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NONLINEARITY AND FRACTIONAL COINTEGRATION ANALYSIS OF THE REAL EXCHANGE RATE: EVIDENCE FROM EUROZONE, CANADA, UNITED KINGDOM AND JAPAN

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ABSTRACT

The application of fractional cointegration technique is very important in resolving the Rogoff's (1996) puzzle of purchasing power parity (PPP). By allowing deviations from equilibrium to follow a fractionally integrated process, the fractional cointegration analysis can explain the persistence of a wider range of mean-reversion behaviour better than the standard cointegration analyses. Our empirical results which based on monthly data from January 1990 to September 2008 period illustrate that the PPP reversion exists and can be characterized by a fractionally integrated process in three out of four moneys. Using these results, we can analyze the modified GPH estimator, which has obviously lower bias. The econometric results obtained from the GPH tests hold PPP as a long-run occurrence, though significant short-run deviations from PPP can exist. Therefore, the fractional cointegration analysis permits the deviations from the equilibrium to follow a fractionally integrated process and hence captures a much wider group of research of parity or mean-reversion behavior.

Keywords: *Purchasing Power Parity, Fractional Cointegration, Modified GPH.*
JEL Classification: *C12, C15*

1. INTRODUCTION

The Purchasing power parity (PPP) is a basis of the exchange rate theory and a standard for evaluating the exchange rates in strategy negotiations. It asserts at least in the long run, the nominal exchange in relation to the units of a chosen foreign currency per unit of the domestic one. The PPP varies proportionally through its relative price ratio; it is defined as the fraction of the foreign price level out of the domestic one. Over the years, the frequent econometric studies have attempted to verify the PPP intention with mean-reversion. The previous studies which were carried out since the 1980s tend to assess that even if the PPP exists, it does so in the short run, through significant long-run deviations. To assess the PPP short run validity, the absolute version was used, but its long-run validity seemed to be out of question to debate. So, the most significant evidence of the PPP was considered by numerous authors to be very 'fragile'[1].

Several studies proposed the hypothesis that the real exchange rates may regress to PPP only over very long periods of time, possibly a century or more. In addition, a long-memory process is more likely to be captured by a fractional cointegration technique than the standard Johansen and Juselius (1990a) or Engle and Granger (1987) ones. Consequently, that mean-reversion to PPP is more likely to be found by using a fractional cointegration, particularly through long-term data, probably of a century or more, than by standard cointegration techniques through shorter data runs. This raises the motivating question of how extensive the data run requirements are for fractional cointegration to exhibit that mean-reversion.

[1] See Taylor (1988, 2003) with Taylor and Taylor (2004) intended for reviews to discuss and confirmation on PPP.

The main idea of this paper is to test the presence of a long memory in the exchange rate of Eurozone countries, Canada, the United Kingdom and Japan. A number of studies contain a variety of measures to be tested on a long-term dependence in compound and ordinary exchange rate. The deductions of this study are various relying on the testing process, sample period, frequencies of the series, composite stock returns or frequent exchange rate and others used in these studies. However, the speed of change may be very sluggish. This is a characteristic of time series processes with long memory; that is, processes that display slow mean reversion. Therefore, the test of long-run PPP requires a long period of observations and an econometric technique able to capture the long memory eventually present in the exchange rate series. Fractional cointegration analysis fulfils this obligation and, for this cause, it is the econometric method to be chosen to test the long run PPP in many countries, namely the Eurozone countries, Canada, United Kingdom and Japan.

Nonlinear behavior and the persistence in real exchange rates are major regions of the PPP study (see, for example, Rogoff (1996), Sarno (2005) and Villeneuve and Handa (2006)) and the emphasis on real exchange rates have permitted the use of univariate methods such as the smooth transition autoregressive (STAR) model and its associated test measures. The three approaches used here result from the search of interest in the possibility of confusing the long memory processes with stationary short memory processes subject to structural change (see, for instance, Smith (2005) and Dolado et al. (2002)) and increase the univariate approach. However, unlike standard tests for nonlinearity, the new tests employ semiparametric rather than parametric estimators. Specifically, they are based on the behaviour of semiparametric estimates of the fractional integration parameter, d , when time series stationary processes contaminated by structural shifts. The first approach uses the GPH estimator of d . The second suggested by Dolado et al (2002) uses the Fractional Augmented Dickey Fuller test; and the third one uses the modified Geweke and Porter-Hudak (1983) (modified-GPH) estimator of d due to Smith (2005). In the present context, each of these procedures may be viewed as a test for spurious long memory and, therefore, as a useful means of examining Bond et al. (2007) findings.

