

## Pseudo-gradient Based Particle Swarm Optimization with Constriction Factor for Multi Objective Optimal Power Flow

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### Abstract

This paper proposes a pseudo-gradient based particle swarm optimization with constriction factor (PG-PSOCF) method for solving multiobjective optimal power flow (MOOPF) problem. The proposed PG-PSOCF is the conventional particle swarm optimization based on constriction factor based on pseudo gradient to enhance its search ability for optimization problems. The proposed method is to deal with the MOOPF problem by minimizing the total cost and emission from generators while satisfying various constraints of real and reactive power balance, real and reactive power limits, bus voltage limits, shunt capacitor limits and transmission limits. Test results on the IEEE 30-bus system have indicated that the proposed method is more efficient than many other methods in the literature. Therefore, the proposed PG-PSOCF can be an effectively alternative method for solving the MOOPF problem.

**Keywords:** Constriction factor; Multiobjective optimal power flow; Particle swarm optimization; Pseudo gradient

### Introduction

The objective of the optimal power flow (OPF) problem is to optimally determine the combination of control variables in power systems such as real power outputs of generators, voltage magnitude at generation buses, position of transformer tap changers, and reactive power outputs of shunt capacitors so that the total cost of thermal generators is minimized [1,2]. In fact, the OPF problem is a nonlinear and large-scale problem since it deals with several variables and nonlinear objective and constraints. Therefore, the OPF problem is always a challenge for solution methods, especially for those with non-differentiable objective functions which cannot be solved by conventional methods. Moreover, the power generation is also a source to release sulphur oxides ( $SO_x$ ), nitrogen oxides ( $NO_x$ ) and carbon dioxides ( $CO_2$ ) into the atmosphere. The US Clean Air Act amendments of 1990 [3] has forced the utilities to adjust their power generation strategies to guarantee a minimum pollution level. Therefore, the OPF problem should also include the emission in its objective to form a multiobjective OPF (MOOPF) problem. The MOOPF problem is to simultaneously minimize total cost and emission of thermal generators while satisfying all unit and system constraints [4].

There have been several conventional methods proposed for solving the OPF problems such as gradient-based method [5], linear programming (LP) [6], non-linear programming (NLP) [1], quadratic programming (QP) [7], Newton-based methods [8], semidefinite programming [9], and interior point method (IPM) [10]. In general, these conventional methods can easily find the optimal solution for a small-scale optimization problem in a very short time. However, the main disadvantage of them is that they suffer difficulty when dealing with non-convex optimization problems with non-differentiable objective functions. Moreover, they are also very difficult for dealing with large-scale problems due to large search space, leading time consuming or no convergence. The meta-heuristic search methods have recently developed shown that they are appropriate for dealing with complicated optimization problems, especially for those with non-differentiable objective functions. Several meta-heuristic search methods have been also widely applied for solving the OPF problem such as genetic algorithm (GA) [11], simulated annealing (SA) [12],

tabu search (TS) [13], evolutionary programming (EP) [14,15], differential evolution (DE) [16], improved particle swarm optimisation (IPSO) [17,18], and modified shuffle frog leaping algorithm (MSFLA) [19]. These meta-heuristic search algorithms can overcome the main drawback suffered by the conventional methods; that means they can deal with the problems which do not require objective functions to be differentiable. However, these meta-heuristic search methods may suffer near optimum solution and the solution quality may not high when dealing with large-scale and complex problems. That is the obtained solutions obtained by the methods may be local optima with long computational time. Therefore, the hybrid methods have also developed to overcome the drawback from the single meta-heuristic methods such as hybrid TS/SA [20], hybrid GA-IPM [21], hybrid differential evolution [22], hybrid of fuzzy and PSO [23], and genetic-based fuzzy mathematical programming technique [24]. The aim of the hybrid methods is to utilize the advantages from each element method to obtain the better optimal solution. Although the hybrid methods can obtain better solution quality than the single methods, they may be suffered slower computational time than the single methods due to combination of many operations. Moreover, the hybrid systems are also usually more complex than the element methods.

In this paper, a pseudo-gradient based particle swarm optimization with constriction factor (PG-PSOCF) method is proposed for solving the MOOPF problem. The proposed PG-PSOCF is the conventional particle swarm optimization based on constriction factor based on pseudo gradient to enhance its search ability for optimization problems. The proposed method is to deal with the MOOPF problem by minimizing the total cost and emission from generators while

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satisfying various constraints of real and reactive power balance, real and reactive power limits, bus voltage limits, shunt capacitor limits and transmission limits. Test results on the IEEE 30-bus system have indicated that the proposed method is more efficient than many other methods in the literature.

The remaining organization of this paper is follows. Section 2 addresses the formulation of MOOPF problem. A PG-PSOCF implementation for the problem is described in Section 3. Numerical results are presented in Section 4. Finally, the conclusion is given.

### The MOOPF Problem Formulation

The objective of the MOOPF problem is to simultaneously minimize the both total cost and emission while satisfying several equality and inequality constraints. Mathematically, the problem is formulated as follows:

$$\text{Minimize} \quad F_1(u,x), F_2(u,x) \quad (1)$$

$$\text{subject to} \quad g(u,x) = 0 \quad (2)$$

$$h(u,x) \leq 0 \quad (3)$$

where  $F_1(u,x)$  and  $F_2(u,x)$  are the objective functions representing total cost and emission, respectively;  $g(u,x)$  represents the equality constraints representing power balance at buses;  $h(u,x)$  represents the inequality constraints representing upper and lower limits of real power outputs, reactive power outputs, bus voltages, transformer tap changers, shunt capacitors, and power flow in transmission lines;  $u$  is the vector of the control variables including active power outputs of generators, magnitudes of generation bus voltage, transformers taps, and shunt capacitors; and  $x$  represents state variables including reactive power output, magnitudes of load bus voltage, bus voltage angles, and power flow in transmission lines.

