

# On the Class of Exact Solutions of MHD Layer Fluid Flow of Second Order Type by Creating Sinusoidal Disturbances on the Porous Boundary

Kulkarni SB\*

Department of Applied Mathematics, Finolex Academy of Management and Technology, Ratnagiri, Maharashtra, India

## Abstract

Exact solution of an incompressible fluid of second order type by causing disturbances in the liquid which is initially at rest due to bottom oscillating sinusoid ally has been obtained in this paper. The results presented are in terms of non-dimensional elastic-viscosity parameter ( $\beta$ ) which depends on the non-Newtonian coefficient and the frequency of excitation ( $\sigma$ ) of the external disturbance while considering the magnetic parameter ( $m$ ) and porosity ( $k$ ) of the medium into account. The flow parameters are found to be identical with that of Newtonian case as  $\beta \rightarrow 0$ ,  $m \rightarrow 0$  and  $k \rightarrow \infty$ .

**Keywords:** Elastico viscous fluid; Second order fluid; Electico viscous parameter; Porous media; Magnetic parameter

## Introduction

The study of flow past porous boundaries has several important applications in the fields of science, engineering, technology, bio physics, and astrophysics and space dynamics. Transpiration cooling of re-entry vehicles and rocket boosters, cross hatching on the ablative surfaces and film vaporization in combustion chambers are few such applications. In the chemical and nuclear reactors the problem assumes greater significance. Over a period of time, in all chemical reactors, slurry adheres to the walls of the reactor and gets consolidated. As a result of which, the chemical compounds within the reactor percolates through the boundaries causing either loss of production and or consuming more reaction time. In many situations, so as to reduce the reaction time which is a subject of prime importance, the reactor chamber is also subjected to sinusoidal vibrations. Further, due to the presence of charged particles either in the in the chemical or nuclear reactor, induced magnetic effects are generated. In which case, the problem is more complicated. Also, the problem assumes greater importance especially in biological systems where the secretion through glands is involved. Many times, either in the chemical processing units or in biological system, the secreted fluid is not only viscous but is also elasto viscous. The presence of elasto viscous nature of the fluid and the presence of magnetic field causes drastic effects in evaluating the characteristic features of the fluid flow. This motivated the study and analysis of the problem in greater detail. The knowledge of flow through porous media is useful in the recovery of crude oil efficiently from the pores of reservoir rocks by displacement with immiscible water. The flow through porous media occurs in the ground-water hydrology, irrigation, drainage problems and also in absorption and filtration processes in chemical engineering. This subject has wide spread applications to specific problems encountered in the civil engineering and agriculture engineering, and many industries. Thus the diffusion and flow of fluids through ceramic materials as bricks and porous earthenware has long been a problem of the ceramic industry. The Scientific treatment of the problem of irrigation, Soil erosion and tile drainage are present developments of porous media. In hydrology, the movement of trace pollutants in water systems can be studied with the knowledge of flow through porous media. The principles of this subject are useful in recovering the water for drinking and irrigation purposes. Thurson was the earliest to recognize the viscoelastic nature of blood and that the viscoelastic behavior is less prominent with increasing

shear rate. Lamb [1] has discussed the viscous flow over an oscillatory bottom earlier in his treatise on hydrodynamics. Berman [2] studied the problem of two-dimensional steady state Newtonian laminar flow in a channel with porous walls. An exact, analytical expression for the dependence of velocity on the pressure gradient has been derived. The response of a Coleman and Noll [3] second order viscoelastic fluid occupying a semi-infinite region due to harmonic oscillation of its bottom has been investigated later by Pattabhiramacharyulu [4,5]. Vidyanidhi and Nigam [6] studied secondary flow in rotating channel. Oscillatory motion of an electrically conducting viscoelastic fluid over a stretching sheet in a saturated porous medium was studied by Rajagopal [7]. A visco-elastic effect of non-Newtonian flow through porous media was studied by Gupta and Sridhar [8]. Pascal and Pascal [9] studied the visco elastic effects in non-Newtonian steady flows through porous media. Flow of a visco elastic fluid over a stretching sheet was studied by Rajagopal [10]. Ariel [11] studied the flow of a visco elastic fluid past a porous plate. Petrov [12] examined analytically the unsteady flow of Bingham fluid caused by an abruptly applied pressure gradient. With respect to the flows of non-Newtonian fluids between two parallel porous walls, Ariel [13] obtained exact analytical solutions of laminar flow of a second grade visco-elastic fluid employing two geometries. An oscillatory plate temperature effect of free convection flow of dissipative fluid between long vertical parallel plates was studied by Narahari [14]. Singh and Paul [15] presented an analysis of the transient free convective flow of a viscous incompressible fluid between two parallel vertical walls occurring as a result of asymmetric heating/cooling of the walls. Turkyilmazoglu [16] examined exact solutions for the incompressible viscous fluid of a porous rotating disk flow. Exact solution corresponding to viscous in compressible Newtonian conducting fluid flow due to a porous rotating disk by Turkyilmazoglu [17]. The effect of suction and blowing on purely analytic solutions of the compressible boundary layer flow due to a porous rotating disk with

