Introduction

With the continued reduction of dimensions of technological devices, the heat produced can be important, component failures can occur. According to NASA, 90% of failures are due to defects and thermal interconnects, according to the US Air Force, 55% of electronic failures are due to thermal effects [1]. The heat transfer control in these systems is a major challenge of miniaturization. Further reducing the size of the systems causes the decreased ability of materials to distribute heat when the scales are of the order of a nanometer or micrometer sometimes due to the spatial confinement effect in the nanostructures. Therefore, measurement of thermal parameters of thin films and heat transfer modeling in these systems are important from the theoretical point of view to understand the physical mechanisms underlying and from the point of view of the application to improve the design [2].

Heat transfer plays an important role in science and also affects the variation in the electrical, thermal and physical properties of materials. Heat transfer in semiconductor samples allows us to study the evolution of the temperature, taking account of the conduction and convection in the materials. The measurement in this area requires very sophisticated experiments because it requires considerable affinity. The temperature has an effect on the concentration of ionized impurity, the mobility and the electric conductivity of a semiconductor [3]. Some researchers [4] assume that the electrical conductivity is a constant at higher temperatures than room temperature. At high temperature, the particles are agitated with a thermally high kinetic energy. These particles cede its kinetic energy to neighboring particles having a low temperature for the entire system reaches thermal equilibrium. The difference between a semiconductor and an insulator is that a semiconductor is low temperature insulation. It becomes conductive when its temperature becomes higher. Indeed, beyond a certain level of thermal motion,

the electrons of an intrinsic semiconductor gain sufficient energy to jump the band gap and move from the valence band to the conduction band. Thus, it exists, an electron pair and an occurrence of conduction by both electrons and holes [3]. At high temperatures, the intrinsic conductivity can be the major factor in the conductivity of a semiconductor. In an oxide insulator or semiconductor, the width of the band gap (energy gap, Eg), which by its magnitude, prevents thermally excited electrons to jump from the valence band to the conduction band. With a thermal agitation, the temperature rises and the heat transfer begins to settle. The fact that the conduction electrons carry the electrical and thermal loads, Weidman Franz [5] established a law linking the thermal conductivity to the electric one. The thermal conductivity dependent on the movement of charge carriers is called conductivity by electrons or holes. It depends on the concentration of charge carriers. In semiconductors, this quantity must greatly depend on the temperature.

Because of its connection to the mechanical and chemical stability of the layers, good adhesion to various substrates, stability under the action of aggressive fluids, its high conductivity, transparency in the visible spectral range, low cost, high sensitivity and rapid response, the nanocrystals of tin dioxide (SnO2) is considered an attractive material for various technological applications include electronic and optoelectronic devices, electroluminescent displays, photocells, detectors, sensors and anti-reflection coatings [6-10]. In the heterostructure SnO2/SiO2/Si, the interfacial layer (SiO2) plays an important role in reducing the dark current, leading to the improvement of the circuit voltage in the cell. It is, in this framework, that this research will be established.

The first investigations on single crystals of SnO2 are due to Jarzelski [11]. He measured the optical absorption, and the electrical conductivity in the crystal. Jarzelski analyzed the pattern of energy levels in single crystals SnO2, with the photoelectric method. Morgan and Wright [12] studied the electrical conductivity in single crystals of SnO2 doped with antimony. An idea that the gas sensor can be
developed using a metal oxide semiconductor used to detect toxic gases was developed using the sintering of SnO₂. These gas sensors have evolved according to the method of obtaining layers of tin dioxide [13].

Mathematical model

A mathematical model that describes the transfer of heat in heterojunction device is proposed. This model is based on the two dimensional heat transfer equation. It may present a roughness shape and heat resistance. The thermal conductivity of the material also has a great influence on the transfer process. Thermo-physical properties of the used materials are determined in order to thermally study the physical system that is formed by several materials in contact. Thus, the structural design of a modern sensor can be schematized by a physical model consisting of an insulating layer between two semiconductor layers. It is presented by the following Figure 1.

