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Orthogonal Left Derivations of Semi-Prime Rings

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Abstract

In this paper we show a few outcomes concerning two remaining deductions on a semi-prime ring are displayed. These outcomes are identified with an outcome which is motivated by Posner's hypothesis. This outcome affirms that if R is a 2-torsion free semi-prime ring, δ and g are non-zero remaining inductions of R with the end goal that g is a surjective on R, and g(y)δ(x)=g(x)δ(y) for all x,y∈R. At that point δ g can't be a non-zero left derivation. A thought of orthogonal left derivations emerges here.

Keywords: Left derivation; Orthogonal left derivations; Prime ring; Semi-prime ring

Introduction

All through R will speak to a cooperative ring. R is said to be 2 - torsion free if 2x=0, x∈R implies x=0 [1]. Review that R is prime if xRy=0 implies x=0 or y=0, and R is semi-prime if xRx=0 suggests x=0. Ref. [2], characterized the accompanying thought. An added substance mapping δ:R→R is known as a left inference if δ(xy)=xδ(y)+yδ(x) holds for all x,y∈R. Different properties of left deductions can be found in refs. [3-8].

Two additive mapping δ,g:R→R is said to be orthogonal if:

(x)Rg(y)=0=g(y)Rδ(x) for all x,y∈R.

Brešar and Vukman [9] presented the idea of orthogonality for two inductions δ and g on a semi-prime ring, and they introduced a few important and adequate conditions for δ and g to be orthogonal. In ref. [10] the creators presented orthogonal summed up inferences on a semi-prime ring and they introduced a few outcomes concerning two summed up determinations on a semi-prime ring. Their outcomes are a speculation of after effects of Brešar and Vukman in ref. [9]. What's more [11], in the creators presented orthogonal (σ,τ)-determinations and orthogonal summed up (,τ)-deductions. Their outcomes dreamy a few aftereffects of Brešar and Vukman [9]. In this paper, our point is to give similar consequences of Brešar and Vukman to orthogonal left derivations [9].

For a generalized semi-prime ring R and a perfect U of R, it is outstanding that the left and right annihilators of U in R agree [12]. We indicate the annihilator of U by Ann (U). Take note of that U∩Ann (U)=0 and U⊕Ann(U) is a fundamental perfect of R [12].

Materials and Methods

In the accompanying, we give the documentation of orthogonal left derivations.

Definition 2.1

Left derivations δ and g are called orthogonal if,

(x)Rg(y)=0=g(y)Rδ(x), for all x,y∈R. (1)

Clearly a non-zero remaining deduction cannot be orthogonal on itself.

Give us a chance to consider a straightforward case of the non-zero orthogonal left derivations.

Example: Give S a chance to be a prime ring and set R=S⊕S. At that point R is a semi-prime ring. Give δ and g a chance to be two non-zero remaining deductions of S. At that point the maps δ_1 and g_1 from R to R, which are characterized by:

$\delta_1((x,y))=(\delta(x),0)$ and $g_1((x,y))=(0,g(y))$, for all x,y∈S, are non-zero left derivations of R.

Then δ_1 and g_1 are orthogonal.

Presently, to get the primary outcomes, we require the accompanying lemmas:

Lemma 2.2 (Lemma 1 [9])

Give R a chance to be a 2-torsion free semi-prime ring and a,b the components of R. At that point the accompanying conditions are proportional:

(i) axb=0, for all x∈R.

(ii) bxa=0, for all x∈R.

(iii) axb+bxa=0, for all x∈R.

On the off chance that one of these conditions is satisfied then abdominal muscle ab=ba=0

Lemma 2.3 (Lemma 2.2 [9])

Give R a chance to be a semi-prime ring. What's more, assume that added substance mappings f and h of R into itself fulfill satisfy f(x)Rh(x)=0, for all x∈R. Then f(x)Rh(y)=0, for all x,y∈R.

Results and Discussion

In Theorem 3.1, we will demonstrate that If δ and g are orthogonal left derivations of a 2-torsion free semi-prime ring R, then there exists a fundamental perfect E of R, with the end goal that the confinements of δ and g to E are fitting direct wholes.

