

Prime Distribution in Pythagorean Triples (1)

Chun-Xuan Jiang*

Institute for Basic Research, Palm Harbor, P.O. Box 3924, Beijing 100854, P.R. China

Mini-Review

Using Jiang function we study the prime distribution in Pythagorean triples.

Pythagorean triples

$$a^2 + b^2 = c^2, \tag{1}$$

In coprime integers must be of the form:

$$a = x^2 - y^2, \quad b = 2xy, \quad c = x^2 + y^2, \tag{2}$$

Where x and y are coprime integers.

Theorem 1: From eqn. (2) we have,

$$a = (x + y)(x - y) \tag{3}$$

Let $x - y = 1$ and $a = x + y = P_1$, we have,

$$P_1^2 = (x + y)^2 = x^2 + y^2 + 2xy = c + b, \tag{4}$$

$$1 = (x - y)^2 = x^2 + y^2 - 2xy = c - b \tag{5}$$

From eqns. (4) and (5) we have,

$$a = P_1, \quad b = \frac{P_1^2 - 1}{2}, \quad c = \frac{P_1^2 + 1}{2} = P_2 \tag{6}$$

There are infinitely many primes P_1 such that P_2 is a prime.

Proof: We have Jiang function [1]

$$J_2(\omega) = \prod_{P>2} (P - 1 - \chi(P)), \tag{7}$$

where $\omega = \prod_{P \geq 2} P$, $\chi(P)$ is the number of solutions of congruence

$$q^2 + 1 \equiv 0 \pmod{P}, \quad q = 1, \dots, P - 1. \tag{8}$$

From (8) we have,

$$\chi(P) = 1 + (-1)^{\frac{P-1}{2}} \tag{9}$$

Substituting (9) into (7) we have

$$J_2(\omega) = \prod_{P>2} (P - 2 - (-1)^{\frac{P-1}{2}}) \neq 0 \tag{10}$$

Since $J_2(\omega) \neq 0$, we prove that there are infinitely many prime P_1 , such that P_2 is a prime.

We have the best asymptotic formula [1].

$$\pi_2(N, 2) = |\{P_1 \leq N : P_2 = \text{prime}\}| \sim \frac{J_2(\omega)\omega}{2\phi^2(\omega)} \frac{N}{\log^2 N} = \left(1 - \frac{1 + P(-1)^{\frac{P-1}{2}}}{(P-1)^2}\right) \frac{N}{\log^2 N}, \tag{11}$$

where $\phi(\omega) = \prod_{P \geq 2} (P - 1)$.

Theorem 2: Let $x + y = P_1$ and $x - y = P_1 - 2$, we have $a = P_1(P_1 - 2)$ and,

$$P_1^2 = (x + y)^2 = c + b, \tag{12}$$

$$(P_1 - 2)^2 = (x - y)^2 = c - b \tag{13}$$

From eqns. (12) and (13) we have,

$$a = P_1(P_1 - 2), \quad b = \frac{P_1^2 - (P_1 - 2)^2}{2}, \quad c = \frac{P_1^2 + (P_1 - 2)^2}{2} = P_2 \tag{14}$$

There are infinitely many primes P_1 such that P_2 is a prime.

Proof: We have Jiang function [1]

$$J_2(\omega) = \prod_{P>2} (P - 1 - \chi(P)), \tag{15}$$

Where $\chi(P)$ is the number of solutions of congruence

$$q^2 + (q - 2)^2 \equiv 0 \pmod{P}, \quad q = 1, \dots, P - 1. \tag{16}$$

From (16) we have,

$$\chi(P) = 1 + (-1)^{\frac{P-1}{2}} \tag{17}$$

Substituting (17) into (15) we have,

$$J_2(\omega) = \prod_{P>2} (P - 2 - (-1)^{\frac{P-1}{2}}) \neq 0 \tag{18}$$

Since $J_2(\omega) \neq 0$, we prove that there are infinitely many prime P_1 such that P_2 is a prime.

We have the best asymptotic formula [1]

$$\pi_2(N, 2) = |\{P_1 \leq N : P_2 = \text{prime}\}| \sim \left(1 - \frac{1 + P(-1)^{\frac{P-1}{2}}}{(P-1)^2}\right) \frac{N}{\log^2 N} \tag{19}$$

Theorem 3: Let $x - y = 1$ and $a = x + y = P_1^2$, we have,

$$a = P_1^2, \quad b = \frac{P_1^4 - 1}{2}, \quad c = \frac{P_1^4 + 1}{2} = P_2. \tag{20}$$

There are infinitely many primes P_1 such that P_2 is a prime.

Proof: We have Jiang function [1],

$$J_2(\omega) = \prod_{P>2} (P - 1 - \chi(P)), \tag{21}$$

Where $\chi(P)$ is the number of solutions of congruence,

$$q^4 + 1 \equiv 0 \pmod{P}, \quad q = 1, \dots, P - 1. \tag{22}$$

From (22) we have,

$$\chi(P) = 4 \text{ if } 8|P - 1, \quad \chi(P) = 0 \text{ otherwise.} \tag{23}$$

*Corresponding author: Chun-Xuan Jiang, Institute for Basic Research, Palm Harbor, P.O. Box 3924, Beijing 100854, P.R. China, Tel: +1-727-688 3992; E-mail: jcxuan@sina.com

Received March 10, 2017; Accepted July 29, 2017; Published July 31, 2017

Citation: Jiang CX (2017) Prime Distribution in Pythagorean Triples (1). J Generalized Lie Theory Appl 11: 276. doi: 10.4172/1736-4337.1000276

Copyright: © 2017 Jiang CX. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

Since $J_2(\omega) \neq 0$, we prove that there are infinitely many prime P_1 such that P_2 is a prime.

We have the best asymptotic formula [1]:

$$\pi_2(N, 2) = |\{P_1 \leq N : P_2 = \text{prime}\}| \sim \frac{J_2(\omega)\omega}{4\phi^2(\omega)} \frac{N}{\log^2 N}, \quad (24)$$

These results are in wide use in biological, physical and chemical fields.

References

1. Chun-Xuan Jiang, Jiang function $J_{n+1}(\omega)$ in prime distribution.

Citation: Jiang CX (2017) Prime Distribution in Pythagorean Triples (1). J Generalized Lie Theory Appl 11: 276. doi: [10.4172/1736-4337.1000276](https://doi.org/10.4172/1736-4337.1000276)

OMICS International: Open Access Publication Benefits & Features

Unique features:

- Increased global visibility of articles through worldwide distribution and indexing
- Showcasing recent research output in a timely and updated manner
- Special issues on the current trends of scientific research

Special features:

- 700+ Open Access Journals
- 50,000+ editorial team
- Rapid review process
- Quality and quick editorial, review and publication processing
- Indexing at major indexing services
- Sharing Option: Social Networking Enabled
- Authors, Reviewers and Editors rewarded with online Scientific Credits
- Better discount for your subsequent articles

Submit your manuscript at: <http://www.omicsonline.org/submission>