\*Corresponding author: Kulkarni SB, Department of Applied Mathematics, Finolex Academy of Management and Technology, Ratnagiri, Maharashtra-415 639, India, Tel: 02352228361; E-mail: [sanjay\\_kulkarni1@yahoo.co.in](mailto:sanjay_kulkarni1@yahoo.co.in)

Received July 09, 2015; Accepted August 21, 2015; Published August 26, 2015

**Citation:** Kulkarni SB (2015) On the Class of Exact Solutions of MHD Layer Fluid Flow of Second Order Type by Creating Sinusoidal Disturbances on the Porous Boundary. J Appl Computat Math 4: 246. doi:10.4172/2168-9679.1000246

**Copyright:** © 2015 Kulkarni SB. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

heat transfer studied by Turkyilmazoglu [18]. Later, Kulkarni [19] had examined the problem of unsteady flow of an incompressible viscous electrically conducting fluid in the tube of elliptical cross section under the influence of the magnetic field. Subsequently, Kulkarni [20] studied the unsteady flow of an incompressible viscous fluid in a tube of spherical cross section on a porous boundary. Recently, Kulkarni [21] had examined the problem of unsteady MHD flow of elastico – viscous incompressible fluid through a porous media between two parallel plates under the influence of magnetic field.

The aim of the present paper is to study a class of exact solutions for the flow of incompressible electrically conducting elastic-viscous fluid of second order fluid by taking into account the magnetic field and porosity factor of the bounding surfaces and compare the results with those in the Newtonian case. A uniform magnetic field is introduced normal to the direction of the flow. Further, it is assumed that, the magnetic Reynolds number is much less than unity, so that the induced magnetic field is neglected in comparison with the applied magnetic field. We study the disturbance due to sinusoidal oscillation of the bottom of a semi-infinite depth. The results are expressed in terms of a non-dimensional porosity parameter  $k$ , which depends on the non-Newtonian coefficient  $\phi_2$  and the frequency of excitation  $\sigma$ . It is noticed that the flow properties are identical with those in the Newtonian case  $\beta \rightarrow 0, k \rightarrow \infty$  and  $m \rightarrow 0$ .

### Mathematical Formulation of the Problem

The momentum equation of the fluid flowing through a generalized porous medium as suggested by Yamamoto [22] and Yamamoto Iwamura [23] is given by

$$\rho \frac{dq}{dt} = \text{div} S - \frac{\mu}{k} \bar{q}$$

and the continuity equation for incompressible homogeneous fluid

$$\text{div} \bar{q} = 0$$

Noll [24] defined a simple material as a substance for which stress can be determined with entire knowledge of the history of the strain. This is called simple fluid, if it has property that all local states, with the same mass density, are intrinsically equal in response, with all observable differences in response being due to definite differences in the history. For any given history  $g(s)$ , a retarded history  $g_\psi(s)$  can be defined as:

$$g_\psi(s) = g(\psi s) : 0 \leq s \leq \infty, 0 \leq \psi \leq 1 \tag{1}$$

$\psi$  being termed as a retardation factor. Assuming that the stress is more sensitive to recent deformation than to the deformations at distant past, Coleman and Noll proved that the theory of simple fluids yields the theory of perfect fluids as  $\psi \rightarrow 0$  and that of Newtonian Fluids as a correction (up to the order of  $\psi$ ) to the theory of the perfect fluids. Neglecting all the terms of the order of higher than two in  $\psi$ , We have incompressible elastico viscous fluid of second order type whose constitutive relation is governed by:

$$S = -PI + \phi_1 E^{(1)} + \phi_2 E^{(2)} + \phi_3 E^{(1)^2} \tag{2}$$

where

$$E_{ij}^1 = U_{i,j} + U_{j,i} \tag{3}$$

and

$$E_{ij}^2 = A_{i,j} + A_{j,i} + 2U_{m,i}U_{m,j} \tag{4}$$

In the above equations,  $S$  is the stress-tensor,  $U_i$  and  $A_i$  are the components of velocity and acceleration in the direction of the  $i^{th}$

coordinate  $X_i$  while  $P$  is indeterminate hydrostatic pressure. The coefficients  $\phi_1, \phi_2$  and  $\phi_3$  are material constants. The constitutive relation for general Rivlin-Ericksen [25] fluid also reduces to equation (2) when the squares and higher orders of  $E^2$  are neglected, while the coefficients being constants. Also the non-Newtonian models considered by Reiner [26] could be obtained from equation (2) when  $\phi_2 = 0$  and naming  $\phi_3$  as the coefficient of cross viscosity. With reference to the Rivlin-Ericksen fluids  $\phi_2$  may be called as the coefficient of viscosity.

It has been reported that a solution of poly-iso-butylene in cetane behaves as a second order fluid and that Markovitz determined the constants  $\phi_1, \phi_2$  and  $\phi_3$ . In many of the chemical processing industries, slurry adheres to the reactor vessels and gets consolidated. As a resultant of this, the chemical compounds within the reactor vessel percolates through the boundaries causing loss of production and consuming more reaction time. In view of such technological and industrial importance wherein the heat and mass transfer takes place in the chemical industry, the problem by considering the permeability of the bounding surfaces in the reactors attracts the attention of several investigators.

