

On the Consecutive Integers $n+i-1=(i+1) P_i$

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Abstract

By using the Jiang's function $J_2(\omega)$ we prove that there exist infinitely many integers n such that $n=2P_1, n+1=3P_2, \dots, n+k-1=(k+1) P_k$ are all composites for arbitrarily long k , where P_1, P_2, \dots, P_k are all primes. This result has no prior occurrence in the history of number theory.

Keywords: Consecutive integers; Jiang's function

Introduction

Theorem 1

There exist infinitely many integers n such that the consecutive integers $n=2P_1, n+1=3P_2, \dots, n+k-1=(k+1) P_k$ are all composites for arbitrarily long k , where P_1, P_2, \dots, P_k are all primes.

Proof: Suppose that $P_i = \frac{m}{i+1}x+1$. We define the prime equations:

$$P_i = \frac{m}{i+1}x+1, \quad (1)$$

Where $i=1, 2, \dots, k$

The Jiang's function [1] is:

$$J_2(\omega) = \prod_{3 \leq P} (P-k-1-\chi(P)) \neq 0 \quad (2)$$

Where $\chi(P)=-k$ if $P^2 m$; $\chi(P)=-k+1$ if Pm ; $\chi(P)=0$ otherwise, $\omega = \prod_{2 \leq P} P$.

Since $J_2(\omega) \rightarrow \infty$ as $\omega \rightarrow \infty$, there exist infinitely many integers x such that P_1, P_2, \dots, P_k are all primes.

We have the asymptotic formula of the number of integers $x \leq N$ [1]

$$\pi_{k+1}(N, 2) \sim \frac{J_2(\omega)\omega^k}{\varphi^{k+1}(\omega)} \frac{N}{\log^{k+1} N}, \quad (3)$$

Where, $\varphi(\omega) = \prod_{2 \leq P} (P-2)$.

From (1) we have,

$$n = mx + 2 = 2\left(\frac{mx}{2} + 1\right) = 2P_1,$$

$$n + 1 = mx + 3 = 3\left(\frac{m}{3}x + 1\right)$$

$$= 3P_2, \dots, n + k - 1 = mx + k + 1 = (k+1)\left(\frac{m}{k+1}x + 1\right) = (k+1)P_k.$$

Example 1: Let $k=5$, we have $n=2 \times 53281, n+1=3 \times 35521, n+2=4 \times 26641, n+3=5 \times 21313, n+4=6 \times 17761$.

Theorem 2

There exist infinitely many integers n such that the consecutive integers $n=(1+2^b) P_1, n+1=(2+2^b) P_2, \dots, n+k-1=(k+2^b) P_k$ are all composites for arbitrarily long k , where P_1, P_2, \dots, P_k are all primes [2].

Proof: Suppose that $m = \prod_{i=1}^k (i+2^b)$. We define the prime equations:

$$P_i = \frac{m}{i+2^b}x+1, \quad (4)$$

Where $i=1, 2, \dots, k$.

The Jiang's function [1] is:

$$J_2(\omega) = \prod_{3 \leq P} (P-k-1-\chi(P)) \neq 0 \quad (5)$$

Where $\chi(P)=-k$ if $P^2 m$; $\chi(P)=-k+1$ if Pm ; $\chi(P)=0$ otherwise.

Since, $J_2(\omega) \rightarrow \infty$ as $\omega \rightarrow \infty$, there exist infinitely many integers x such that P_1, P_2, \dots, P_k are all primes.

We have the asymptotic formula of the number of integers $x \leq N$ [1]

$$\pi_{k+1}(N, 2) \sim \frac{J_2(\omega)\omega^k}{\varphi^{k+1}(\omega)} \frac{N}{\log^{k+1} N}, \quad (6)$$

From (4) we have:

$$n = mx + 1 + 2^b = (1+2^b)\left(\frac{m}{1+2^b}x + 1\right) = (1+2^b)P_1,$$

$$n + 1 = mx + 2 + 2^b = (2+2^b)\left(\frac{m}{2+2^b}x + 1\right) = (2+2^b)P_2, \dots,$$

$$n + k - 1 = mx + k + 2^b = (k+2^b)\left(\frac{m}{k+2^b}x + 1\right) = (k+2^b)P_k.$$

Example 2: Let $b=1$ and $k=4$, we have $n=3 \times 27361, n+1=4 \times 20521, n+2=5 \times 16417, n+3=6 \times 13681$.

Theorem 3

There exist infinitely many integers n such that the consecutive integers $n=3P_1, n+2=5P_2, \dots, n+2=5P_2, \dots, n+2(k-1)=(2k+1) P_k$ are all composites for arbitrarily long k , where P_1, P_2, \dots, P_k are all primes [3].