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Theorem 3.1

Give R a chance to be a 2-torsion free semi-prime ring. Give δ and g a chance to be left derivations of R. At that point the accompanying conditions are equal:

(i) δ and g are orthogonal.

(ii) There exist standards E_1 and E_2 of R with the end goal that:

$E_1 \cap E_2=0$ and $E=E_1 \oplus E_2$ is a basic perfect of R.

(a) maps R into E_1 and g maps R into E_2.

(b) The restriction of δ to $E=E_1 \oplus E_2$ is a direct sum $\delta_1 0_2$, where $\delta_1: E_1 \to E_1$ is a left derivation of E_1 and $0_2: E_2 \to E_2$ is zero. If $\delta_1=0$ then $\delta=0$.

(c) The restriction of g to $E=E_1 \oplus E_2$ is a direct sum $0_1 \oplus g_2$, where $0_1: E_1 \to E_1$ is zero and $g_2 : E_2 \to E_2$ is a left derivation of E_2. If $g_2=0$ then g=0.

In Theorem 3.2, we give a few vital and adequate conditions for the orthogonality of two left derivations.

Theorem 3.2

Give R a chance to be a 2-torsion free semi-prime ring. Let δ and g be left derivations of R such that g is a surjective on R, and $g(y)\delta(x)=g(x)\delta(y)$, for all $x,y \in R$. Then δ and g are orthogonal if and only if one of the following conditions holds:

(i) $g(x)\delta(x)=0$, for all $x \in R$.

(ii) $\delta(x)g(x)=0$, for all $x \in R$.

(iii) $g(x)\delta(x)+\delta(x)g(x)=0$, for all $x \in R$.

(iv) $\delta g=0$.

(v) $g\delta=0$.

(vi) $\delta g+g\delta=0$.

(vii) δ g is a left derivation.

(viii) $g\delta$ is a left derivation.

(ix) There exist a, b in R such that $(\delta g)(x)=xa+xb$, for all $x \in R$.

For the evidence of the Theorem 3.1 and Theorem 3.2, we require the accompanying lemma:

Lemma 3.3

Give R a chance to be a 2-torsion free semi-prime ring. Give δ and g a chance to be left derivations of R. In the event that δ and g are orthogonal then the accompanying relations hold.

(i) $g(x)\delta(x)=0$, for all $x \in R$.

(ii) $\delta(x)g(x)=0$, for all $x \in R$.

(iii) $g(x)\delta(x)+\delta(x)g(x)=0$, for all $x \in R$.

(iv) $\delta g=0$.

(v) $g\delta=0$.

(vi) $\delta g+g\delta=0$.

Proof: (i) By the hypothesis we have $\delta(x)Rg(x)=0$, for all $x \in R$. By Lemma 2.2, we get $g(x)\delta(x)=0$, for all $x \in R$.

(ii) By the hypothesis we have $\delta(x) R g(x)=0$, for all $x \in R$.

By Lemma 2.2, we get $\delta(x) g(x)=0$, for all $x \in R$.

(iii) By the hypothesis we have $\delta(x)Rg(x)=0$, for all $x \in R$.

By Lemma 2.2, we ge $\delta(x)g(x)=g(x)\delta(x)=0$, for all $x \in R$.

Thus $g(x)\delta(x)+\delta(x)g(x)=0$, for all $x \in R$.

(iv) We have $\delta(x)yg(z)=0$, for all $x,y,z \in R$. Hence,

$0=\delta(\delta(x)yg(z))$

$=\delta(x)\delta(yg(z))+yg(z)\delta^2(x)$

$=\delta(x)y(\delta g)(z)+\delta(x)g(z)\delta(y)+yg(z)\delta^2(x)$

The second two summands are zero since δ and g are orthogonal. Therefore, this relation reduces to $\delta(x)y(\delta g)(z)=0$, where x, y, z are arbitrary elements in R. But then also $(\delta g)(z)R(\delta g)(z)=0$, for all $z \in R$. Since R is semi-prime, we get $(\delta g)(z)=0$.