The remainder of the paper is organized as follows. Section 2 proposes a conceptual framework for the analysis of PPP in the extended run. Section 3, presents empirical evidence from fractional cointegration analysis as the main basis and describe the data. Section 4 introduces the GPH and the modified GPH tests. Section 5 outlines the cointegration tests for long-run PPP and reports the empirical results. Section 6 concludes.

2. TESTING "PPP" IN THE EXTENDED RUN

The power of a long-run PPP can be examined through testing stationarity in a real exchange rate which is measured by

$$Q_t = S_t P_t^* / P_t \tag{2.1}$$

Where Q_t is the real exchange rate, S_t is the nominal exchange rate (currency/dollar), P_t and P_t^* are, respectively, domestic and foreign price levels. Denoting logs by lower container letters, we reach the linear relationship

$$q_t = s_t + p_t^* - p_t \tag{2.2}$$

The series s_t , p_t and p_t^* are expected to be non-stationary. However, PPP theory tells us that these non-stationary series are related through a stationary error term. That is, they are cointegrated as defined by Engle and Granger (1987). For testing reasons, instead of equation (2.2), we test the subsequent PPP relationship

$$s_t = \alpha_0 + \alpha_1 p_t - \alpha_2 p_t^* + e_t \tag{2.3}$$

Where e_t is a stationary error term incarcerating deviations from PPP, α_0 is some constant, α_1 and α_2 are coefficients to estimate. By imposing the regularity and proportionality restrictions[2], the cointegration test container can be carried out in a bivariate setting. We have tested the strength of the two restrictions with the chi-square test supposed by Johansen and Juselius (1990). The cointegration tests are therefore conducted in a trivariate situation.

[2] The symmetry restriction implies that $\alpha_1 = \alpha_2$ in equation (2.3), and the proportionality restriction requires $\alpha_1 = \alpha_2 = 1$.

Since Mandelbrot's study (1972) on fractional processes, the motivating properties of these processes have been investigated in the literature; for example, see Granger and Joyeux (1980), Hosking (1981, 1982) and Sowell (1990). A general group of fractional processes ARFIMA (*p*, *d*, *q*) is described as follows:

$$\Phi(B)(1-B)^d s_t = \Theta(B)e_t \tag{2.4}$$

Where $\{s_1,...,s_T\}$ is a place of time series process, $\Phi(B) = 1-\varphi_1 B - ... - \varphi_p B^p$ and $\Theta(B) = 1+\theta_1 B + ... + \theta_q B^q$ are polynomials in the lag operator B by all roots of $\Phi(B)$ and $\Theta(B)$ being stable, and e_t is a white noise term that will be relaxed in the data. When $p = q = 0$, s becomes a simple fractional noise process. The fractional parameter d enables us to obtain nointeger values. The variance of the procedure is finite when $d < 0.5$, but infinite when $d \geq 0.5$. The process is stationary designed for $d < 0.5$ and invertible for $d > -0.5$. Long memory is linked with $0 < d < 0.5$. If $d < 0$, the procedure is supposed to exhibit an intermediate memory. Approximating $(1-B)^d$ expansion, we obtain

$$(1-B)^d = \sum_{k=0}^{\infty} \Gamma(k-d) B^k / \Gamma(k+1)\Gamma(-d) \tag{2.5}$$

Where $\Gamma(.)$ is the standard gamma function. Both autocorrelation functions obey a hyperbolic prototype of decomposition contrary to the exponential decline for the usually used stationary ARMA process (2.4) with $d = 0$. In addition to have nointeger values of d increases the flexibility in modelling the long-term dynamics by allowing a comfortable class of spectral behaviour at lower frequencies than those applied by ARMA models. This can be seen as _GPH of the spectral density of s_t.