Introducing the following non dimensional variables as:

$$U_i = \frac{\phi_1 u_i}{\rho L} \quad T = \frac{\rho L^2 t}{\phi_1} \quad \phi_2 = \rho L^2 \beta \quad P = \frac{\phi_1^2 p}{\rho L^2}$$

$$\frac{X_i}{L} = x_i \quad \frac{Y_i}{L} = y_i \quad \varphi \quad \rho \quad \nu \quad A_i = \frac{\phi_1^2 a_i}{\rho^2 L^3}$$

$$S_{i,j} = \frac{\phi_1^2 s_{i,j}}{\rho L^2} \quad E_{i,j}^{(1)} = \frac{\phi_1 e_{i,j}^{(1)}}{\rho L^2} \quad E_{i,j}^{(2)} = \frac{\phi_1^2 e_{i,j}^{(2)}}{\rho^2 L^4} \quad K = \frac{k L^2}{\phi_1} \quad M = \frac{m \phi_1}{L^2}$$

where  $T$  is the (dimensional) time variable, and  $\rho$  the mass density and  $L$  a characteristic length.

We consider a class of plane flows given by the velocity components

$$u_1 = u(y, t) \quad \text{and} \quad u_2 = 0 \tag{5}$$

in the directions of rectangular Cartesian coordinates  $x$  and  $y$ . The velocity field given by (5) identically satisfies the incompressibility condition. The stress can now be obtained in the non-dimensional form as:

$$s_{xx} = -p + \nu_c \left( \frac{\partial u}{\partial y} \right)^2 \tag{6}$$

$$s_{yy} = -p + (\nu_c + 2\beta) \left( \frac{\partial u}{\partial y} \right)^2 \tag{7}$$

$$s_{xy} = \frac{\partial u}{\partial y} + \beta \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial t} \right) \tag{8}$$

In view of the above, the equations of motion in the present case of porous boundary will yield

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \beta \frac{\partial}{\partial t} \left( \frac{\partial^2 u}{\partial y^2} \right) - \left( \frac{1}{k} + m \right) u \tag{9}$$

and

$$0 = -\frac{\partial p}{\partial y} + (2\beta + \nu_c) \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right)^2 \tag{10}$$

Eqn (9) shows that  $-\frac{\partial p}{\partial x}$  must be independent of space variables

and hence may be taken as  $\xi(t)$ . Eqn (10) now yields

$$p = p_0(t) - \xi(t)x + (v_c + 2\beta)\left(\frac{\partial u}{\partial y}\right)^2 \quad (11)$$

Considering  $\xi(t) = 0$ , the flow characterized by the velocity is given by:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \beta \frac{\partial}{\partial t} \left( \frac{\partial^2 u}{\partial y^2} \right) - \left( \frac{1}{k} + m \right) u \quad (12)$$

where  $k$  is the non-dimensional porosity constant. It may be noted that the presence of  $\beta$  changes the order of differential from two to three.

### Disturbance of a Liquid at Rest due to the Sinusoidal Oscillations of the Bottom

The oscillations of a classical viscous liquid on the upper half of the plane  $y \geq 0$  with the bottom oscillating with the velocity  $\alpha e^{i\sigma t}$  are examined in the present case. The motion of the second order fluid is governed by Equation (12) with boundary conditions

$$u(0, t) = \alpha e^{i\sigma t} \quad (13)$$

$$u(\infty, t) = 0 \quad (14)$$

Assuming the trial solution as:

$$u(y, t) = \alpha e^{i\sigma t} f(y) \quad (15)$$

$$f''(y) = p^2 f(y) \quad (16)$$

$$\text{Where } p^2 = \frac{i\sigma + \frac{1}{K} - (\beta\sigma^2 + \frac{1}{k} + m) + i(\sigma - (\frac{\beta\sigma}{k} + m\beta\sigma))}{1 + i\beta\sigma} \quad (17)$$

When expressed in polar form

$$p = r \left( \cos\left(\frac{\pi}{4} - \frac{\varepsilon}{2}\right) + i \sin\left(\frac{\pi}{4} - \frac{\varepsilon}{2}\right) \right) \quad (18)$$

$$r = \frac{[(\beta\sigma^2 + \frac{1}{k} + m)^2 + (\sigma - (\frac{\beta\sigma}{k} + m\beta\sigma))^2]^{1/2}}{\sqrt{(1 + \beta^2\sigma^2)}}, \quad \varepsilon = \tan^{-1}(Q) \quad \text{and}$$

$$Q = \frac{\frac{1}{k} + \beta\sigma^2 + m}{\sigma - (\frac{\beta\sigma}{k} + m\beta\sigma)}$$

Also the conditions satisfied by  $f(y)$  are

$$f(0) = 1, f(\infty) = 0 \quad (19)$$

This yields the solution

$$f(y) = \exp\left(-yr \left( \cos\left(\frac{\pi}{4} - \frac{\varepsilon}{2}\right) + i \sin\left(\frac{\pi}{4} - \frac{\varepsilon}{2}\right) \right)\right) \quad (20)$$

and hence

$$u(y, t) = \alpha \exp\left(i\sigma t - yr \left( \cos\left(\frac{\pi}{4} - \frac{\varepsilon}{2}\right) + i \sin\left(\frac{\pi}{4} - \frac{\varepsilon}{2}\right) \right)\right) \quad (21)$$

The flow is thus represented by standing transverse wave with its amplitude rapidly diminishing with increasing distance from the plane. This phenomenon is independent of  $v_c$  as noticed for all two-dimensional flows.

