B)\nsubseteq b^*\text{-}int(A)\cup b^*\text{-}int(B)$

(2) If C={b}, B={d} and $(B\cap C)=\phi$, then $b^*\text{-}cl\ (C)=\{b,d\}$ $b^*\text{-}cl\ (B)=B$ and $b^*\text{-}cl(B\cap C)=\phi$, there for, $b^*\text{-}cl\ (B)\cap b^*\text{-}cl\ (C)\nsubseteq b^*\text{-}cl(B\cap C)$

Example 2.5: *Let X={a,b,c,d} with topology τ={X,Φ {a},{b,c},{a,b,c}} then*

(1) If A={a,b}, B={a,c} and$(A\cup B)=\{a,b,c\}$, then $b^*\text{-}cl\ (A)=A$ $b^*\text{-}cl\ (B)=B$ and $b^*\text{-}cl(A\cup B)=X$ So, $b^*\text{-}cl(A\cup B)\nsubseteq b^*\text{-}cl(A)\cup b^*\text{-}cl(B)$

(2) If C={a,d}, D={b,d} and $(B\cap C)=\{d\}$, then $b^*\text{-}int(C)=C$ $b^*\text{-}int(D)=D$ and $b^*\text{-}int(C\cap D)=\phi$ $b^*\text{-}int(C)\cup b^*\text{-}int\ (D)$ $b^*\text{-}int(C\cap D)$

3 Some Topological Operations.

Definition 3.1: *Let (X,τ) be a space and $A \subset X$. Then the b^*-boundary of A (briefly, b^*-b(A)) is given by b^*-b(A))=b^*-cl (A)) ∩ b^*-cl(X/A)*

Example 3.1: *From Example (2.1) we have A={a} B={a,b} C={a,b,d} then b^*-b(A))={b,c,d}, b^*-b(B))={c,d}and b^*-b(C))={c}*

Remark 3.1: *For any subset A of a space (X,τ) we have b^*-b(A)⊆ b(A) and b^*-b(A)⊆ p-b(A).*

The inclusion of the above remark can be replaced as shows in the following example.

Example 3.2: *From Example (2.3) and A={a,b} then b^*-b(A)=ϕ, p-b(A)={b,d} we have p-b(A)$\nsubseteq$ b^*-b(A).*

Theorem 3.1: *If A is a sub sets of a space (X,τ), then the following statement are hold:*

(1) $b^* - b(A)) = b^* - b(X \setminus A))$.

(2) $b^* - b(A)) = b^* - cl(A)) \setminus b^* - int(A)$.

(3) $b^* - b(A)) \cap b^* - int(A) = \Phi$.

(4) $b^* - b(A)) \cup b^* - int(A) = b^* - cl(A)$.

Proof: (1) Since $b^* - b(A)) = b^* - cl(A)) \cap b^* - cl(X \setminus A) = b^* - b(X \setminus A)) = b^* - cl(X \setminus A)) \cap b^* - cl(A)$

(2)Since, $b^* - b(A)) = b^* - cl(A)) \cap b^* - cl(X \setminus A) = b^* - cl(A)) \cap (X \setminus b^* - intl(A)) = (b^* - cl(A) \cap X) \setminus (b^* - cl(A) \cap X) \setminus (b^* - cl(A) \cap b^* - intl(A)) = b^* - cl(A) \setminus b^* - int(A)$.

(3) Also, by using(2) $b^* - b(A)) \cap b^* - int(A) = (b^* - cl(A) \setminus b^* - int(A)) \cap b^* - int(A) = (b^* - cl(A) \cap b^* - int(A)) \setminus b^* - int(A) = b^* - int(A) \setminus b^* - int(A) = \Phi$.

(4) By using(3) $b^* - b(A)) \cup b^* - int(A) = (b^* - cl(A) \setminus b^* - int(A)) \cup b^* - int(A) = b^* - cl(A)$.