The general structure of the spectral density of s_t in equation (2.4) is given by

$$f_s(\lambda) = \left|1-\exp(-i\lambda)\right|^{-2d} g_u(\lambda) \tag{2.6}$$

Where $u_t = \Phi^{-1}(B)\Theta(B)e_t$ is a stationary process and $g_u(\lambda)$ is its spectral density at frequency λ. The spectral density $g_u(\lambda)$ is defined as

$$g_u(\lambda) = \left(\sigma^2 / 2\pi\right) \frac{\left|\Theta(B)\right|^2}{\left|\Phi(B)\right|^2} \tag{2.7}$$

Granger and Joyeux (1980) and Hosking (1981) show that the spectral density function of s_t, denoted by $f_s(\lambda)$ and defined in equation (2.6) below is proportional to λ^{-2d} as $\lambda \to 0^+$. Thus, the fractional parameter d crucially determines the low-frequency dynamics of the process. For $d > 0$, $f_x(\lambda)$ is unbounded at frequency $\lambda = 0$, but bounded for ARMA processes with $d = 0$. It becomes clear that when $d > 0$, s_t is a long memory process performance with a slow autocorrelation decay, at a hyperbolical rate (Hosking, 1981), which is contrary to an exponential decay of a short memory process. Note that the more outsized the value of *d*, the stronger is the long-term dependence.

3. EMPIRICAL EVIDENCE FROM FRACTIONAL COINTEGRATION ANALYSIS

Diebold et al. (1991) have examined the PPP using a sample of Belgium, France, Germany, Sweden, U.K and U.S during the period 1791-1913. They verified that parity reversion occurs $(i.e.\ c_\infty = 0)$ [3] when $d < 1$. Conversely $(c_\infty = \infty)$ when $d > 1$; c_∞ is finite and nonzero simply in the unit root case, $d = 1$. Diebold, et al. (1991) found out that purchasing power parity holds in the long run for each of the currencies studied and that the typical half-life of a shock to parity is approximately 3 years. Cheung and Lai (1993) have examined the

[3] $c_j = \sum_{i=0}^{j} a_i$; j=0,1,2,... where $(1-L)r_t = A(L)\varepsilon_t = \sum_{i=0}^{\infty} a_i \varepsilon_{t-i}$

PPP using a sample of Canada, France, Italy, Japan, U.K and U.S during the period 1914-1989. They have used the GPH test for fractional cointegration because Diebold and Rudebusch (1991) and Sowell (1990a) observed that the standard unit-root tests such as the Dickey-Fuller test may have a low power against fractional alternative. The results of formal hypothesis testing designate significant evidence of $d<1$ in all cases, though the evidence for Canada appear relatively marginal.

Bailie and Bollerslev (1994) have examined the PPP using a sample of West German, UK, Japan, Canada, French, Italy and Switzerland during the period March 1, 1980 to January 28, 1985. Because of the difficulty to distinguish between unit root processes and fractional alternatives, it seemed reasonable to consider the error correction term $\alpha' y_t$, to be possibly fractionally-integrated. The deviations from the estimated cointegrating relationship reveal that the exchange rates may well be joined together through a long memory. Soofi (1998) proposed that $\alpha' Y_t$[4] is stationary for the cointegrating vector $\alpha=(1,-1,-1)'$. Accordingly, vector α is used to construct a test for fractional cointegration.

Alves et al. (2001) used the GPH-Tapered (GPHT) estimator because it is less biased than the Geweke and Porter-Hudak (1983) estimator when the series are generated by non stationary or noninvertible fractionally integrated processes, as it is the case with almost all series. The disadvantage of this procedure (hereafter referred to as the GPHT method) is the increase in the variance of the estimator for no stationary processes.

Nielsen (2004) proposes a Lagrange Multiplier (LM) test of the null hypothesis of cointegration in fractionally cointegrated models. The asymptotic Gaussian which is derived from it includes the one-sided and two-sided testing problems and shows that they coincide with the local asymptotic power functions of the one-sided and two-sided LM tests. Masih and Masih (2004) used the previous studies by relaxing the strict $I(0)$ or distinction of the equilibrium error, since cointegration requires that where $z_t=\beta[x_{1t},x_{2t}]$, is $I(d-b)$ with $b>0$, then x_{1t}, x_{2t} forms a cointegration relationship of order $I(d,b)$ the equilibrium error being only mean-reverting.

Villeneuve and Handa (2006) use cointegration and fractional cointegration techniques to test purchasing power parity. This raises the interesting question of how long the data run needs to be for fractional cointegration to display mean-reversion. Testing for fractional cointegration, using the GPH procedure, we estimate the parameter of integration d, and we test whether the residuals of the cointegrating relationship are $I(d)$ with $0<d<1$.

4. ECONOMETRIC METHODOLOGY

As a concept, fractional cointegration was initially suggested by Granger (1986), though fractional differencing was previously introduced into the literature by Granger and Joyeux (1980) and Hosking (1981), and others. Fractional cointegration was formally proposed for the first time by Cheung and Lai (1993) in an applied study on exchange rates time series.