In this system, the heat transfer that develops in the heated tin dioxide layer having a thickness of 0.2 microns coupled with another layer of cold silicon, having a thickness of 1 micron, is studied numerically. A third layer of silicon dioxide, SiO₂ is considered between these two layers.

To complete the mathematical model, the relation that links the two thermal (λ) and electrical (σ) (conductivities given by Weidman Franz [5] is used:

\[ \lambda = \sigma L T \] where \( L = \pi^2 k^2 / (3e^2) = 2.44 \times 10^{-8} \) (W/K)

(1)

The tin dioxide thermal conductivity, that is function of temperature, is given by this curve Figure 2.

Using the least squares method, correlation between the thermal conductivity and the temperature of the tin dioxide is given by the following Equation:

\[ \lambda = -2.136 \times 10^{-4} T^2 + 0.078 T + 27.484 \] (2)

In this case, silicon is a substrate followed by an insulating layer. Then, the silicon layer is not strongly influenced by the heating temperature. As a result, the thermal conductivity of the silicon is considered constant (149W/(mK)). In addition, the SiO₂ layer serving as an insulator, has a very low electrical conductivity. Then, its thermal conductivity is constant (1.4 W/(mK)). Finally, the mathematical model is represented in the following Figure 3.

To simplify the mathematical model, the following simplifying conditions are used: Heat transfer is carried out in a two-dimensional transient manner, the transfer occurs without heat generation and without chemical reaction, there is no generated heat flow, the pressure is the atmospheric one, the radiation effects are considered negligible. Using these simplifying conditions, the heat equation is the following:

\[ \rho C_p \partial T / \partial t = \partial (\lambda \partial T / \partial x) / \partial x + \partial (\lambda \partial T / \partial y) / \partial y \] (3)

With \( \rho \) is the density and \( C_p \) is the specific heat of the used materials.

Taking into account the variability of the thermal conductivity as a function of the temperature in the tin dioxide layer, the equation (3) becomes:

\[ (-4.272 \times 10^{-10} T + 0.078)(\partial^2 T / \partial x^2 + \partial^2 T / \partial y^2) + (-2.136 \times 10^{-10} T + 0.078 T + 27.484)(\partial^2 T / \partial x^2 + \partial^2 T / \partial y^2) = \rho C_p \partial T / \partial t \] (4)

To complete the mathematical model, the simplified heat equation is linked to the initial and boundary conditions (Figure 4) as the following:

Initial conditions for \( t<t_o \)

For: \( x=x_o \) and \( y=y_o \) then \( T=T_o=298.15 \) K

Boundary conditions for \( t>t_o \)

For \( x=x_o \) \quad For \( x=x_u \)
The schemed form of the two-dimensional heat transfer equation is given by the following relationship:

\[
\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{Q}{\rho C} \frac{\partial T}{\partial t}
\]

Where, \(Q\) is the heat quantity and \(h\) is the convective heat transfer coefficient.

**Conduction at the interfaces:***

At the SnO2/air and air/SiO2 interfaces, the continuity of the conduction and the convection is given by the following relationships:

\[
\lambda_{SnO2} \frac{\partial T}{\partial y}|_{SnO2} = \lambda_{air} \frac{\partial T}{\partial y}|_{air} + h(T_a - T)
\]

(6)

\[
\lambda_{SiO2} \frac{\partial T}{\partial y}|_{SiO2} = \lambda_{air} \frac{\partial T}{\partial y}|_{air} + h(T_a - T)
\]

(7)

Where, \(T_a\) is the ambient medium temperature.

At the interface Si/SiO2, the continuity of the thermal conductivities is given by the following relationship:

\[
\lambda_{Si} \frac{\partial T}{\partial y}|_{Si} = \lambda_{SiO2} \frac{\partial T}{\partial y}|_{SiO2}
\]

(8)

At the interface of the two layers SiO2 and SnO2, the continuity of the thermal conductivities is given by the following relationship:

\[
\lambda_{SnO2} \frac{\partial T}{\partial y}|_{SnO2} = \lambda_{SiO2} \frac{\partial T}{\partial y}|_{SiO2}
\]

(9)

**Numerical treatment:**

The equations of the mathematical model that describes the physical phenomenon are schemed by two numerical methods: the explicit finite difference method and the finite element method.

