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Mini-Review
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Prime Distribution in Pythagorean Triples (1)

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Mini-Review

Using Jiang function we study the prime distribution in Pythagorean triples.

Pythagorean triples

$$a^2+b^2=c^2, \tag{1}$$

In comprime integers must be of the form:

$$a=x^2-y^2,\quad b=2xy,\quad c=x^2+y^2, \tag{2}$$

Where x and y are coprime integers.

Theorem 1: From eqn. (2) we have,

$$a=(x+y)(x-y) \tag{3}$$

Let $x-y=1$ and $a=x+y=P_1$, we have,

$$P_1^2=(x+y)^2=x^2+y^2+2xy=c+b, \tag{4}$$

$$1=(x-y)^2=x^2+y^2-2xy=c-b \tag{5}$$

From eqns. (4) and (5) we have,

$$a=P_1,\quad b=\frac{P_1^2-1}{2},\quad c=\frac{P_1^2+1}{2}=P_2 \tag{6}$$

There are infinitely many primes P_1 such that P_2 is a prime.

Proof: We have Jiang function [1]

$$J_2(\omega)=\prod_{P>2}(P-1-\chi(P)), \tag{7}$$

where $\omega=\prod_{P\geq 2}P$, $\chi(P)$ is the number of solutions of congruence

$$q^2+1\equiv 0(\bmod P),\ q=1,\cdots,P-1. \tag{8}$$

From (8) we have,

$$\chi(P)=1+(-1)^{\frac{P-1}{2}} \tag{9}$$

Substituting (9) into (7) we have

$$J_2(\omega)=\prod_{P>2}(P-2-(-1)^{\frac{P-1}{2}})\neq 0 \tag{10}$$

Since $J_2(\omega)\neq 0$, we prove that there are infinitely many prime P_1, such that P_2 is a prime.

We have the best asymptotic formula [1].

$$\pi_2(N,2)=\left|\{P_1\leq N:P_2=prime\}\right|\sim\frac{J_2(\omega)\omega}{2\phi^2(\omega)}\frac{N}{\log^2 N}=\left(1-\frac{1+P(-1)^{\frac{P-1}{2}}}{(P-1)^2}\right)\frac{N}{\log^2 N}, \tag{11}$$

where $\phi(\omega)=\prod_{P\geq 2}(P-1)$.

Theorem 2: Let $x+y=P_1$ and $x-y=P_1-2$, we have $a=P_1(P_1-2)$ and,

$$P_1^2=(x+y)^2=c+b, \tag{12}$$

$$(P_1-2)^2=(x-y)^2=c-b \tag{13}$$

From eqns. (12) and (13) we have,

$$a=P_1(P_1-2),\quad b=\frac{P_1^2-(P_1-2)^2}{2},\quad c=\frac{P_1^2+(P_1-2)^2}{2}=P_2 \tag{14}$$

There are infinitely many primes P_1 such that P_2 is a prime.

Proof: We have Jiang function [1]

$$J_2(\omega)=\prod_{P>2}(P-1-\chi(P)), \tag{15}$$

Where $\chi(P)$ is the number of solutions of congruence

$$q^2+(q-2)^2\equiv 0(\bmod P),\ q=1,\cdots,P-1. \tag{16}$$

From (16) we have,

$$\chi(P)=1+(-1)^{\frac{P-1}{2}} \tag{17}$$

Substituting (17) into (15) we have,

$$J_2(\omega)=\prod_{P>2}(P-2-(-1)^{\frac{P-1}{2}})\neq 0 \tag{18}$$

Since $J_2(\omega)\neq 0$, we prove that there are infinitely many prime P_1 such that P_2 is a prime.

We have the best asymptotic formula [1]

$$\pi_2(N,2)=\left|\{P_1\leq N:P_2=prime\}\right|\sim\left(1-\frac{1+P(-1)^{\frac{P-1}{2}}}{(P-1)^2}\right)\frac{N}{\log^2 N} \tag{19}$$

Theorem 3: Let $x-y=1$ and $a=x+y=P_1^2$, we have,

$$a=P_1^2,\ b=\frac{P_1^4-1}{2},\quad c=\frac{P_1^4+1}{2}=P_2. \tag{20}$$

There are infinitely many primes P_1 such that P_2 is a prime.

Proof: We have Jiang function [1],

$$J_2(\omega)=\prod_{P>2}(P-1-\chi(P)), \tag{21}$$

Where $\chi(P)$ is the number of solutions of congruence,

$$q^4+1\equiv 0(\bmod P),\ q=1,\cdots,P-1. \tag{22}$$

From (22) we have,

$$\chi(P)=4 \text{ if } 8|P-1,\quad \chi(P)=0 \text{ otherwise}. \tag{23}$$

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Since $J_2(\omega)\neq 0$, we prove that there are infinitely many prime P_1 such that P_2 is a prime.

We have the best asymptotic formula [1]:

$$\pi_2(N,2) = \left|\{P_1 \le N : P_2 = prime\}\right| \sim \frac{J_2(\omega)\omega}{4\phi^2(\omega)} \frac{N}{\log^2 N}, \tag{24}$$

These results are in wide use in biological, physical and chemical fields.

References

1. Chun-Xuan Jiang, Jiang function $J_{n+1}(\omega)$ in prime distribution.