Theorem 3.2: *If A is a sub sets of a space (X,τ), then the following*

statement are holds:

(1) *A* is a b^*-*open* set if and only if $A \cap b^*$-$b(A)=\Phi$

(2) *A* is a b^*-*closed* set if and only if b^*-$b(A) \subset A$

(3) *A* is a b^*-*clopen* set if and only if b^*-$b(A)=\Phi$.

Proof: (1) let A is a b^*-*open* set. Then $A=b^*$- $int(A)$ hence by theorem (3.1)

$A \cap b^*$-$b(A)=b^*$-$int(A) \cap b^*$-$b(A)=\Phi$ Conversely, let $A \cap b^*$-$b(A)=\Phi$ then by theorem (3.1),

$A \cap (b^* - cl(A) \setminus b^* - int(A)) = (A \cap (b^* - cl(A)) \setminus (A \cap b^* - int(A)) = A / b^* - int(A) = \Phi$. so, $A=b^*$-$int(A)$ and hence A is b^*-*open*.

(2) let A is a b^*-*closed* set. Then $A=b^*$-$cl(A)$, by thearom (3.1), but $b^* - b(A) = (b^* - cl(A) \setminus b^* - int(A)) = A \setminus b^* - int(A)$, then b^*-$b(A) \subset A$ Conversely let b^*-$b(A) \subset A$. Then by theorem(3.1), $b^* - cl(A) = b^* - b(A)) \cup b^* - int(A) \subset A \cup b^* - int(A) = A$ thus b^*-$cl(A) \subset A$ and $A \subset b^*$-$cl(A)$ there for, $A=b^*$-$cl(A)$,

(3) let *A* is a b^*-*clopen* set. Then $A=b^*$-$int(A)$, and $A=b^*$-$cl(A)$, hence by thearom (3.1), $b^* - b(A) = (b^* - cl(A) \setminus b^* - int(A)) = A \setminus A = \Phi$ Conversely, suppose that b^*-$b(A)=\Phi$.Then $b^* - b(A) = (b^* - cl(A) \setminus b^* - int(A)) = \Phi$, and hence, *A* is a b^*-*clopen* set.

Definition 3.2: *Let (X,τ) be a space and $A \subset X$.Then the setX\(b^*-cl(A)) is called the b^*-exterior of A and is denoted by b^*-ext(A). Each point p∈ X is called an b^*-exterior point of A, if it is a b^*-interior point of X\A.*

Example 3.3: *let X={a,b,c,d} with topology τ={X,Φ {a},{b},{a,b},{ b,c,d}}*

If *A={a} B={a,c} C={ b,c,d}* then we have

b^*-ext(A)={ b,c,d}, b^*-ext(B))={ b, d} and b^*-b(C))={a}

Remark 3.2: *For any topology space (X,τ) and $A \subset X$, we have ext(A)⊆p-ext(A)⊆b^*-ext(A)*

Proof: Since b^*-*cl* (*A*) ⊆ *cl*(*A*), then $X \setminus cl(A) \subseteq X \setminus b^* - cl(A)$ and $int(X \setminus A \subseteq b^* - int(X \setminus A)$ i.e $ext(A) \subseteq b^* - ext(A)$. Since $ext(A) \subseteq p - ext(A)$, then we have $p - ext(A) \subseteq b^* - ext(A)$. This implies that the relation hold.

Example 3.4: *Let X={a,b,c,d} with topology τ={X,Φ {c},{d}{c,d},{b,c}{b,c,d}}. And A={b,d}, B={c} we have* $b^* - ext(A) = \{a,c\}, p - ext(A) = \{c\}, p - ext(B) = \{b,d\}, ext(B) = \{d\}$

Theorem 3.3: *If A and B is two sub sets of aspace (X,τ), then the following statements are hold: ext(A) ∪ b^**

(1) b^*-$ext(A)=b^*$-$int(A)$.

(2) b^*-$ext(A)$ is b^*-*open*

(3) b^*-$ext(A) \cup b^*$-$int(A)$=.