**Scheming with the explicit finite difference method:**

The computational domain is divided into computing network nodes \((\Delta x, \Delta y, \Delta t)\). \(\Delta x\) and \(\Delta y\) are the space steps, and \(\Delta t\) is the time step.

Scheming with the explicit finite difference method:

\[T(i, j, n+1) = T(i, j, n) + \frac{A}{\rho C} \Delta t \frac{Q}{\Delta x^2} \frac{\partial^2 T}{\partial x^2}
\]

(10)

**Boundary conditions:**

At the interface SnO2/Air/SiO2/Si, the continuity of the thermal conductivities is given by the following relationship:

\[\lambda_{SnO2} \frac{\partial T}{\partial y}|_{SnO2} = \lambda_{air} \frac{\partial T}{\partial y}|_{air} + h(T_a - T)
\]

When there is porosity (air layer) between SnO2 and SiO2:

\[
T(i, j, n+1) = \frac{\lambda_{SnO2} T(i,j,n) + \lambda_{SiO2} T(i,j,n)}{\lambda_{SnO2} + \lambda_{SiO2}}
\]

Conduction & convection continuities at the interface SnO2/Air:

\[
T(i, j, n+1) = \frac{\lambda_{SnO2} T(i,j,n) + \lambda_{air} T(i,j,n)}{\lambda_{SnO2} + \lambda_{air}}
\]

**Scheming with the finite element method:**

To solve the Equation (3) by the finite element method, the residue weighed in the Galerkin formulation as reported by Poulain et al. [14] and Hatta et al. [15] is used. The heat equation is multiplied by an arbitrary function \(T^*\), called the weighing function, and integrated along the volume. Thus, the formulation of the integral of the low thermal problem is obtained:

\[
\int_T \left[ \rho C \frac{\partial T}{\partial t} - \lambda \nabla^2 T \right] dV = 0
\]

(12)

The analytical solution of equation (12) is generally inaccessible. The numerical solution by the finite element method, which is a particular case of the Galerkin method, (the temperature field and the test functions are belonging to the same finite-dimensional space), is used.

The medium is divided into a finite number of elements interconnected only at nodal points. For each element occupying the space, the temperature at any point can be expressed as a function of their values to their nodal point:

\[
T(x, y, z) = \left[ N \right] \left[ T \right]
\]

(13)

Where \([N]\) is known as a form of matrix and \([T]\) is a vector which contains the values of the temperature at the nodal points of the element. It follows that the temperature gradient at any point can be written as:

\[
\nabla T = \left[ N \right] \left[ \frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z} \right] = [B]\left[ T \right]
\]

(14)

Where \([B]=\left[ L \right][N]\) and \([L]\) designates a differential operator.

Substituting equation (14) in equation (3), the heat flux in the...
element can be obtained as follows:

\[ \{ q \} = T \{ q_1, q_2 \} = -\lambda \{ B \} T \]  

(15)

Assume that the medium undergoes a virtual temperature change, \( \delta T \). Then, multiplying Equation (12) by \( \delta T \) and by incorporating into the whole of the space domain, and with the substitution of the Equations (13) and (15), the finite element approximation of the heat equation, can be obtained by:

\[ w(T, T') = \{ T \}'^T \left( [D] \{ T \} + [C] \{ N \} \{ T \} - [M] \right) = 0 \]  

(16)

Where, the dot represents differentiation, while respecting time.

Using the finite difference scheme, Equation (16) becomes:

\[ [D] \{ T \}'^T - [C] \{ N \} \{ T \} - [M] = 0 \]  

(21)

Equation (21) is solved using an explicit finite difference method and Gauss-Legendre integration.

Results and Discussion

Numerical model validation

After both numerical models reach the calculation stability condition, comparisons (Figure 5) between the results for a sample of silicon coupled to silicon dioxide on the one hand, and a sample of silicon coupled to silicon dioxide, and tin dioxide, on the other hand, are performed. The results show satisfactory qualitative and quantitative agreements. Unlike in some profiles, comes from the numerical method (the finite difference method), which is considered less perfect than the finite element one.