The magnification factor  $A^*$  of the amplitude of this wave, with respect to the amplitude of the disturbance ( $a$ ), may be written as

$$(A^*)^2 = (\text{Real part of } u(y, t))^2 + (\text{Imaginary part } u(y, t))^2 \quad (22)$$

$$A^* = \alpha \exp\left(-y\sqrt{r} \cos\left(\frac{\pi}{4} - \frac{\varepsilon}{2}\right)\right) \quad (23)$$

which is in the form of  $A^* = e^{-\chi y^*}$

$$\text{where, } \chi y^* = \frac{y\sqrt{r}}{2} \left[ \cos\left(\frac{\varepsilon}{2}\right) + \sin\left(\frac{\varepsilon}{2}\right) \right] \quad (24)$$

Where

$$\chi = \frac{1}{(1 + \beta^2\sigma^2)^{1/4}} \sqrt{\frac{Q + \sqrt{1 + Q^2}}{1 + Q^2}} \quad (25)$$

and

$$y^* = \frac{y(1 + \beta^2\sigma^2)^{1/4}}{\sqrt{2}} \left[ \frac{\left(\frac{1}{k} + m + \beta\sigma^2\right)^2 + \left(\sigma - \left(\frac{\beta\sigma}{k} + m\beta\sigma\right)\right)^2}{(1 + \beta^2\sigma^2)^2} \right]^{1/4} \quad (26)$$

$$\text{The flow rate is given by } \int_0^1 u(y, t) dy \quad (27)$$

The skin friction on the plate is given by

$$= \left\{ \frac{\partial u}{\partial y} + \beta \left[ \frac{\partial^2 u}{\partial y \partial t} - \frac{\partial^2 u}{\partial y^2} \right] \right\}_{y=0} \quad (28)$$

### Discussions and Conclusions

The variation of velocity profiles for various values of the porosity factor ( $k$ ) are illustrated in Figure 1. It is seen that the velocity decreases as we move away from the plate. Further, as the porosity factor ( $k$ ) increases, the velocity also increases.

Figure 2 illustrates the velocity profiles for different values of magnetic parameter ( $m$ ). It is observed that, as  $m$  increases, there is a decreasing trend in the velocity profiles. This can be attributed due to the strong intra molecular forces.

The variations of velocity for various values of the time are

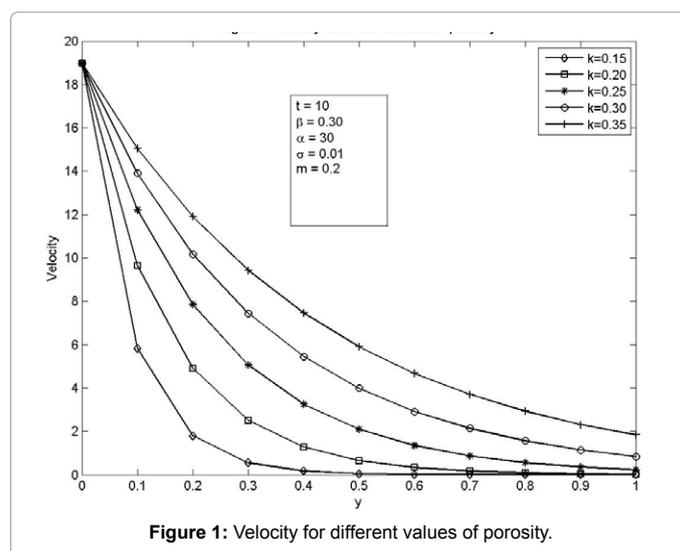


Figure 1: Velocity for different values of porosity.

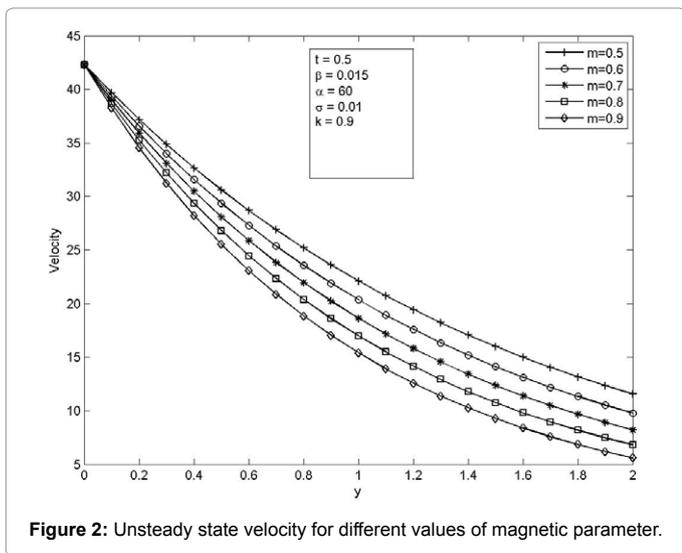


Figure 2: Unsteady state velocity for different values of magnetic parameter.