(4) b^*-$ext(A) \cap b^*$-$b(A)$=.

(5) b^*-$ext(A) \cup b^*$-b^*-$b(A)$=$b^*\ cl(X \setminus A)$.

(6) $\{b^* - int(A), b^* - b(A) and\ b^* - ext(A)\}$ from apartition of X.

(7) If $A \subseteq B$, then $b^* - ext(B) \subseteq b^* - ext(A)$

(8) $b^* - ext(A \cup B) \subseteq b^* - ext(A) \cup b^* - ext(B)$.

(9) $b^* - ext(A \cap B) \supseteq b^* - ext(A) \cap b^* - ext(B)$.

(10) $b^* - ext(X) = \Phi\ and\ b^* - ext(\phi) = X$.

Proof: (1) by Definition (3.2) $b^* - ext(A) = X \setminus b^* - cl(A) = b^* - int(X \setminus A)$.

(2) From (1) $b^* - ext(A) = b^* - int(X \setminus A)$. Since b^*-$int(A)$ is the union of all b^*-*open* sets of X contained in A thus b^*-$ext(A)$ is b^*-*open*

(3) Since $b^* - ext(A) \cap b^* - int(A) = X \setminus b^* - cl(A) \cap b^* - int(A) = b^* - int(X \setminus A) \cap b^* - int(A) = \phi$

(4) By theorem (3.1), $b^* - ext(A) \cap b^* - b(A) = b^* - int(X \setminus A) \cap b^* - b(X \setminus A) = \Phi$.

(5) Also, by theorem (3.1)

$b^* - ext(A) \cup b^* - b(A) = b^* - int(X \setminus A) \cup b^* - b(X \setminus A) = b^* - cl(X \setminus A)$.

(6) From (3),(4) we have $b^* - ext(A) \cap b^* - int(A) = \Phi$ and $b^* - ext(A) \cap b^* - b(A) = \Phi$. Then by theorem (3.1) then $b^* - b(A)) \cap b^* - int(A) = \Phi$. Now, we need to prove that $b^* - int(A) \cup b^* - b(A) \cup b^* - ext(A) = X$ hence from(5) $b^* - ext(A) \cup b^* - b(A) = b^* - cl(X \setminus A)$ then $b^* - int(A) \cup b^* - cl(X \setminus A) = b^* - int(A) \cup X \setminus b^* - int(A) = X$.

(7) let $A \subseteq B$ then $(b^* - cl(A)) \subseteq (b^* - cl(B))$ and hence

$X \setminus (b^* - cl(B)) \subseteq X \setminus (b^* - cl(A))$. So $b^* - ext(B) \subseteq b^* - ext(A)$.

(8) $b^* - ext(A \cup B) = X \setminus (b^* - cl(A \cup B)) \subseteq X \setminus (b^* - cl(A) \cup (b^* - cl(B)) == (X \setminus (b^* - cl(A))) \cap (X \setminus (b^* - cl(B))) = b^* - ext(A) \cap b^* - ext(B) \subseteq b^* - ext(A) \cup b^* - ext(B)$.

(9) $b^* - ext(A \cap B) = X \setminus (b^* - cl(A \cap B)) \supset X \setminus (b^* - cl(A) \cap (b^* - cl(B)) = (X \setminus (b^* - cl(A))) \cup (X \setminus (b^* - cl(B))) = b^* - ext(A) \cup b^* - ext(B) \supseteq b^* - ext(A) \cap b^* - ext(B)$.

(10) $b^* - ext(X) = X \setminus (b^* - cl(X)) = X \setminus X = \phi$ and $b^* - ext(\phi) = X \setminus (b^* - cl(\phi)) = X \setminus \phi = X$

Remark 3.3: *The inclusion relation in part (5),(6) of the above theorem cannot by replaced by equality as is show by the following example.*

Example 3.5: *From Example (2.1) we have A={b,c}andB={a,c} then* b^*-$ext(A)$={a,b},b^*-$ext(B)$=Φ *but* $b^* - ext(A \cup B) = \Phi$. *Therefor,* $b^* - ext(A) \cup b^* - ext(B) \nsubseteq b^* - ext(A \cup B)$. *Also.* $b^* - ext(A \cap B) = \{a,b,d\}$, hence $b^* - ext(A \cap B) \nsubseteq b^* - ext(A) \cap b^* - ext(B)$.