1.1. GPH method

A combination of the Engle-Granger two-step procedure and the usefulness of the fractionally integrated processes and their implications correspond quite intuitively to tests of the long-run PPP, since PPP requires that s_t in equation (2.4) to be a mean-reverting process. The estimating procedure can be summarized using the following steps:

(i) Determining the univariate properties of each of the time series involved with respect to integration via a series of unit root tests;

(ii) Given that the variables do share common integration processes, conducting OLS regressions and computing the $z_t s$ or the equilibrium errors;

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[4] $Y_t=(p^*,s,p)$, where p^*, s and p are, respectively, logs of the foreign price level, nominal exchange rate and the US price index.

(iii) Examining whether the $z_t s$ are $I(d)$ with $d<1$ using an appropriate estimation technique designed to detect fractionally integrated processes.

Since (iii) requires a direct estimation of the integration parameter, d , standard unit root tests cannot be used for this exercise as they frequently tend to exhibit low power against fractional alternatives (see Diebold and Rudebusch (1991)). Consistent with other studies that have investigated fractional integration properties of time series (see Cheung and Lai (1993), Cheung (1993)), used a semi-parametric procedure owing to GPH. The primary equation of interest behind the estimation procedure relies on an OLS regression as follows:

$$\ln\left[I\left(w_j\right)\right]=c-d\ln\left[4\sin^2\left(w_j/2\right)\right]+\eta_j \quad \forall \text{j=1,....,n} \tag{2.8}$$

For $w=2\pi j/T$ $(\forall \text{ j=1,...,T-1})$, $\text{n}=\text{g}(\text{T})<T$, where $I\left(w_j\right)$ is the periodogram of χ^2 at frequency w_j defined by

$$I(w)=\frac{1}{2\pi T}\left|\sum_{t=1}^{T}e^{itw}\left(X_t-\bar{X}\right)\right|^2 \tag{2.9}$$

The GPH test can also be used as a test of the unit root hypothesis with $I(1)$ processes imposing a test on d (GPH) from the first-differenced form of the series being significantly different from zero. In this respect, the GPH procedure poses an alternative view point with which to scrutinise the unit root hypothesis.

1.2. Modified GPH of Andrews and Guggenberger (2003)

The method is as similar as that of the GPH estimator but that it includes frequencies of the power $2k$ for $k=1,...,r$, for some positive integer r, as additional regressors in the pseudo-regressive representation that yields the GPH estimators.

We consider a semiparametric method for a stationary Gaussian long memory times series $\{Y_t : t=1,...,n\}$. The spectral density of the time series is

$$f(\lambda)=|\lambda|^{-2d}\, g(\lambda) \tag{2.10}$$

Where d is the integrated parameter, $g(.)$ an even function on $[-\pi,\pi]$ that is continuous at zero with $0<g(0)<\infty$.

GPH considered a d as an estimator based on the model parametrization

$$f(\lambda)=\left|1-\exp(-i\lambda)\right|^{-2d} f^*(\lambda), \tag{2.11}$$

Where $f^*(\lambda)$ satisfies the identical conditions as $g(.)$, and on first m periodogram ordinates I_j

$$I_j=\frac{1}{2\pi n}\left|\sum_{t=1}^{n}Y_t\exp\left(i\lambda_j t\right)^2\right|, \quad \text{for} \quad j=1,...,m \tag{2.12}$$

Where $\lambda_j=\dfrac{2\pi}{n}$ and m is a positive integer slighter than n. The GPH estimator is given by (−1/2) times the OLS estimator of the slope parameter in a regression of $\{\log I_j : j=1,...,m\}$ on a constant and regressor variable $\tilde{\lambda}_j$ $\log|-\exp\left(-i\lambda_j\right)|$

The modified GPH[5] uses the Robinson (1995) specification which reinstates the regressor $\tilde{\lambda}_j$ by X_j in the model parametrization (2.4)

$$X_j=-2\log\lambda_j \tag{2.13}$$

[5] See Andrews and Guggenberger (2003) for an absolute presentation.

The specificity of the modified GPH is that it adjoins the regressors $\lambda_j^2, \lambda_j^4, ...\lambda_j^{2r}$ to the pseudo-regression representation of the GPH estimator. Thus, authors define bias-reduced estimator d_r. The new estimator is better than the standard GPH. When $r = 0$, d_r is asymptotically equivalent to the standard GPH.