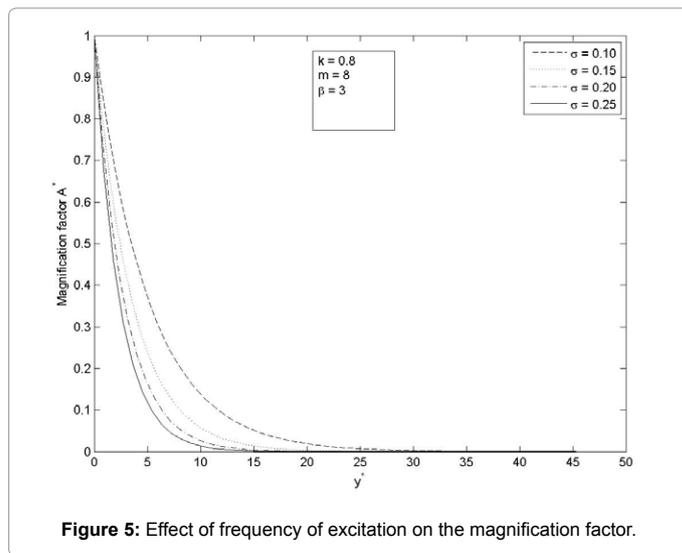


Figure 5: Effect of frequency of excitation on the magnification factor.

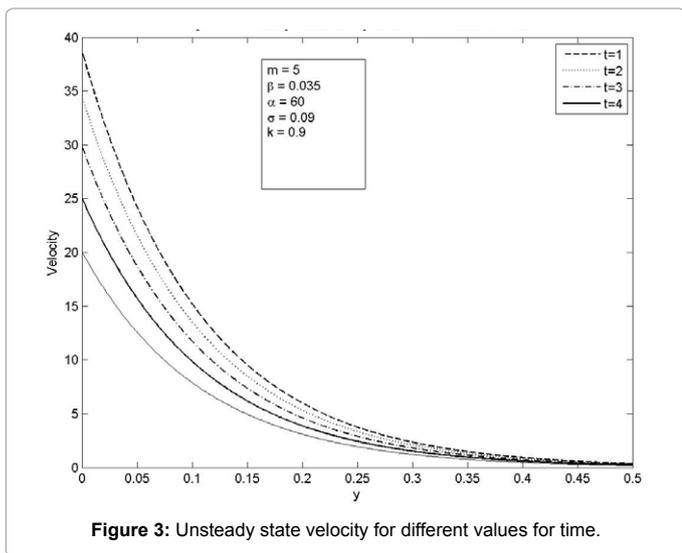


Figure 3: Unsteady state velocity for different values for time.

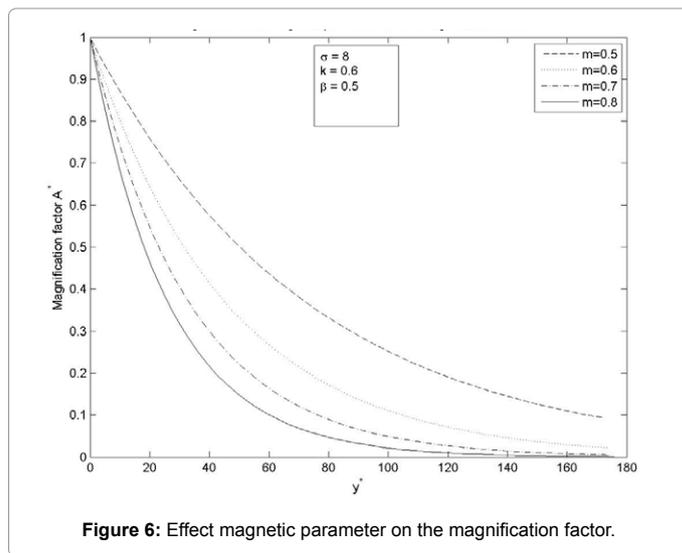


Figure 6: Effect magnetic parameter on the magnification factor.