Definition 3.3: *If A is a subset of a space* (*X*,*τ*), *then a point pX is called a* b^*-*limit point of a set A ⊂ X if every* b^*-*open set G∈X containing p contains a point of A other than p. The set of all* b^*- *limit point of A is called an* b^*-*derived set of A and is denoted by* b^*-$d(A)$

Example 3.6: *let X={a,b,c,d} with topology τ={X,Φ {a},{c,d},{a,c,d}} and If A={a,d} B={a,c,d} the* b^*-$d(A)$)={Φ}, *and* b^*-$d(B)$)={b}.

Theorem 3.4: *If A and B is two sub sets of aspace* (*X*,*τ*), *then the following* statements are hold:

(1) $If A \subset B, then\ b^* - d(A) \subset b^* - d(B)$.

(2) *A* is a b^*-*closed* set if and only if it cantains each of its b^*-limit point.

(3) $b^* - cl(A) = A \cup b^* - d(A)$.

(4) $b^* - d(A \cup B) \supset b^* - d(A) \cup b^* - d(B)$

(5) $b^* - d(A \cap B) \subset b^* - d(A) \cap b^* - d(B)$

Proof: (1) By definition (3.3), we have $p \in b^*$-$d(A)$ if and only if $G \cap (A \setminus \{P\}) \neq \phi$, for every b^*-open set G containing p. But $A \subset B$, then $G \cap (B \setminus \{P\}) \neq \phi$, for every b^*-open set G containing p. Hence, so $p \in b^*$-$d(B)$

There for b^*-$d(A) \subset b^*$-$d(B)$

(2) Let A be b^*-*closed* set and $p \notin A$ *then* $p \in (X/A)$ which is b^*-*open*, hence there exists b^*-*open*($X \setminus A$) such that

$(X/A) \cap A = \phi$ so $p \notin b^*$-$d(A)$, there for b^*-$d(A) \subset A$. Conversely, suppose that b^*-$d(A) \subset A$ and $p \notin A$. Then $p \notin b^*$-$d(A)$, hence there exists b^*-*open* set G containing p such that $G \cap A = \phi$ and hence

$X \setminus A = \bigcup_{p \in A} \{G, G\ is\ b^*$ open there for A is b^*-*closed*

(3) Since, $b^* - d(A) \subset b^* - cl(A)$ and $A \subset b^* - cl(A)$ $b^* - d(A) \cup A \subset b^* - cl(A)$.

Conversely, suppose that $p\notin b^*\text{-}d(A)\cup A$ Then $p\notin b^*\text{-}d(A)$, $p\notin A$ and hence there exists b^*-*open* set G containing p such that $G\cap A=\phi$. Thus $p\notin b^*\text{-}cl(A)$ which implies thet $b^*-cl(A)\subset b^*-d(A)\cup A$, there for, $b^*-cl(A)=b^*-d(A)\cup A$.

(5) Since $A\supseteq A\cap B$ and $B\supseteq A\cap B$ we have $b^*\text{-}d(A)\supseteq b^*\text{-}d(A\cap B)$ and $b^*\text{-}d(B)\supseteq b^*\text{-}d(A\cap B)$. There for $b^*\text{-}d(A)\cap b^*\text{-}d(B)\supseteq b^*\text{-}d(A\cap B)$.

Definition 3.4: *Let (X,) be a space and $A\subseteq X$. Then the b^*-border of A (briefly, $b^*\text{-}Bd(A)$) is given by* $b^*\text{-}Bd(A))=A\backslash b^*\text{-int}(A)$.