Physical parameters influence on the thermal transfer phenomenon

In the following, the finite element method and the thermal conductivity of tin dioxide which has a parabolic profile relative to the temperature are used. This latter strongly influences on the heat transfer mechanisms of the system.

Effect of the imposed temperature: The study is carried out on heterojunction device SnO\(_2\)/SiO\(_2\)/Si, of different depths (0.2 \( \mu \)m/50 nm/1 \( \mu \)m), and a length of 1\( \mu \)m, with different temperatures applied to the surface of SnO\(_2\).

It is noted that the initial temperature begins to increase along the depth of the sample, and a function of time, to reach the corresponding heating temperature. If the heat flux applied to the surface of SnO\(_2\), increases, the minimum temperature will decrease. To better observe this phenomenon, the results in the form of linear temperature profiles are presented:

The curves of the Figure 6 show the evolution of the temperature during the heat transfer in the multilayer nanomaterials SnO\(_2\)/SiO\(_2\)/Si that is subject only to the conduction. After 100ns, if the applied temperature increases, the applied heat flux also increases. Then, the profile of the thermal conductivity of SnO\(_2\) can no longer withstand this temperature increase. According to Figure 6, if the applied temperature is between 300 K and 423 K, the temperature starts from 298.08 K and 384.23 K respectively, and increases with time to reach the heating temperature. But if the applied temperature is beyond 423 K, the system temperature breakdown and it remains at its initial temperature. In the following, the applied temperature of 423 K will be used.

Tin dioxide thickness effect: Similarly, the study is carried out on heterojunction device SnO\(_2\)/SiO\(_2\)/Si, where the depth of the SnO\(_2\) is variable (x \( \mu \)m/50 nm/1 \( \mu \)m). The applied temperature is taken equal to 423K.
It is noted that the initial temperature, begins to increase along the depth of the sample, and a function of time, in order to reach the heating temperature (423K). Quantitatively, if the thickness of the SnO₂ layer increases, the temperature profile will become less important because the thickness of the first layer (SnO₂) prevents heat easily diffuse into the depth of the sample, leading to the temperature decrease in the deeper layers. Qualitatively, if the thickness of the SnO₂ layer increases, the parabolic evolution of the temperature profile appears clearly. This evolution is deducted, from the impact of the parabolic profile of the thermal conductivity, on the heat transfer phenomenon, in the SnO₂ layer. It is noted that, in the case where the thermal conductivity is variable, the increase of the initial temperature to the heating one is less than in the case where the thermal conductivity is constant. Figure 7 represents the temperature linear profiles:

These curves show the evolution of the temperature during the heat transfer into the multilayer SnO₂/SiO₂/Si, which is subject only to the conduction. During 100ns, the heterojunction device will heat up more quickly if the thickness of SnO₂ is less than 0.2 µm. If the thickness of SnO₂ is 10 nm and for a time of 100 ns, the temperature increases from its initial value of 395.3 K to reach its hot value (423 K). But, for a thickness of SnO₂, above 0.2 µm (e.g., 500 nm), and a time of 100 ns, the temperature increases from 369.191 K, to reach its heating temperature (423 K). Therefore, the decrease of the initial value of the temperature
in the deep layer of the sample is particularly important, if the thickness of the tin dioxide layer is large. Then, to exhaust heat through the depth of the sample, it is better to choose a thickness of the thin films of tin dioxide lower than 0.2 µm.

**Porosity effect on the heat transfer:** The effect of the porosity on the heat transfer in the heterojunction device is studied numerically. Porosity is an air layer defined by the two layers of SnO₂ and SiO₂. The thermal analysis is performed on the sample SnO₂/Air/SiO₂/Si in the form of a sandwich. A heating temperature, equal to 423K, is applied to the surface of SnO₂. The dimensions and the thermal conductivities of the various layers are presented in the following Table 1.

Similarly, the results are presented as temperature profiles, as the following Figure 8.