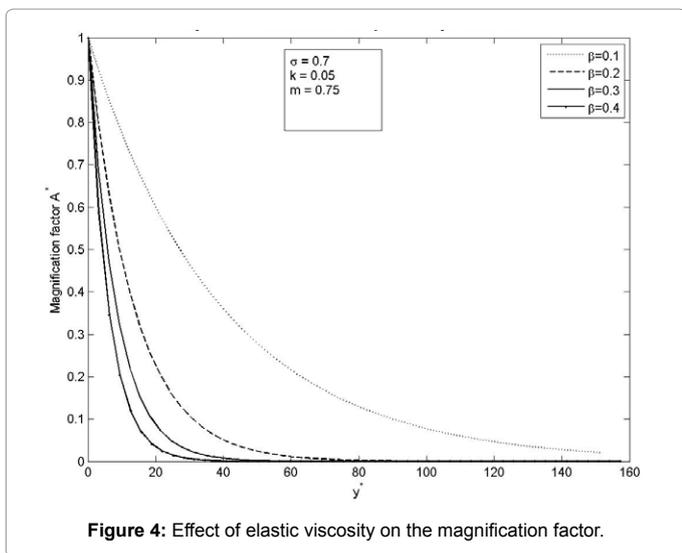


Figure 4: Effect of elastic viscosity on the magnification factor.