The second two summands are zero since δ and g are orthogonal. In this manner, this connection decreases to $\delta(x)y(\delta)(z)=0$, where x,y and z are discretionary components in R. Be that as it may, then likewise $(\delta g)(z)R(\delta g)(z)=0$, for all $z \in R$. Since R is semi-prime, we get $(\delta g)(z)=0$.

(v) By a similar way in (iv) we get the result.

(vi) From (iv) and (v), Lemma 2.4, we have $\delta g+g\delta=0$.

We need the following lemma to proof Theorem 3.2.

Lemma 3.4

Let R be a 2-torsion free semi-prime ring. Let δ and g be left derivations of R such that g is a surjective on R, and $g(y)\delta(x)=g(x)\delta(y)$, for all $x,y \in R$. Then δ and g are orthogonal if and only if $g(y)\delta(x)+g(x)\delta(y)=0$, for all $x,y \in R$.

Proof: Suppose that $g(y)\delta(x)+g(x)\delta(y)=0$, for all $x,y \in R$. By the assumption, we have $g(y)\delta(x)=0$, for all $x,y \in R$. Since R is 2-torsion free, we have $g(y)\delta(x)=0$, for all $x,y \in R$. Take $x=g(z)x$ in the above relation, where z in R, we get,

$0=g(y)\delta(g(z)x)$

$=g(y)g(z)\delta(x)+g(y)x\delta(g(z))$

$=g(y)x\delta(g(z))$, for all $x,y,z \in R$.

Since g is surjective, we get $g(y)x\delta(z)=0$, for all $x,y,z \in R$.

Then $g(y)R\delta(z)=0$, for all $y,z \in R$. Using Lemma 2.2, we see that δ and g are orthogonal.

Conversely, if δ and g are orthogonal, we have,

$\delta(x)Rg(y)=0$, for all $x,y \in R$.

By Lemma 2.2, we get $g(y)\delta(x)=g(x)\delta(y)=0$, for all $x,y \in R$.

Thus $g(y)\delta(x)+g(x)\delta(y)=0$, for all $x,y \in R$.

Let δ and g be left derivations of any ring R. By a direct computation, we verify the following identities:

$$(\delta g)(xy)=x(\delta g)(y)+g(y)\delta(x)+g(x)\delta(y)+y(\delta g)(x) \quad (2)$$

$$(g\delta)(xy)=x(g\delta)(y)+\delta(y)g(x)+\delta(x)g(y)+y(g\delta)(x) \quad (3)$$

We now have enough information's to prove Theorem 3.1.

Proof of Theorem 3.1: (i) $\Rightarrow$(ii). Let E_1 be an ideal of R generated by all $\delta(x)$, $x \in R$, and let E_2 be Ann (E_1), the annihilator of E_1. From eqn. (1) we see that $g(x)$, $x \in E_2$, for all $x \in R$. Whenever E_1 is an ideal in a semi-prime ring we have $E_1 \cap E_2=0$ and $E=E_1 \oplus E_2$ is an essential ideal. Thus (a) and (b) are proved.

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Our next goal is to show that δ is zero on E_2. Take $e_2 \in E_2$. Then $e_1e_2=0$, for all $e_1 \in E_1$. Hence $0=\delta(e_1e_2)=e_1\delta(e_2)+e_2\delta(e_1)$.

It is obvious from the definition of E that δ leaves E_1 invariant, hence $e_2\delta(e_1)=0$. Then the relation above reduces to $e_1\delta(e_2)=0$. Since in a semi-prime ring the left and right and two-sided annihilators of an ideal coincide, then we have $\delta(e_2)\in Ann(E_1)=E_2$. But on the other hand $\delta(e_2)$ belongs to the set of generating elements of E_1. Thus $\delta(e_2)\in E_1 \cap E_2=0$, which means that δ is zero on E_2.

As we have mentioned above δ leaves E_1 invariant. Therefore we may define a mapping $\delta_1:E_1\rightarrow E_1$ as a restriction of δ to E_1. Suppose that $\delta_1=0$. Then δ is zero on $E=E_1\oplus E_2$. Take $e\in E$ and $x\in R$. We have $\delta(ex)=e\delta(x)+x\delta(e)$. But $\delta(ex)=\delta(e)=0$ since $ex, e\in R$.