Example 3.7: *Let $X=\{a,b,c,d\}$ with topologies $\tau=\{X,,\{d\},\{a,d\},\{c,d\},\{b,d\},\{b,c,d\},\{a,b,d\},\{a,c,d\}\}$.* If $A=\{a,c\}$, $B=\{c,d\}$ and $C=\{b,c\}$, then $b^*\text{-}Bd(A)=\{a,c\}$, $b^*\text{-}Bd(B)=\phi$ and $b^*\text{-}Bd(c)=\{c\}$.

Theorem 3.5: *For a subset A of a space and X, the following statements are hold:*

(1) $A=b^*-int(A)\cup b^*-Bd(A))$,

(2) $b^*-int(A)\cap b^*-Bd(A))=\phi$,

(3) $b^*-Bd(X))=b^*-Bd(\phi))=\phi$,

(4) $b^*-Bd(b^*-int(A))=\phi$,

(5) $b^*-int(b^*-Bd(A))=\phi$,

(6) $b^*-Bd(b^*-Bd(A))=b^*-Bd(A))$,

(7) $b^*-Bd(A))=A\cap b^*-cl(X\setminus A)$,

(8) $b^*-Bd(A))=b^*-d(X\setminus A)$,

Proof: (1) $b^*-int(A)\cup b^*-Bd(A))=b^*-int(A)\cup(A\setminus b^*-int(A))=(b^*-int(A)\cup A)\setminus(b^*-int(A)\cup b^*-int(A))=A\setminus b^*-int(A)=A,$

(2) $b^*-int(A)\cap b^*-Bd(A))=b^*-int(A)\cap(A\setminus b^*-int(A)=(b^*-int(A)\cap A)\setminus(b^*-int(A)\cap b^*-int(A))$ $=b^*-int(A)\setminus b^*-int(A)=\phi$

(3) $b^*-Bd(X))-X\setminus b^*-int(X)=X\setminus X=\phi$ and $b^*-Bd(\phi))=\phi\setminus b^*-int(\phi)=\phi\setminus\phi=\phi$.

(4) $b^*-Bd(b^*-int(A))=b^*-int(A)\setminus b^*-int(A)=\phi$.

(5)Since, $b^*-int(b^*-Bd(A))=b^*-int(A\setminus b^*-int(A))=b^*-int(A)\setminus b^*-int(b^*-int(A))=b^*-int(A)\setminus b^*-int(A)=\phi$

(6) Since, $b^*-Bd(b^*-Bd(A))=b^*-Bd(A)\setminus b^*-int(b^*-Bd(A))=b^*-Bd(A)\setminus\phi=b^*-Bd(A)$,

(7) Also, from Theorem (2.5), $b^*-Bd(A))=A\setminus b^*-int(A))=A\setminus(X\setminus b^*-cl(A))=A\cap b^*-cl(X\setminus A)$.

(8) Further, from Theorem 2.3.1 $b^*-Bd(A))=A\setminus b^*-int(A))=A\setminus(A\setminus b^*-d(A))=b^*-d(X\setminus A)$.

Theorem 3.6: *For a subset A of a space and X, the following statements are equivalent*

(1) A is b^*-*open*,

(2) $A=b^*\text{-}int(A)$,

(3) $b^*\text{-}Bd(A)=\phi$.

Proof: (1)→(2)Obvious from Theorem (2.4).

(2)→(3). Suppose that $A=b^*\text{-}int(A)$. Then by Definition (3.4),

$b^*\text{-}Bd(A)=b^*\text{-}int(A)\backslash b^*\text{-}int(A)=\phi$

(3)→(1). Let $b^*\text{-}Bd(A)=\phi$. Then by Definition (3.4), $A\backslash b^*\text{-}int(A)=\phi$ and hence $A=b^*\text{-}int(A)$.