These curves show the evolution of the temperature during the heat transfer in the SnO₂/Air/SiO₂/Si, taking account of the conduction and convection, to interfaces SnO₂/Air and Air/SiO₂. The presence of the air layer (porosity) coming from a deposition problem of a semiconductor thin layer on a substrate prevents the temperature of the latter to evacuate quickly. This kind of transfer leads to a rapid temperature increase in the semiconductor layer. The three presentations of Figure 8 show the existence of two levels. The first level corresponds to the heating of the SnO₂ layer, and the second one corresponds to the heating of the Si/SiO₂ layer. Unfortunately, the intermediate portion between the two levels insulates thermally, the semiconductor of the substrate, and prevents the semiconductor temperature to evacuate quickly [16-19].

Figure 9 indicates clearly the difference between the sample of SnO₂/Air/SiO₂/Si and that of SnO₂/SiO₂/Si. The difference between the two curves is becoming more important with time. This difference is due to the presence of porosity. It traps the temperature in the SnO₂ layer. For a time t=100 ns remarkable jump between the two curves is presented. Then, the presence of the porosity between the thin layer and the substrate highly insulates the top of the sample (thin layer) and entraps the temperature in the latter; this reduces the heat transfer phenomenon from the top to the bottom of the sample.

**Conclusion**

The two-dimensional heat transfer equation, that is coupled to the initial and boundary conditions at different interfaces of heterojunction device (SnO₂/SiO₂/Si) and/or (SnO₂/Air/SiO₂/Si), taking into account the variability of the thermal conductivity of the tin dioxide, is studied numerically. If the contact between the tin dioxide layer and the substrate is made without the presence of a porosity (air layer), the

<table>
<thead>
<tr>
<th>Materials</th>
<th>Dimensions</th>
<th>Thermal Conductivities</th>
</tr>
</thead>
<tbody>
<tr>
<td>SnO₂</td>
<td>1 µm*0.2 µm</td>
<td>-2.136 × 10⁻⁴ × T²+0.078 × T+27.484 W/(m.K)</td>
</tr>
<tr>
<td>SiO₂</td>
<td>1 µm*50 nm</td>
<td>1.4 W/(m.K)</td>
</tr>
<tr>
<td>Si</td>
<td>1 µm*1 µm</td>
<td>149 W/(m.K)</td>
</tr>
<tr>
<td>Air</td>
<td>1 µm*25 nm</td>
<td>3.7 × 10⁻⁴T(0.32 × 10⁻⁴-2.59 × 10⁻⁸ × T)</td>
</tr>
</tbody>
</table>

**Table 1:** Dimensions and thermal conductivities of SnO₂/Air/SiO₂/Si.

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**Figure 8:** Axial evolution of the temperature in SnO₂/Air/SiO₂/Si in 1,100 and 1000 ns.
coupling is done by the continuity of the conduction at the interfaces. Otherwise, the coupling is realized by the continuity of the conduction and convection at the interfaces. The mathematical model is schemed by two different methods: the explicit finite difference method and the finite element method. The results are the following:

1. Comparison between both results of the two numerical methods showed satisfactory qualitative and quantitative agreements to both Si/ 
   SiO₂ and Si/SiO₂/SnO₂ and that the results of the finite element method is more efficient than that of the finite difference ones.

2. The results of the finite element method show that the variable thermal conductivity of tin dioxide does not permit to apply a high thermal gradient between the surface of the sample and the other layers, which prevents to apply a temperature higher than 423 K.

3. The influence of layer thickness of tin dioxide on the heat transfer through the heterojunction device is studied. Therefore, the decrease of the initial value of the temperature in the deep layer of the sample is particularly important, if the thickness of the SnO₂ layer is large. Then, to exhaust heat through the depth of the sample, it is better to choose a thickness of the thin films of tin dioxide lower than 0.2 µm.

4. Finally, during the deposition of the thin layer, the presence of the porosity (air layer) between the thin layer and the substrate highly insulates the top of the sample (tin dioxide layer) and entraps the temperature in the latter; this reduces the heat transfer phenomenon from the top to the bottom of the sample. Thus, a thermal fatigue can contribute an electrical strain which decreases the electrical efficiency of the system.

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References


Figure 9: Comparison between the two sandwiches SnO₂/Air/SiO₂/Si and SnO₂/SiO₂/Si.