Consequently $e\delta(x)=0$, for all $x\in R$. Thus $\delta(x)\in Ann(E)$. But ideal E is essential and therefore $Ann(E)=0$. Hence $\delta(x)=0$, for all $x\in R$. Then (c) is thereby proved.

It remains to prove (d). First we show that g is zero on E_1. Take x, y, z $\in$ R and set $e_1=x\delta(y)z$. Then $g(e_1)=g(x\delta(y)z)=xg(\delta(y)z)+\delta(y)zg(x)$

$=x\delta(y)g(z)+xz(g\delta)(y)+\delta(y)zg(x)$.

Since δ and g are orthogonal we have $\delta(y)g(z)=0$, $\delta(y)zg(x)=0$ and $g\delta=0$ by Lemma 3.3. Hence $g(e_1)=0$. In a similar fashion we see that $g(x\delta(y))=0$, $g(\delta(y)z)=0$ and $g(\delta(y))=0$ by Lemma 3.3. Then g is zero on E_1. Recall that g maps R into E_2. In particular, it leaves E_2 invariant. Thus, we may define $g_2:E_2\rightarrow E_2$ as a restriction of g to E_2. The proof that $g_2=0$ implies $g=0$ is the same as the proof that $\delta_1=0$ implies $\delta=0$.

(ii) $\Rightarrow$(i). Clear.

Proof of Theorem 3.2: "δ and g are orthogonal" $\Rightarrow$(i), (ii), (iii), (iv), (v) and (vi) are proved by Lemma 3.

(i) $\Rightarrow$"δ and g are orthogonal". A linearization of $g(x)\delta(x)=0$ gives,

$g(x)\delta(y)+g(y)\delta(x)=0$, for all $x,y\in R$.

Hence δ and g are orthogonal by Lemma 4.

(ii) $\Rightarrow$"δ and g are orthogonal". A linearization of $\delta(x)g(x)=0$ gives,

$(x)g(y)+\delta(y)g(x)=0$, for all $x,y\in R$.

Left multiplication by g (y) in the above relation gives,

$g(y)\delta(x)g(y)+g(y)\delta(y)g(x)=0$, for all $x,y\in R$.

By the assumption, we get,

$g(x)\delta(y)g(y)+g(y)\delta(y)g(x)=0$, for all $x,y\in R$.

Hence,

$g(y)\delta(y)g(x)=0$, for all $x,y\in R$.

Since g is surjective, we get,

$g(y)\delta(y)x=0$,

where x, y are arbitrary elements in R.

Since R is semi-prime, we get,

$g(y)\delta(y)=0$, for all $y\in R$.

Therefore, by (i), Theorem 3.2, we get the result.

(iii) $\Rightarrow$"δ and g are orthogonal". Suppose that $g(x)\delta(x)+\delta(x)g(x)=0$, for all $x\in R$. Then,

$g(x)\delta(x)=-\delta(x)g(x)$,

for all $x\in R$(.),

A linearization of $g(x)\delta(x)+\delta(x)g(x)=0$ gives,

$g(x)\delta(y)+g(y)\delta(x)+\delta(x)g(y)+\delta(y)g(x)=0$, for all $x,y\in R$.

By the assumption, we have $2g(y)\delta(x)+\delta(x)g(y)+\delta(y)g(x)=0$, for all $x,y\in R$. Left multiplication by g (y) in the above relation, we get $2g(y)g(y)\delta(x)+g(y)\delta(x)g(y)+g(y)\delta(y)g(x)=0$, for all $x,y\in R$. By (.) and the assumption, we obtain $2g(y)g(x)\delta(y)+g(x)\delta(y)g(y)-\delta(y)g(y)g(x)=0$, for all $x,y\in R$.

Hence $2g(y)g(x)\delta(y)+[g(x), \delta(y)g(y)]=0$, Take $g(x)=\delta(y)g(y)$ in the above relation, we get $2g(y)g(x)\delta(y)=0$, for all $x,y\in R$. Since R is 2-torsion free and g is surjective, we have $g(y)x\delta(y)=0$, for all $x,y\in R$. Then $g(y)R\delta(y)=0$, for all $y\in R$.