Proof: Suppose that $m = \prod_{i=1}^k (2i+1)$. We define the prime equations:

$$P_i = \frac{m}{2i+1}x+1, \quad (7)$$

Where $i=1, 2, \dots, k$.

The Jiang's function [1] is:

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$$J_2(\omega) = \prod_{3 \leq P} (P - k - 1 - \chi(P)) \neq 0 \tag{8}$$

Where $\chi(P) = -k$ if $P^2 \mid m$; $\chi(P) = -k+1$ if $P \mid m$; $\chi(P) = 0$ otherwise.

Since, $J_2(\omega) \rightarrow \infty$ as $\omega \rightarrow \infty$, there exist infinitely many integers x such that P_1, P_2, \dots, P_k are all primes.

We have the asymptotic formula of the number of integers $x \leq N$ [1]

$$\pi_{k+1}(N, 2) \sim \frac{J_2(\omega)\omega^k}{\varphi^{k+1}(\omega)} \frac{N}{\log^{k+1} N} \tag{9}$$

From (7) we have:

$$n = mx + 3 = 3\left(\frac{m}{3}x + 1\right) = 3P_1,$$

$$n + 2 = mx + 5 = 5\left(\frac{m}{5}x + 1\right) = 5P_2, \dots,$$

$$n + 2(k-1) = mx + 2k + 1 = (2k+1)\left(\frac{m}{2k+1}x + 1\right) = (2k+1)P_k$$

Example 3: Let $k=4$, we have $n=3 \times 631$, $n+2=5 \times 379$, $n+4=7 \times 271$, $n+6=9 \times 211$.

Theorem 4

There exist infinitely many integers n such that the consecutive integers $n+2=3P_2, \dots, n+2=3P_2, \dots, n+2(k-1)=(2k+1)P_k$ are all composites for arbitrarily long k , where P_1, P_2, \dots, P_k are all primes [4].

Proof: Suppose that $m = \prod_{i=1}^k (2i-1)$. We define the prime equations:

$$P = \frac{m}{2}x + 1 \tag{10}$$

Where $i=1, 2, \dots, k$.

The Jiang's function [1] is:

$$J_2(\omega) = \prod_{3 \leq P} (P - k - 1 - \chi(P)) \neq 0 \tag{11}$$

Where $\chi(P) = -k$ if $P^2 \mid m$; $\chi(P) = -k+1$ if $P \mid m$; $\chi(P) = 0$ otherwise.

Since, $J_2(\omega) \rightarrow \infty$ as $\omega \rightarrow \infty$, there exist infinitely many integers x such that P_1, P_2, \dots, P_k are all primes.

We have the asymptotic formula of the number of integers $x \leq N$ [1]

$$\pi_{k+1}(N, 2) \sim \frac{J_2(\omega)\omega^k}{\varphi^{k+1}(\omega)} \frac{N}{\log^{k+1} N} \tag{12}$$

From (10) we have:

$$n = P_1 = mx + 1,$$

$$n + 2 = mx + 3 = 3\left(\frac{m}{3}x + 1\right) = 3P_2, \dots,$$

$$n + 2(k-1) = mx + 2(k-1) = (2k-1)\left(\frac{m}{2k-1}x + 1\right) = (2k-1)P_k.$$

Example 4: Let $k=4$, we have $n=9661$, $n+2=3 \times 3221$, $n+4=5 \times 1933$, $n+6=7 \times 1381$.

Theorem 5

There exist infinitely many integers n such that the consecutive integers $n=3P_1, \dots, n+4=7P_2, \dots, n+4(k-1)=(4k+1)P_k$ are all composites for arbitrarily long k , where P_1, P_2, \dots, P_k are all primes [5].

Example 5: Let $k=4$, we have $n=3 \times 2311$, $n+4=7 \times 991$, $n+8=11 \times 631$, $n+12=15 \times 463$.

Theorem 6

There exist infinitely many integers n such that the consecutive integers $n=5P_1, \dots, n+4=9P_2, \dots, n+4(k-1)=(4k+1)P_k$ are all composites for arbitrarily long k , where P_1, P_2, \dots, P_k are all primes [6].

Conclusion

Jiang's function $J_2(\omega)$ prove that there exist infinitely many integers n such that $n=2P_1, n+1=3P_2, \dots, n+k-1=(k+1)P_k$ are all composites for arbitrarily long k , where P_1, P_2, \dots, P_k are all primes which result has no prior occurrence in the history of number theory.

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