The modified-GPH estimator, articulated $\hat{d}_r$, can be used to investigate whether the apparent fractional character of a series is really due to mean shift. If $\hat{d}_r < \hat{d}$ then it is probably that the series contains a mean shift. If $\hat{d}_r \geq \hat{d}$, then it is unlikely that the confirmation for fractional behaviour is due to mean shifts. Importantly, Smith (2005) indicated that $\hat{d}_r$ should not be viewed as an estimation of the 'true' value of d as this requires a nontrivial modelling. It must also be noted that there are no critical values for Smith's process. It is not a formal significance test but rather a helpful diagnostic check.

5. EMPIRICAL RESULTS

Our sample contains monthly data for four countries over the period 1990:1-2008:9 on nominal exchange rate which is defined as the domestic consumer price indices over the foreign consumer price indices. The nominal exchange rates used are all bilateral at the end of the period concerning US dollar rates in relation to the Canadian dollar, the Eurozone countries, the UK pound and the Japanese yen. These data are obtained from the International Financial Statistics (IFS).

The estimate of the cointegrating vector, $\hat{\alpha} = (\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2)$, is obtained from an OLS regression of equation (2.3) using the currencies of Eurozone countries, Canada, United Kingdom and Japan (see table 1).

Reversing the signs of p^* and p in the cointegrating vector for the coefficients of the long-run PPP relationships estimated, the signs of the coefficients in Table 1 are consistent with the PPP hypothesis: an increase in p^* will increase e, while a decrease of the domestic price level p will decrease e [6].

Findings from table 2, show that the ADF test cannot reject the hypothesis of a unit root in all variables. The ADF test suggests that the underlying variables are stationary in first difference while they possess a unit root in terms of level. The ADF statistics are obtained from regression (2.3) with the lag parameter p selected using both the AIC and the SIC[7]. As can be seen in table 2, the statistical results are apparently mixed.

The GPH test rejects the unit root null hypothesis, a result that agrees in the context of ARIMA models. The estimated value of d is in this case higher than 1 for all currencies, lying in the nonstationary range of d. We want to test whether the fundamental process generating the time series $\{s_t\}$ has a unit root or not. If this is the case the spectral density of the first difference is $G_t = (1-L)s_t$.

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[6] Note: so as our cointegrating equation is approximately consistent with the answer of Johnson (1991), Kim (1990) and Kugler and Lenz (1993).

[7] We tried diverse ARFIMA (p, d, q) specifications and used the Log-likelihood value, Schwartz Information Criterion (SIC) and Akaike Information Criterion (AIC) to distinguish between models. As Teyssière (1997) points out, the statistical properties of the AIC and SIC have not been established for the class of a long memory ARCH process; however we think that these statistics offer good reasonable leadership. They are designed herein as:

$$AIC = -2\ln\left(L\left(\hat{\theta}\right)\right) + 2 \times n_\theta$$

$$SIC = -2\ln\left(L\left(\hat{\theta}\right)\right) + n_\theta \times \ln(n)$$

Where $L(\hat{\theta})$ is the optimized likelihood value, n_θ is the number of estimated parameters and n is the sample size.

The GPH test for cointegration is then performed, and its results are reported in table 4. The results differ slightly across the different values of u under consideration. Table 3 shows that all of the estimates of d, decline lower to 1, suggesting possible fractional integration behaviour. Moreover, results of official hypothesis testing indicate significant evidence of $d < 1$, in all cases. Moreover, in all cases except the UK and Eurozone countries, the estimates of d are significantly greater than 0. The results on the whole indicate the presence of cointegration and possibly fractional cointegration between sp_t, and p_t. The GPH test results provide a wider and more significant support for a long-run PPP.

We also note that the long sample period involves data start different exchange-rate regimes. In general, flexible exchange-rate periods are more inclined to be associated with an advanced variability in the exchange rate and the price changes than the fixed exchange rate periods. Hence the variances of our data series may not be stationary across the exchange-rate regimes. Nonetheless, this class of non-stationarity will not pose any problem to the present analysis. Monte Carlo results reported by Cheung (1993) propose that the GPH test is robust to variance shifts and conditional heteroscedastic effects.

The modified-GPH method, which is perfect and attractive for the performance of the three approaches, is rather a more general underpinning representation that strongly supports the case of the nonlinearity of the real exchange rate for Eurozone countries and Japan. It also designates the same way for Canada and the UK. If the root mean square error minimizing value of m is used, it powerfully maintains the case for long memory of the real exchange rate for Eurozone countries.

