On the Consecutive Integers $n+i-1=(i+1)P_i$

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Abstract

By using the Jiang’s function $J_i(x)$ we prove that there exist infinitely many integers $n$ such that $n=2P_i$, $n+1=3P_i$, $n+k-1=(k+1)P_i$ are all composites for arbitrarily long $k$, where $P_i$, $P_2$, $P_3$, $P_4$, $P_5$ are all primes. This result has no prior occurrence in the history of number theory.

Keywords: Consecutive integers; Jiang’s function

Introduction

Theorem 1

There exist infinitely many integers $n$ such that the consecutive integers $n=2P_i$, $n+1=3P_i$, $n+k-1=(k+1)P_i$ are all composites for arbitrarily long $k$, where $P_i$, $P_2$, $P_3$, $P_4$, $P_5$ are all primes.

Proof: Suppose that $P_i = \frac{m}{i+1}x + 1$. We define the prime equations:

\begin{equation}
P_i = \frac{m}{i+1}x + 1,
\end{equation}

where $i = 1, 2, \ldots, k$.

The Jiang’s function [1] is:

\begin{equation}
J_i(x) = \Pi(P - k - 1 - \chi(P)) \neq 0
\end{equation}

Where $P = k$ if $Pm$, $\chi(P) = k + 1$ if $Pm$; $\chi(P) = 0$ otherwise, $\omega = \frac{m}{i+1}P_i$.

Since $J_i(x) \rightarrow \infty$ as $\omega \rightarrow \infty$, there exist infinitely many integers $x$ such that $P_i$, $P_2$, $P_3$, $P_4$, $P_5$ are all primes.

We have the asymptotic formula of the number of integers $x \leq N$ [1]

\begin{equation}
\pi_{i+1}(N, 2) \sim \frac{J_i(x)}{\varphi^{\omega+1}(x)} N \log\frac{x}{N},
\end{equation}

(3)

Where, $\varphi(x) = \Pi(P - 2)$.

From (1) we have,

\begin{align*}
n &= mx + 2 \left( \frac{m}{2}x + 1 \right) = 2P_i, \\
n + 1 &= mx + 3 = 3\left( \frac{m}{3}x + 1 \right) = 3P_2, \\
&\vdots \\
n + k - 1 &= mx + k + 1 = (k + 1)\left( \frac{m}{k+1}x + 1 \right) = (k + 1)P_3.
\end{align*}

Example 1: Let $k = 5$, we have $n = 2 \times 53281$, $n+1=3 \times 35521$, $n+2=4 \times 26641$, $n+3=5 \times 21313$, $n+4=6 \times 17761$.

Theorem 2

There exist infinitely many integers $n$ such that the consecutive integers $n=(1+2^k)P_i$, $n+1=(2+2^k)P_i$, $\ldots$, $n+k-1=(k+2^k)P_i$ are all composites for arbitrarily long $k$, where $P_i$, $P_2$, $P_3$, $P_4$, $P_5$ are all primes [2].

Proof: Suppose that $m = \Pi(i + 2^k)$. We define the prime equations:

\begin{equation}
P_i = \frac{m}{i+2}x + 1,
\end{equation}

Where $i = 1, 2, \ldots, k$.

The Jiang’s function [1] is:

\begin{equation}
J_i(x) = \Pi\left(P - k - 1 - \chi(P)\right) \neq 0
\end{equation}

Where $\chi(P) = k$ if $Pm$; $\chi(P) = k + 1$ if $Pm$; $\chi(P) = 0$ otherwise.

Since, $J_i(x) \rightarrow \infty$ as $\omega \rightarrow \infty$, there exist infinitely many integers $x$ such that $P_i$, $P_2$, $P_3$, $P_4$, $P_5$ are all primes.

We have the asymptotic formula of the number of integers $x \leq N$ [1]

\begin{equation}
\pi_{i+1}(N, 2) \sim \frac{J_i(x)}{\varphi^{\omega+1}(x)} N \log\frac{x}{N},
\end{equation}

(4)

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The Jiang’s function $J_1(\omega)$ is:

\[ J_1(\omega) = \prod_{P \leq \omega} (P - k - 1 - \chi(P)) \neq 0 \]  

(8)

Where $\chi(P) = -1$ if $P^m m; \chi(P) = -k + 1$ if $P m; \chi(P) = 0$ otherwise.