By Lemma 2.3, we then have $g(y)R\delta(z)=0$, for all $y,z\in R$.

Using Lemma 2.2, we see that δ and g are orthogonal.

(iv) $\Rightarrow$"δ and g are orthogonal". Suppose that $\delta g=0$. According to eqn. (2), we have,

$g(y)\delta(x)+g(x)\delta(y)=0$, for all $x,y\in R$.

Hence, we get δ and g are orthogonal by Lemma 3.4.

(v) $\Rightarrow$"δ and g are orthogonal". Suppose that $g\delta=0$. According to eqn. (3), we have,

$\delta(y)g(x)+\delta(x)g(y)=0$, for all $x,y\in R$.

Take y=x in the above relation, we get,

$2\delta(x)g(x)=0$, for all $x\in R$.

Since R is 2-torsion free, we have,

$\delta(x)g(x)=0$, for all $x\in R$.

Therefore, by (ii), Theorem 3.2, we get the result.

(vi) $\Rightarrow$"δ and g are orthogonal". If δ and g are any left derivations then we have by eqns. (2) and (3) that,

$(\delta g+g\delta)(xy)=x(\delta g+g\delta)(y)+g(y)\delta(x)+g(x)\delta(y)+\delta(y)g(x)+\delta(x)g(y)+y(\delta g+g\delta)(x)$

Thus, if $\delta g+g\delta=0$, then the above relation reduces to,

$g(y)\delta(x)+g(x)\delta(y)+\delta(y)g(x)+\delta(x)g(y)=0$, for all $x,y\in R$.

Take x=y in the above relation, then we have $2(g(x)\delta(x)+\delta(x)g(x))=0$, for all $x\in R$. Since R is 2-torsion free, we have $g(x)\delta(x)+\delta(x)g(x)=0$, for all $x\in R$. Therefore, by (iii), Theorem 3.2, we get the result.

(iv) $\Rightarrow$(vii). Clear.

(vii) $\Rightarrow$"δ and g are orthogonal". Since δg is a left derivation. We have,

$(\delta g)(xy)=x(\delta g)(y)+y(\delta g)(x)$, for all $x,y\in R$. Comparing this express with eqn. (2), we obtain $g(y)\delta(x)+g(x)\delta(y)=0$, for all $x,y\in R$.

Now apply Lemma 3.4.

(v) $\Rightarrow$(viii). Clear.

(viii) $\Rightarrow$"δ and g are orthogonal". Since $g\delta$ is a left derivation. We have,

$(g\delta)(xy)=x(g\delta)(y)+y(g\delta)(x)$, for all $x,y\in R$. Comparing this express with eqn. (3), we obtain $\delta(y)g(x)+\delta(x)g(y)=0$, for all $x,y\in R$.

Let y=x in the above relation, we get $2\delta(x)g(x)=0$, for all $x\in R$.

Since R is a 2-torsion free, we have $\delta(x)g(x)=0$, for all $x\in R$.

Therefore, by (ii), Theorem 3.2, we get the result.

(iv) ⇒(ix). Clear.

(ix) ⇒"δ and g are orthogonal". For every $x, y \in R$ we have $(\delta g)(x y)=xya+xyb$.

That is, $x(\delta g)(y)+g(y)\delta(x)+g(x)\delta(y)+y(\delta g)(x)=xya+xyb$.

Using $(\delta g)(x)=xa+xb$ and $(\delta g)(y)=ya+yb$,

we get $2g(x)\delta(y)+yx(a+b)=0$. Replacing x by y, x in the above relation yields that $y\{2g(x)\delta(y)+yx(a+b)\}+2xg(y)\delta(y)=0$

Then we have $2xg(y)\delta(y)=0$, for all $x,y\in R$.

Since R is 2-torsion free, we have $x\, g(y)\delta(y)=0$, where x,y are arbitrary elements in R. Since R is semi-prime, we get $g(y)\delta(y)=0$, for all $y\in R$.

Therefore, by (i), Theorem 3.2, we get the result.