Table 5 gives the GPH estimates, $\hat{d}$, and the modified-GPH estimates, $\hat{d}_r$, for the two real exchange rate series. For each series the parameter is estimated with both the root mean square error minimising "Plugin" value and the $T^{1/2}$ value for m, and a series of values for k. For both series there is significant evidence in favour of nonlinearity, $\hat{d}_r$ being less than $\hat{d}$ in the large majority of cases. For both series, when m is set to the "Plugin" value, $\hat{d}_r < \hat{d}$ for all cases except r =3, 4 and 5 for Canada and r=3 for U.K. Using the "fixed" $m = T^{1/2}$, the suggestions are that real exchange rate between Canada/USA and U.K/USA are nonlinear. Therefore the modified-GPH test supports the findings of Bond et al. (2007) with respect to nonlinearity of the real exchange rate between Canada /USA and U.K/USA but also proposes the possibility of nonlinearity in the real exchange rate between USA/Japan, which was much less clear in their results.

6. SUMMARY AND CONCLUSION

The generalized notion of cointegration, which is to identify fractional cointegration, has been introduced and used to examine the empirical relevance of a long-run PPP. The analysis of fractional cointegration allows the equilibrium error to follow a fractionally integrated process and avoids the stringent $I(1)$ and $I(0)$ difference maintained in previous empirical studies. By allowing fractionally integrated equilibrium errors, a flexible and parsimonious method to model the small frequency dynamics of deviations from the equilibrium is made possible. Accordingly, the fractional cointegration analysis can identify a wide range of mean reversion behaviour, which turns out to be important for a new evaluation of long-run PPP. The empirical results show that the evidence from the fractional cointegration analysis is a great deal more favourable to long run PPP than that from the standard cointegration analysis using unit-root tests and three (Japan and Canada) out of 4 relationships that investigated the equilibrium error can be characterized as a fractionally integrated process.

The answer of the fractionally integrated property of PPP deviations for many different countries raises questions concerning the source of such property. Johansen and Juseluis (1990b) have suggested that deviations from PPP for the United Kingdom can be accounted for by the interactions between exchange rate and the interest rates. It is possible that the behaviour of PPP deviations may in general reproduce the statistical property of the economic fundamentals such as the levels of output, money supply and interest rates.

Although the full maximum likelihood estimation technique can be statistically powerful, the maintained assumptions of normality and known means appear to be not appropriate in the empirical application at this time. Furthermore, the implementation of Sowell's (1988) multivariate maximum likelihood technique is not straightforward in terms of hypothesis testing. In constructing the multivariate likelihood function, the cointegrating vector is not identifiable under the null hypotheses of no cointegration. Like Sowell's (1988) study, this leads to a non standard hypothesis testing problem such that one may enchanter only between the upper and the lower bound but distribution of the test statistic is not precise.

REFERENCES

Andrews, D.W.K., et Guggenberger, P., (2003), “A bias-reduced log-periodogram regression estimator for the long-memory parameter”, Econometrica, 71 (2), 675-712.

Baillie, R.T. and Bollerslev, T., (1994), “Cointegration, fractional cointegration, and exchange rate Dynamics”, Journal of Finance, 49, 737-45.

Bond, D., Harrison, M. J. and O'Brien , E. J (2007), “Modelling Ireland's exchange rates: from EMS to EMU,” Working Paper No. 823, Europeanpean Central Bank.

Cheung, Y., (1993), “Tests for fractional integration: A Monte Carlo investigation”, Journal of Time Series Analysis 14, 331-345.

Cheung, Y.W. and Lai, K.S., (1993), “A fractional cointegration analysis, of purchasing power parity”, Journal *of Business and Economic Statistics*, 11, 103-12.

Costa, A. A. and Crato, N., (2001), “Long-run versus shortrun behaviour of the real exchange rates”, Applied Economics, 33, 683-88.

Diebold, F.X., Husted, S. and Rush, M., (1991), “Real exchange rates under the gold standard”, *Journal of Political Economy*, 99, 1252-71.

Diebold. F. X., and Rudebusch, G. D., (1991), “On the Power of Dickey-Fuller Tests Against Fractional Alternatives”, *Economics Letters,* 35, 155-160.

Dolado J., Gonzalo, J. and Mayoral, L., (2002), “A Fractional Dickey-Fuler test For Unit Roots”, Econometrica, Vol. 70, No.5, 1963-2006.

Engle, R. F., and Granger, C. W. J. (1987), “Cointegiation and Error Correction; Representation. Estimation, and Testing”, *Economctrica.* 55, 251-276.