Since, $J_1(\omega) \rightarrow \infty$ as $\omega \rightarrow \infty$, there exist infinitely many integers $x$ such that $P_1, P_2, \ldots, P_k$ are all primes.

We have the asymptotic formula of the number of integers $x \leq N$ for $[1]$

\[ \pi_{\omega,x}(N, 2) \sim \frac{J_1(\omega)\omega^k N}{\varphi^{k+1}(\omega) \log^{k+1} N}. \]  

(9)

From (7) we have:

\[ n = mx + 3 = 3\left(\frac{m}{3}\right)x + 1) = 3P_1, \]

\[ n + 2 = mx + 5 = 5\left(\frac{m}{5}\right)x + 1) = 5P_1, \ldots, \]

\[ n + 2(k - 1) = mx + 2k + 1 = (2k + 1)\left(\frac{m}{2k + 1}\right)x + 1) = (2k + 1)P_k. \]

Example 3: Let $k = 4$, we have $n = 3 \times 631, n + 2 = 5 \times 379, n + 4 = 7 \times 271, n + 6 = 9 \times 211$.

Theorem 4

There exist infinitely many integers $n$ such that the consecutive integers $n + 2 = 3P_1, n + 4 = 3P_2, n + 4 = 3P_3, \ldots, n + 4 = 3P_k$ are all composites for arbitrarily long $k$, where $P_1, P_2, \ldots, P_k$ are all primes.

Proof: Suppose that $m = \Pi_{i=1}^{k}(2i - 1)$. We define the prime equations:

\[ P = \frac{1}{2}x^2 + \frac{1}{2}x + 1 \]  

(10)

Where $i = 1, 2, \ldots, k$.

The Jiang’s function $[1]$ is:

\[ J_1(\omega) = \prod_{P \leq \omega} (P - k - 1 - \chi(P)) \neq 0 \]  

(11)

Where $\chi(P) = -1$ if $P^m m; \chi(P) = -k + 1$ if $P m; \chi(P) = 0$ otherwise.

Since, $J_1(\omega) \rightarrow \infty$ as $\omega \rightarrow \infty$, there exist infinitely many integers $x$ such that $P_1, P_2, \ldots, P_k$ are all primes.

We have the asymptotic formula of the number of integers $x \leq N$ for $[1]$

\[ \pi_{\omega,x}(N, 2) \sim \frac{J_1(\omega)\omega^k N}{\varphi^{k+1}(\omega) \log^{k+1} N}. \]  

(12)

From (10) we have:

\[ n = P_1 = mx + 1, \]

\[ n + 2 = mx + 3 = \frac{3m}{3}x + 1) = 3P_2, \ldots, \]

\[ n + 2(k - 1) = mx + 2k + 1 = (2k + 1)\left(\frac{m}{2k + 1}\right)x + 1) = (2k + 1)P_k. \]

Example 4: Let $k = 4$, we have $n = 9661, n + 2 = 3 \times 3221, n + 4 = 5 \times 1933, n + 6 = 7 \times 1381$.

Theorem 5

There exist infinitely many integers $n$ such that the consecutive integers $n + 2 = 3P_1, n + 4 = 3P_2, n + 4 = 3P_3, n + 4 = 3P_4 (k - 1) = (4k + 1) P_k$ are all composites for arbitrarily long $k$, where $P_1, P_2, P_3, P_k$ are all primes.

Example 5: Let $k = 4$, we have $n = 3 \times 2311, n + 4 = 7 \times 991, n + 8 = 11 \times 631, n + 12 = 15 \times 463$.

Theorem 6

There exist infinitely many integers $n$ such that the consecutive integers $n + 2 = 3P_1, n + 4 = 3P_2, n + 4 = 3P_3, n + 4 = 3P_4 (k - 1) = (4k + 1) P_k$ are all composites for arbitrarily long $k$, where $P_1, P_2, P_3, P_k$ are all primes.

Conclusion

Jiang’s function $J_1(\omega)$ prove that there exist infinitely many integers $n$ such that $n = 2P_1, n + 1 = 3P_2, n + 8 = 5P_3, n + 12 = 7P_4 (k - 1) = (4k + 1) P_k$ are all composites for arbitrarily long $k$, where $P_1, P_2, P_3, P_k$ are all primes which result has no prior occurrence in the history of number theory.

References