Definition 3.5: *A subset N of a space (X,τ) is called a b^*-neighbourhood (briefly, b^*-nbd.) of a point $p\in X$ if there exists a b^*-open set W such that $p\in X\subseteq N$. The class of all b^*-nbds of pX is called the b^*-neighbourhood system of p and denoted by b^*-Np.*

Example 3.8: *Let $X=\{a,b,c,d\}$ with topology $\tau=\{X,\Phi,\{a\},\{c\},\{a,b\},\{a,c\},\{a,b,c\}\}$*, then $b^*\text{-}N_a=\{a,c\}$.

Remark 3.4: *For any topology spase (X,) and for each $x\in X$ we have $N_x\subseteq p\text{-}N_x\subseteq b^*\text{-}N_x$.*

Example 3.9: *From Example (2.2). We have $\{a,c\}\in b^*\text{-}N_c$ but it is not in $p\text{-}N_c$ and not in N_c.*

Theorem 3.7: *A subset G of a space X is b^*-open if and only if it is b^*-nbd, for every point $p\in G$.*

Proof: Necessity. Let G be an b^*-*open* set.Then G is a b^*-*nbd*. for each $p\in G$.

Sufficiency. Let G be a b^*-*nbd*, for each $p\in G$. Then there exists a b^*-*open* set W containing p such that $p\in W\subseteq G$, so $G=\cup\{p{:}p\in W\}$. Therefore, G is b^*-*open*.

Theorem 3.8: *For a space (X,τ). If b^*-Np is the b^*-nbd. systems of a point $p\in X$, then the following statements are hold:*

(1) b^*-N_p is not empty and p belongs to each member of b^*-N_p

(2) Each superset of the members of b^*-N_p belongs to b^*-N_p,

(3) Each member $N\in$ b^*-Np is a superset of the member $W\in b^*$-Np, where W is b^*-*nbd* of each point $p\in W$.

Proof: (1) Since X is a b^*-*open* set containing p, then $X\in b^*$-Np. So, b^*-$Np\neq\phi$. Also, if Nb^*-Np, then there exists a b^*-*open* set G such that $p\in G\subseteq N$. Therefore, p belongs to each member of b^*-Np.

(2) Let M be a superset of $N\in$ b^*-N_p, then there exists a b^*-*open* set G such that $p\in G\subseteq N\subseteq M$ which implies $p\in G\subseteq M$ and hence, M is a b^*-*neighbourhood* of p. Therefore, $M\in$ b^*-N_p

(3) Let N be a b^*-*neighbourhood* of $p\in X$, then there exists a b^*-*open* set W such that $p\in W\subseteq N$. Then by Theorem 2.5.1, W is a b^*-*neighbourhood* of each of its points.

Definition 3.6: *For a space (X,τ), a subset A of X is said to be b^*-dense in X if and only if b^*-cl (A)=X The family of all b^*-dense sets in (X,τ) will be denoted by b^*-D(X,τ)*

Example 3.10: *Let X={a,b,c} with topology τ={X,Φ, {a,b} If, A={a,b}, and b^*-cl (A)=X than b^*-dense in X.*

Remark 3.5: *Every b^*-dense set in a space (X,τ) is dense in (X,τ) by the fact that b^*-cl (A) ⊆ cl (A), while the converse may not be true.*

Example 3.11: *Let X={a,b,c,d} with topology τ={X,φ,{a,c},{ b,d},{a,c,d}}. If A={ b,c,d }, then cl(A)=X but b^*-Cl(A)={b,c,d} Therefore, A is dense in X but not b^*-dense in X.*

Theorem 3.9: *For a space (X,τ) and $E\subseteq X$, the following statements are equivalent:*

(1) E is b^*-*dense* in X

(2) If F is an b^*-*closed* set in X containing E, then, $F=X$

(3) M-$int(X/E)=\phi$.

Proof: (1)→(2). Let E be an b^*- *dense* set of X. Then b^*-$Cl(E)=X$. But F is an b^*-*closed* set contains E, then b^*-$Cl(E)\subseteq F$ and therefore $F=X$.