Geweke, J. and Porter-Hudak, S. (1983), “The estimation and application of long memory time series Models”, Journal of Time Series Analysis, 4, 221-38.

Granger, C. W. J. (1986), “Developments in the study of cointegrated economic variables”, Oxford Bulletin of Economics and Statistics, 48, 213-28.

Granger CWJ, Joyeux R. (1980), “An introduction to long-memory time models and fractional differencing”, *Journal of Time Series Analysis* 1: 15-29.

Hosking, J.R.M., (1981), “Fractional differencing”, *Biometrika ,* 68, 165-176.

Hosking J.R.M., (1982), “Some models of persistence in time series”, In *Time Series Analysis : Theory and Practice* 1, Enderson OD (ed.). North-Holland: Amsterdam; 641-653.

Johansen, S. (1991), “Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models”, *Econometrica,* 59, 1551-1580.

Johansen, S., and Juselius, K. (1990a), “Maximum Likelihood Estimation and Inference on Cointegration-With Applications to the Demand for Money”, *Oxford Bulletin of Economics and Statistics,* 52, 169-210.

Johansen, S., arid Juselius, K. (1990b), “Some Structural Hypotheses in a Multivariate Cointegration Analysis of the Purchasing Power Parity and the Uncovered Interest Parity for the UK”,

Preprint 1, Institute of Mathematical Statistics, University of Copenhagen. Kim, Y. (1990), "Purchasing Power Parity: Another Look at the Long-Run Data," *Economics Letters,* 334-339.

Kugler, P. and Lenz, C. (1993) Multivariate cointegration analysis and the long run validity of PPP, Review of Economics and Statistics, 75, 180-84.

Mandelbrot, B., (1972), “Statistical methodology for nonperiodic cycles: from the covariance to R/S Analysis”, *Annals of Economic and Social Measurements* 1: 259-290.

Masih, A. M. M and Masih, R. (2004), “Fractional cointegration, low frequency dynamics and long-run purchasing power parity: an analysis of the Australian dollar over its recent float”, Applied Economics, 36, 593-605.

Nielsen, M., (2004) “Optimal residual-based tests for fractional cointegration and exchange rate dynamics”, Journal of Business & Economic Statistics, 22, 3, 331-345.

Rogoff, K. (1996): \The purchasing power parity puzzle," Journal of Economic Literature, 34, 647-668.

Sarno, L. (2005): \Towards a solution to the puzzles in exchange rate economics: where do we stand?," Canadian Journal of Economics, 35, 673-708.

Smith, A. (2005): \Level shifts and the illusion of long memory in economic time series," Journal of Business & Economic Statistics, 23, 321-335.

Soofi, A. (1998), “A fractional co-integration test of purchasing power parity: the case of selected members of OPEC”, *Applied Financial Economics* 8, 559-566.

Sowell, F. (1988), “Maximum Likelihood Estimation of Fractionally Integrated Time Series Models”, Working Paper. Carnegie Mellon University, GSIA.

Sowell F. (1990), "Maximum likelihood estimation of stationary univariate fractionally integrated time series models"., *Journal of Econometrics* 53, 165-188.

Sowell, F. (1990a), "The Fractional Unit Root Distribution", *Econometrica,* 58, 495-505.

Teyssière, G. (1997), "Microeconomic Models for Long Memory in the Volatility of Financial Time Series", Studies in nonlinear Dynamics and Econometrics, vol. 5, No. 1.

Tsay, R. (1989), "Testing and Modelling Threshold Autoregressive Processes", Journal of the American Statistical Association, vol 84, pp. 231-240.

Villeneuve, J. and Handa, J. (2006), "Purchasing Power Parity as a long-term memory process: evidence from Canada". Applied Financial Economics, 16, 109-117.

Appendix

Table 1:OLS estimate of PPP equations

	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$
Eurozone countries	-2.32	3.74	-4.84
Canada	-0.92	3.35	-2.14
UK	3.75	2.14	-1.96
Japan	-20.65	5.02	0.44