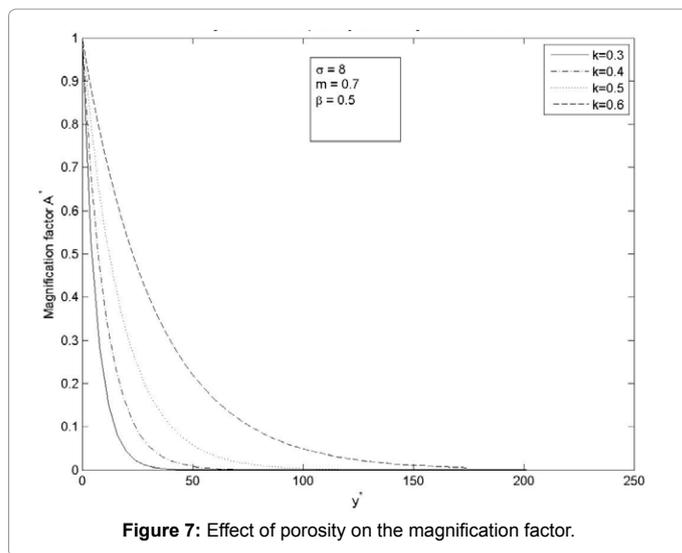
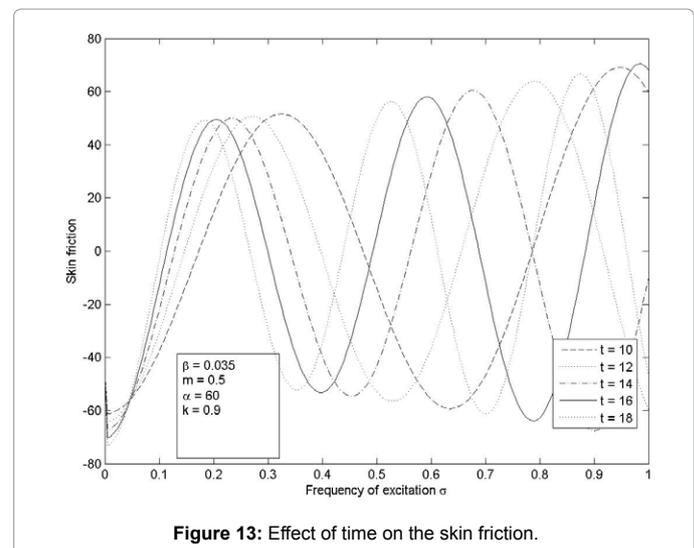
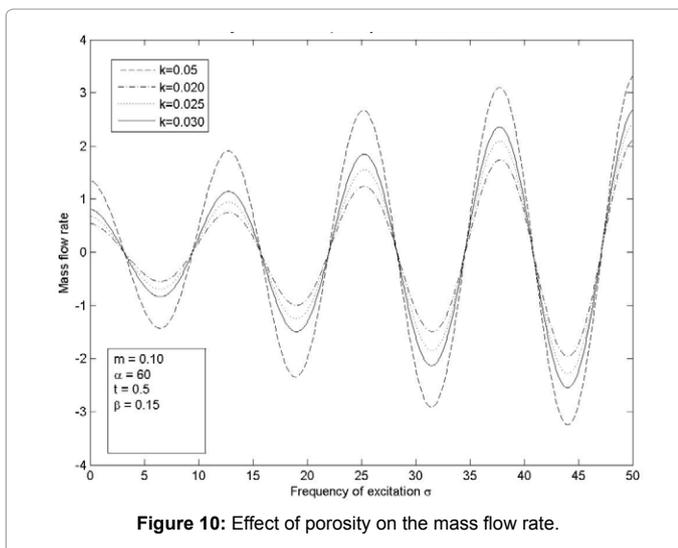
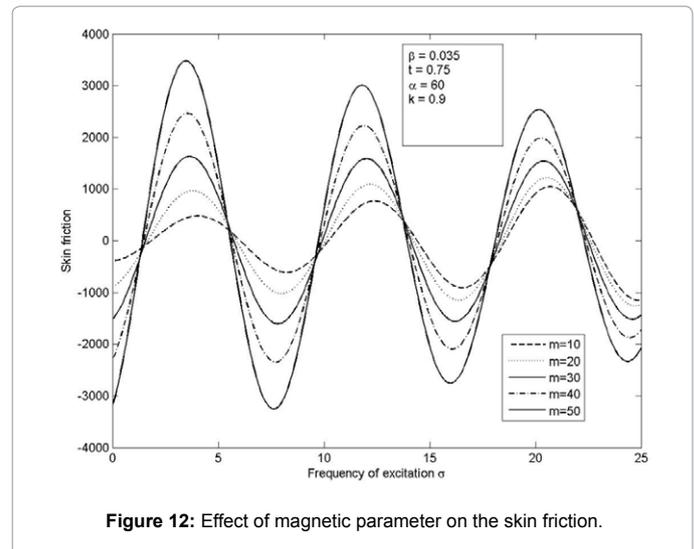
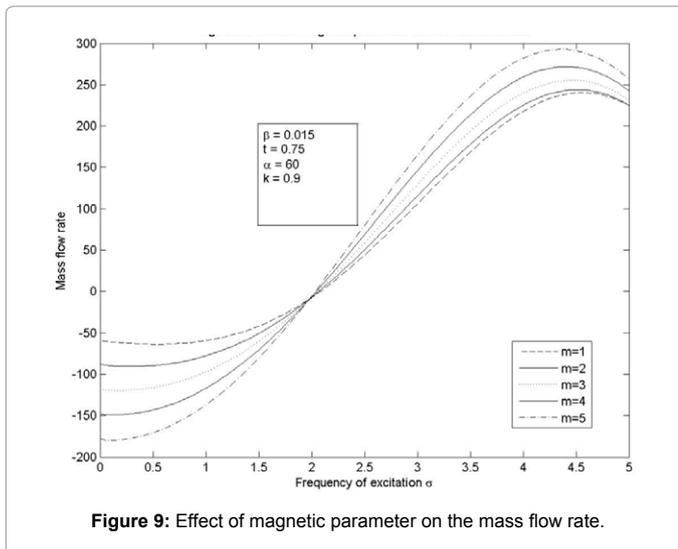
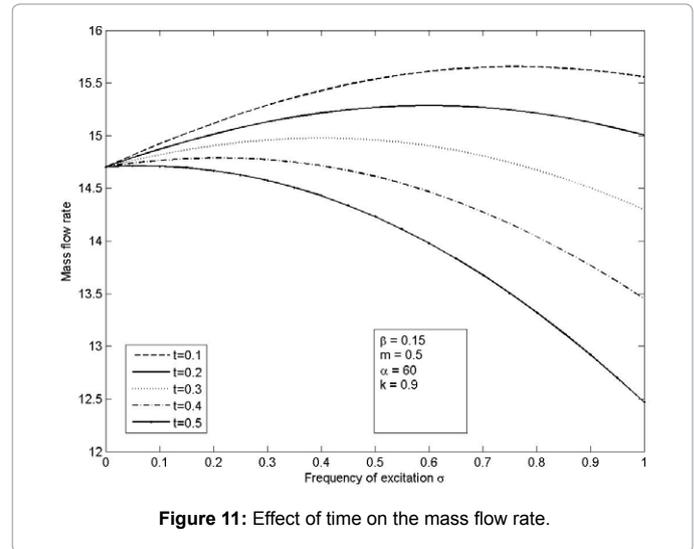
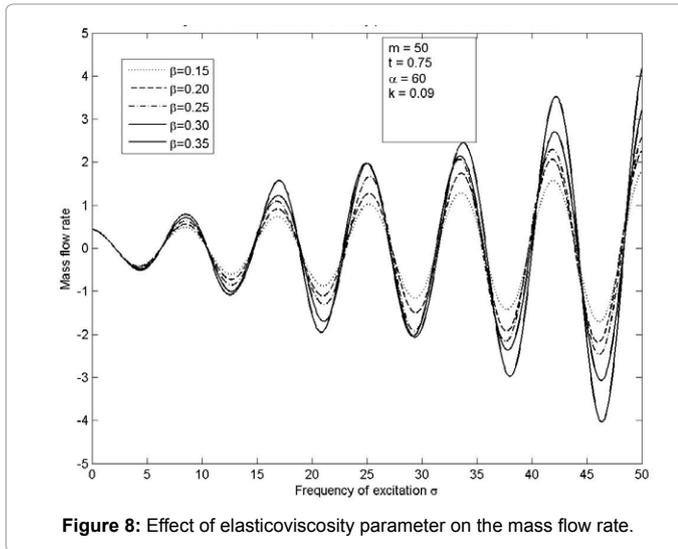


Figure 7: Effect of porosity on the magnification factor.



illustrated in Figure 3. It is seen that velocity decreases as time increases.

It is noticed from the Figures 4-6 the effect of elastic-viscosity parameter ( $\beta$ ), frequency of excitation ( $\sigma$ ) and magnetic parameter ( $m$ ) on the magnification factor ( $A^*$ ). It is seen that as  $\beta$  increases, there is a decreasing trend in the magnification factor. From the Figure 7 it is observed that porosity factor increases the magnification factor also increases.