(2)→(3). Since b^*-$Cl(E)$ is an b^*-*closed* set contains E, By (2) we have b^*-$Cl(E)=X$. Hence $\phi = X\setminus b^* p-cl(E)=b^*-int(X\setminus E)$.

(3)→(1). Since b^*-$int(X/E)=\phi$. Then b^*-$Cl(E)=X$ Hence E is b^*-*dense* in X.

Proposition 3.1: *For a space (X,τ), if $E\in b^*$-D(X,τ), then the following statements are hold:*

(1) $b^*-b(E)=b^*-cl(X\setminus E)$,

(2) $b^*-ext(E)=\phi$.

Proof: (1) From Definition (3.1), we have $b^*-b(E)=b^*-cl(E)\cap b^*-cl(X\setminus E)$ and since $E\in b^*$-$D(X,\tau)$, then $b^*-b(E)=b^*-cl(X\setminus E)$

(2) Also, by From Definition (3.2), $b^*-ext(E)=X\setminus b^*-cl(E)$ but $E\in b^*$-$D(X,\tau)$, then b^*-$ext(E)=\phi$.

Definition 3.7: *For a space (X,τ), $A\subseteq X$ is called:*

(1) b^*- nowhere dense if $int(A)\subseteq b^*$-int (b^*-$cl(A)$)=ϕ

(2) b^*- residual if b^*-$cl(X\setminus A)=X$ or b^*-int $(A)=\phi$

b^*- nowhere dense is b^*-*iresidual* from the fact that b^*-$int(A)\subseteq$ b^*-int (b^*-$cl(A)$) for every $A\subseteq X$

Example 3.12: *Let X={a,b,c}with topology τ={X,Φ,{a},{a,b}} and A={b} than b^*-int (b^*-cl(A))=φ. and b^*-int (A)=φ so A is b^*- nowhere dense and b^*-residual.*

Proposition 3.2: *A subset A of a space (X,τ), $A\subseteq X$ is b^*-nowhere dense of X if $A\subseteq b^*$-Cl(X/b^*-cl (A)).*

Proof: Let A is b^*- nowhere dense then b^*-int (b^*-$cl(A)$)=ϕ.

Hence $X\setminus b^*-int(b^*-cl(A))=b^*-cl(X\setminus b^*-cl(A))=b^*-cl(b^*-int(X\setminus A))=X\supseteq A$

Theorem 3.10: *The b^*-boundary of each b^*-open (resp. b^*-closed) set is b^*-nowhere dense.*

Proof: Let $A\in b^*O(X)$ then

$$b^*-int(b^*-cl(b^*-b(A)))=b^*-int(b^*-cl(b^*-cl(A)\cap b^*-cl(X\setminus A))=b^*-$$
$$int(b^*-cl(b^*-cl(b^*-int(A)\cap(X\setminus b^* int(A))))\subseteq b^*-int(b^*-cl(b^*-int(A)\cap(X\setminus b^*-int(A))\subseteq b^*-int(b^*$$
$$-cl(b^*-int(A)\cap(X\setminus b^*-cl(b^*-int(A)))))\subseteq b^*-cl(b^*-int(A)\cap(X\setminus b^*-cl(b^*-int(A))))=\phi$$

Also if $A\in b^*C(X)$ Then

$b^* - int(b^* - cl(b^* - b(A)) = b^* - int(b^* - cl(b^* - cl(A) \cap b^* - cl(X \setminus A)) = b^* -$
$int(b^* - cl(A) \cap X \setminus b^* - cl(b^* - int(b^* - cl(A))) \subseteq b^* - int(b^* - cl(A) \cap (X \setminus b^* - cl(b^* - int(b^* - cl(A))) \subseteq b^* - int(b^* -$
$cl(A)) \cap (X \setminus b^* - int(b^* - cl(A))) = \phi$

Proposition 3.3: *For a space (X,τ), $A \subseteq X$, then the sets $A\cap b^*\text{-}cl(X\setminus A)$ and $b^*\text{-}cl(A)\cap (X\setminus A)$ are b^*- residual.*