Table 2: The Augmented Dickey-Fuller test

		ADF			
		Lag(AIC)	nct	wc	ct
Eurozone countries	q_t	6	-	-	-4.872
	Δq_t	5	-1.631	-1.560	-1.463
	p_t	5	-1.386	-1.386	-0.975
	Δp_t	4	-	-3.034	-3.362
Canada	q_t	2	0.214	-1.321	-1.556
	Δq_t	1	-	-	-3.541
	p_t	6	0.854	-1.455	-1.444
	Δp_t	5	-0.917	-1.225	-1.690
U.K.	q_t	5	-0.225	-1.584	-3.155
	Δq_t	3	-2.368	-2.318	-3.123
	p_t	7	1.599	-1.615	-0.930
	Δp_t	6	-1.807	-2.298	-2.731
Japan	q_t	6	1.631	-0.820	-2.605
	Δq_t	5	-1.429	-2.514	-2.433
	p_t	8	0.436	-2.607	-0.466
	Δp_t	7	-0.622	-0.569	-2.496
	p_t^*	3	1.591	-2.302	-1.325
	Δp_t^*	8	-	-	-4.002

Notes: (nct) indicate the Dickey-Fuller Tests with no constant and no trend, (wc) indicate the Dickey-Fuller Tests with constant and no trend and (ct) indicate the Dickey-Fuller Tests with constant and with trend. The critical value is -1.95, -2.86 and -3.41 respectively for nct, wt and ct.

Table 3:The GPH test for the real exchange rate

	$d(u=0.45)$	$d(u=0.50)$	$d(u=0.55)$
Eurozone countries	0.850 (0.155)	0.835 (0.107)	0.751 (0.087)
Canada	0.886 (0.143)	1.080 (0.141)	1.083 (0.109)
UK	0.958 (0.183)	0.819 (0.155)	0.675 (0.136)
Japan	0.858 (0.102)	0.906 (0.071)	0.880 (0.078)

Notes: we report results for $u = 0.45,\ 0.50$ and 0.55 this set of choice of u also yielded good test performance in our simulation research. The figures in parentheses are the t-statistics for the corresponding GPH parameter d estimates, computed based on the identified hypothetical error variance $\left(\pi^2/6\right)$.

Table 4: GPH tests for fractional cointegration

	$\hat{d}\ (u=0.45)$		$\hat{d}\ (u=0.50)$		$\hat{d}\ (u=0.55)$	
	$H_0: d=1$	$H_1: d=0$	$H_0: d=1$	$H_1: d=0$	$H_0: d=1$	$H_1: d=0$
Eurozone countries	-0.967	5.535**	-1.542*	7.803**	-2.862**	8.632**
Canada	-0.797	6.195**	0.567	7.659**	0.443	5.79**
U.K	-0.229	5.382**	-1.167	5.283**	-2.389**	4.963**
Japan	-1.392	8.411**	-1.323	12.76**	-1.538*	11.28**

Note: the sample size for the GPH test is given by $n = T^{\mu}$ .the statistic of GPH is given by $GPH_T = \hat{d}\left(\frac{\pi^2}{6\sum_{j=1}^{m}\left(Z_j - \bar{Z}\right)^2}\right)^{-1/2}$, $GPH_T \xrightarrow{L} N(0,1)$. Significance is indicated by * at the 10% level or ** at the 5% level.

Table 5: Modified-GPH of Real Exchange Rate

countries	m	GPH	Modified GPH				
			r=1	r=2	r=3	r=4	r=5
Eurozone countries	Plugin	0.966 (0.076)	0.682 (0.311)	0.647 (0.448)	0.253 (0.597)	-0.168 (0.762)	-0.093 (0.949)
	Fixed	0.705 (0.230)	0.559 (0.429)	0.266 (0.652)	-0.617 (0.915)	-0.414 (1.238)	0.279 (1.646)
Canada	Plugin	1.068 (0.076)	1.060 (0.311)	0.793 (0.448)	1.330 (0.597)	1.640 (0.762)	2.122 (0.949)
	Fixed	0.891 (0.230)	0.952 (0.429)	1.414 (0.652)	2.091 (0.915)	2.131 (1.238)	1.591 (1.646)
U.K	Plugin	1.255 (0.076)	0.776 (0.311)	0.959 (0.448)	1.259 (0.597)	1.178 (0.762)	1.179 (0.949)
	Fixed	0.928 (0.230)	1.004 (0.429)	1.168 (0.652)	1.247 (0.915)	1.369 (1.238)	2.134 (1.646)
Japon	Plugin	1.122 (0.076)	0.970 (0.311)	1.015 (0.448)	1.082 (0.597)	0.971 (0.762)	0.673 (0.949)
	Fixed	0.885 (0.230)	1.089 (0.429)	0.947 (0.652)	0.928 (0.915)	0.310 (1.238)	-0.588 (1.646)

Notes: standard errors in parentheses. We report results for this set of choice of $r = 1,\ldots,5$ also yielded good test performance in our simulation research.