A notable consequence of Posner [1] states that, if R is a prime ring of trademark not 2, δ and g are non-zero inductions of R, then δ g cannot be a derivation. The outcome which is enlivened by a hypothesis of E. Posner, states that, if R is a 2-torsion free semi-prime ring, δ and g are non-zero left derivations of R such an extent that g is a surjective on R, and $g(y)\delta(x)=g(x)\delta(y)$, for all $x,y\in R$. At that point δ g cannot be a non-zero left derivation. One can consider (vii) and (iv), Theorem 3.2 as a proof of this outcome.

We now express a few outcomes of Theorem 3.2.

Corollary 3.3

Give R a chance to be a prime ring of trademark not equivalent 2. Give δ and g be left derivations of R with the end goal that g is a surjective on R, and $g(y)\delta(x)=g(x)\delta(y)$, for all $x,y\in R$. On the off chance that δ and g are fulfill one of the states of Theorem 3.2, then either δ=0 or g=0.

Since a non-zero left derivation cannot be orthogonal on itself we see that (i), Theorem 3.2 yield the accompanying outcome.

Corollary 3.4

Let R be a 2-torsion free semi-prime ring. And let δ be a left derivation of R such that δ is a surjective on R, and $\delta(y)\delta(x)=\delta(x)\delta(y)$, for all $x,y\in R$. If $\delta(x)^2=0$, for all $x\in R$, then δ=0.

According to (vii), Theorem 3.2, we have,

Corollary 3.5

Let R be a 2-torsion free semi-prime ring. And let δ be a left derivation of R such that δ is a surjective on R, and $\delta(y)\delta(x)=\delta(x)\delta(y)$, for all $x, y \in R$. If δ^2 is also a left derivation, then δ=0.

Similarly, using (ix), Theorem 3.2, we obtain,

Corollary 3.6

Let R be a 2-torsion free semi-prime ring. And let δ be a left derivation of R such that δ is a surjective on R, and $\delta(y)\delta(x)=\delta(x)\delta(y)$, for all $x,y\in R$. If there exist $a,b\in R$ such that $\delta^2(x)=xa+x\,b$, for all $x\in R$, then δ=0.

It is normal to inquire as to whether there is any association between left derivations δ and g of a ring R, If $\delta^2=g^2$ or if $\delta(x)^2=g(x)^2$, for each $x\in R$. In the accompanying hypotheses, we give certifiable answer of this question.

Theorem 3.7

Let R be a 2-torsion free semi-prime ring. Let δ and g be left derivations of R such that g is a surjective on R, and $g(y)\delta(x)=g(x)\delta(y)$, for all $x,y\in R$. If $\delta^2=g^2$, then δ+g and δ-g are orthogonal.

Proof: From $\delta^2=g^2$ it follows immediately that, $(\delta+g)(\delta-g)+(\delta-g)(\delta+g)=0$. Hence δ+g and δ-g are orthogonal by (vi), Theorem 3.2.

Corollary 3.8

Let R be a prime ring of characteristic not equal 2. Let δ and g be left derivations of R such that g is a surjective on R, and $g(y)\delta(x)=g(x)\delta(y)$, for all $x,y\in R$. If $\delta^2=g^2$ then either δ=g or δ=g.

Theorem 3.9

Let R be a 2-torsion free semi-prime ring. Let δ and g be left derivations of R such that g is a surjective on R, and $g(y)\delta(x)=g(x)\delta(y)$, for all $x,y\in R$. If $\delta(x)^2=g(x)^2$, for all $x\in R$, then δ+g and δ-g are orthogonal.

Proof: Note that $(\delta+g)(x)(\delta-g)(x) + (\delta-g)(x)\,(\delta+g)(x)=0$, for all $x\in R$. Now apply (iii), Theorem 3.2.

Corollary 3.10

Let R be a prime ring of characteristic not equal 2. Let δ and g be left derivations of R such that g is a surjective on R, and $g(y)\delta(x)=g(x)\delta(y)$, for all $x,y\in R$. If $\delta(x)^2=g(x)^2$, for all $x\in R$, then either δ=-g or δ=g.

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