Proof: Since

$b^* - int(A \cap b^* - cl(X \setminus A)) \subseteq b^* - int(A) \cap b^* - int(b^* - cl(X \setminus A)) \subseteq b^* - int(A) \cap b^* -$
$cl(X \setminus A) = b^* - int(A) \cap (X \setminus b^* - int(A)) = \phi$,

Then $A\cap b^*\text{-}cl(X\setminus A)$ is residual. Similarly

$b^* - int(b^* - cl(A) \cap (X \setminus A)) \subseteq b^* - int(b^* - cl(A)) \cap b^* - int(X \setminus A) = b^* - cl(A) \cap (X \setminus b^* - cl(A)) = \phi$, and hence $b^*\text{-}c(A)\cap(X\setminus A)$ is b^*- residual.

Theorem 3.11: *The b^*-boundary of any set contains the union of two b^*-residual sets.*

Proof: Let (X,τ) be a space and $A \subseteq X$. Then by Proposition (3.3), we have

$(A \cap b^* - cl(X \setminus A)) \cup (b^* - cl(A) \cap (X \setminus A)) = ((A \cap b^* - cl(X \setminus A) \cup b^* - cl(A)) \cap ((A \cup b^* -$
$cl(X \setminus A)) \cup (X \setminus A) = ((A \cup b^* - cl(A)) \cap (b^* - cl(X \setminus A) \cup b^* - cl(A)) \cap ((A \cup (X \setminus A) \cap (b^* -$
$cl(X \setminus A) \cup (X \setminus A)) = b^* - cl(A) \cap (b^* - cl(A) \cup b^* - cl(X \setminus A)) \cap b^* - cl(X \setminus A) \subseteq (b^* - cl(A) \cap b^* -$
$cl(A \cup (X \setminus A))) \cap b^* - cl(X \setminus A) = b^* - cl(A) \cap b^* - cl(X \setminus A) = b^* - b(A).$

References

1. Bhattacharya P, Lahiri BK (1987) Semi generalized closed sets in topology. Indian J Math 29: 373-382.
2. Skowron A (1989) On topology information systems. Indian Bull Polish Acta Sci Math 36178: 87-90.
3. Pawlak Z (2012) Rough sets theatrical aspects of reasoning about data volume 9 of system theory knowledge Engineering and problems solving Kluwer Dordrecht.
4. Stone MH (1987) Applications of the theory of Boolean rings to general topology. Indian J Math 29: 373-382.
5. Levine N (1963) Semi open sets and semi continuous mappings in topological spaces. Amr Math Monthly 70: 36-41.
6. Najastad O (1965) On some classes of nearly open sets. Pacific J Math 15: 961-970.
7. Abd El-Monsef ME (1980) Studied on Some pre topological concepts. *Ph.D. Thesis.*
8. Mashhour AS, Abd El-Monsef ME, El-Deeb SN (1982) On pre continuous and 3. weak pre continuous mappings. Proc Math and phys Soc Egypt 53: 47-53.
9. Abd EL-Monsef ME, El-Deeb SN, Mahmoud RA(1983) β-open sets and β-continuous mappings. Bull Fac Sci Assiut Univ 12: 77-90.
10. Andrijevic D (1996) On b-open sets. Mat Vesnik 48: 59-64.
11. Ibrahim HZ Bc (2013) Open Sets in Topological Spaces. Advances in Pure Mathematics 3: 34-40.
12. Anitha M, Thangavelu P (2005) On Pre-Generalized Pre-Regular-Closed sets. Acta Ciencia Indica 31M: 1035-1040.
13. Thangavelu P, Rao KC (2002) p-sets in topological spaces. Bulletin of Pure and Applied Sciences 21E: 341-355.
14. Navalgi GB (1998) pre–neighborhood. Matematics Edu. *XXXII:* 201-206.
15. Andrijevic D (1986) Semi preopen sets. Math. Vesnik 38: 24-32.