It is observed from the Figures 8-11 the effect of elastic-viscosity parameter ( $\beta$ ), magnetic parameter ( $m$ ), porosity factor ( $k$ ) and time on the mass flow rate. It is seen that the constant values of  $\beta$ ,  $m$ ,  $k$  and  $t$  the frequency of excitation ( $\sigma$ ) increases the mass flow rate is also increases.

It is observed from the Figures 12 and 13 the effect of magnetic parameter ( $m$ ), and time on the skin friction. It is seen that the constant value of  $m$  the frequency of excitation ( $\sigma$ ) increases the skin friction is decreases the reverse trend is observed for time parameter.

As  $k \rightarrow \infty$ , the results obtained for the velocity field in agreement to that of Murthy et al. [27]. The case of Newtonian fluid can be realized as  $\beta \rightarrow 0$ ,  $k \rightarrow \infty$  and  $m \rightarrow 0$ .

## References

1. Lamb H (1932) Hydrodynamics. Journal of Engg Mathematics 10: 41-54.
2. Berman AS (1958) Laminar flow in an annulus with porous walls. Journal of Appl Phy 29: 71-75.
3. Coleman BD, Noll W (1960) Archives Rational Mechanics and analysis 6: 355-370.
4. Ramacharyulu PN (1964) Applied Science Research 14: 268.
5. Ramacharyulu PN (1966) The problem in the study of of non-Newtonian fluid dynamics. Phd thesis, submitted OU, Hyderabad, India.
6. Vidyanidhi V, Nigam, SD (1967) Secondary flow in rotating channel. Journal of Maths and Physical Sciences 1: 85-97.
7. Rajagopal KR, Na TY, Gupta AS (1984) Flow of a viscoelastic fluid over a stretching sheet. Journal of Rheology 23: 213-215.
8. Gupta RK, Sridhar T (1985) Viscoelastic effects in non-Newtonian flow through porous media. Rheo Acta 24: 148-151.
9. Pascal H, Pascal F (1989) On viscoelastic effects in non-Newtonian steady flows through porous media. Transport in Porous Media 4: 17-35.
10. Rajagopal K, Veena PH, Pravin VK (2006) Oscillatory motion of an electrically conducting viscoelastic fluid over a stretching sheet in saturated porous medium with suction/blowing. Mathematical problems in Engg: 1-14.
11. Ariel PD (1994) The flow of a viscoelastic fluid past a porous plate. Acta Mechanica 107: 199-204.
12. Petrov AG (2000) The development of the flow of viscous and viscoelastic media between two parallel plate. Journal of Appl Mathematics Mech 64:123-132.
13. Ariel PD (2002) On exact solutions of flow problems of a second grade fluid through two parallel walls. International Journal of Engg Science 40: 913-941.
14. Narahari M (2005) Oscillatory plate temperature effects of free convection flow of dissipative fluid between long vertical parallel plates. International Journal of App I Math and Mech 5: 30-46.
15. Singh AK, Paul T (2006) Transient natural convection between two vertical walls heated/cooled asymmetrically. International Journal of Appl Mechanics and Engineering 11: 143-154.
16. Turkyilmazoglu M (2009) Exact solutions for the incompressible viscous fluid of a porous rotating disk flow. International journal of non-linear mechanics 44: 352-357.
17. Turkyilmazoglu M (2009) Exact solution corresponding to viscous incompressible Newtonian conducting fluid flow due to a porous rotating disk. Journal of Heat Transfer 131: 1-7.
18. Turkyilmazoglu M (2009) Purely analytic solutions of the compressible boundary layer flow due to a porous rotating disk with heat transfer. Physics of Fluids 21: 106104.
19. Kulkarni SB (2014) Unsteady flow of an incompressible viscous electrically conducting fluid in tub of elliptical cross section under the influence of magnetic field. International Journal of Mathematical, Computational, Physical and Quantum Engineering 8: 1311-1317.
20. Kulkarni SB (2015) Unsteady Poiseuille flow of an incompressible Elastico-Viscous fluid in a tube of spherical cross section on a porous boundary. International Journal of Mechanical Aero Space Industrial and Mechatronics Engineering 9: 240-246.
21. Kulkarni SB (2015) Unsteady MHD flow of Elastico-Viscous incompressible fluid through a porous media between two parallel plates under the influence of magnetic field. Defence Science Journal 65: 119-125.
22. Yamamoto K, Yosida Z (1974) Flow through a porous wall with convective acceleration. Journal of Physical society of Japan 37: 774-779.
23. Yamamoto K, Iwamura N (1976) Flow with convective acceleration through a porous. Journal of Engineering Mathematics 10: 41-54.
24. Noll WA (1958) Mathematical theory of mechanical behaviour of continuous media. Arch Ratl Mech and Anal 2: 197-226.
25. Rivlin RS, Ericksen JL (1955) Stress relaxation for isotropic materials. J Rat Mech and Anal 4: 350-362.
26. Reiner MA (1945) Mathematical theory of diletancy. Amer J of Maths 67: 350-362.
27. Murthy RV, Kulkarni SB (2007) On the class of exact solutions of an incompressible fluid flow of second order type by creating sinusoidal disturbances. Def Sci J 2: 197-209.