The fuel cost function (\$/h) of generators in form of quadratic function is represented by:

$$F_1(u,x) = \sum_{i=1}^{N_g} (a_i + b_i P_{gi} + c_i P_{gi}^2) \quad (4)$$

where  $N_g$  is the number of generators including the slack bus;  $P_{gi}$  is the active power output of generator at bus  $i$ ;  $a_i$ ,  $b_i$  and  $c_i$  are the cost coefficients of generator  $i$ .

The total emission (ton/h) from generators is represented by:

$$F_2(u,x) = \sum_{i=1}^{N_g} (\alpha_i + \beta_i P_{gi} + \gamma_i P_{gi}^2 + \xi_i \exp(\lambda_i P_{gi})) \quad (5)$$

where  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_i$ ,  $\xi_i$ , and  $\lambda_i$  are emission coefficients of generator  $i$ .

The equality and inequality constraints of the problem represented mathematical model as follows:

a) Real and reactive power flow equations at each bus:

$$P_{gi} - P_{di} = V_i \sum_{j=1}^{N_b} V_j [G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j)]; \quad i = 1, \dots, N_b \quad (6)$$

$$Q_{gi} - Q_{di} = V_i \sum_{j=1}^{N_b} V_j [G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j)]; \quad i = 1, \dots, N_b \quad (7)$$

b) Voltage and reactive power limits at generation buses:

$$V_{gi,\min} \leq V_{gi} \leq V_{gi,\max}; \quad i = 1, \dots, N_g \quad (8)$$

$$Q_{gi,\min} \leq Q_{gi} \leq Q_{gi,\max}; \quad i = 1, \dots, N_g \quad (9)$$

c) Capacity limits for switchable shunt capacitor banks:

$$Q_{ci,\min} \leq Q_{ci} \leq Q_{ci,\max}; \quad i = 1, \dots, N_c \quad (10)$$

d) Transformer tap settings constraint:

$$T_{k,\min} \leq T_k \leq T_{k,\max}; \quad k = 1, \dots, N_t \quad (11)$$

e) Security constraints for voltages at load buses and transmission lines:

$$V_{li,\min} \leq V_{li} \leq V_{li,\max}; \quad i = 1, \dots, N_d \quad (12)$$

$$P_l \leq P_{l,\max}; \quad i = 1, \dots, N_l \quad (13)$$

where  $Q_{gi}$  is reactive power outputs of generating unit  $i$ ;  $P_{di}$  and  $Q_{di}$  are real and reactive load demand at bus  $i$ , respectively;  $N_b$  is the number of buses;  $V_i$  and  $\theta_i$  are voltage magnitude and angle at bus  $i$ , respectively;  $G_{ij}$  and  $B_{ij}$  are transfer conductance and susceptance between bus  $i$  and bus  $j$ , respectively;  $V_{gi}$  is voltage at generation bus  $i$ ;  $Q_{ci}$  is reactive power compensation source at bus  $i$ ;  $N_c$  is the number of shunt capacitors;  $T_k$  is tap-setting of transformer branch  $k$ ;  $N_t$  is the number of transformers;  $V_{li}$  is voltage magnitude at load bus  $i$ ;  $N_d$  is the number of load buses;  $P_l$  is power flow in transmission line  $l$  connecting between bus  $i$  and bus  $j$ ; and  $N_l$  is the number of transmission lines.

For the MOOPF problem formulation, the vector of control variables  $u$  is represented by:

$$u = [P_{g2}, \dots, P_{gN_g}, V_{g1}, \dots, V_{gN_g}, Q_{c1}, \dots, Q_{cN_c}, T_1, \dots, T_{N_t}]^T \quad (14)$$

where bus 1 is selected as the reference bus and the vector of the state variables  $x$  represented by:

$$x = [Q_{g1}, \dots, Q_{gN_g}, V_{l1}, \dots, V_{lN_d}, P_1, \dots, P_{N_l}]^T \quad (15)$$

## Pseudo-Gradient Based Particle Swarm Optimization with Constriction Factor

### Particle swarm optimization with constriction factor

The conventional PSO was developed in 1995 by Kennedy and Eberhart [25]. So far, this method has become one of the most popular meta-heuristic search methods implemented in the optimization problems of many fields due to its simplicity in application and efficiency in finding near optimum solution. The principle of PSO for searching the optimal solution for a problem is based on a population of particles which moves in the search space of the problem. The movement of the particles is determined via its location and velocity. During the movement, the position of particles will be updated according to the change of their velocity.

For application of PSO to find the optimal solution of an  $n$ -dimension problem, a population of  $N_p$  particles will be used where the position and velocity vectors of particle  $d$  are represented by  $x_d = [x_{1d}, x_{2d}, \dots, x_{nd}]$  and  $v_d = [v_{1d}, v_{2d}, \dots, v_{nd}]$ , respectively, where  $d = 1, \dots, N_p$ . At each step, the best position of each particle represented by  $pbest_d = [p_{1d}, p_{2d}, \dots, p_{nd}]$  ( $d = 1, \dots, N_p$ ) based on the valuation of the fitness function and the best particle in the population represented by  $gbest$  will be stored for the next step. The velocity of each particle in the next iteration ( $k+1$ ) for fitness function evaluation is calculated by:

$$v_{id}^{(k+1)} = w^{(k+1)} \times v_{id}^{(k)} + c_1 \times rand_1 \times (pbest_{id}^{(k)} - x_{id}^{(k)}) + c_2 \times rand_2 \times (gbest_{id}^{(k)} - x_{id}^{(k)}) \quad (16)$$

where the constants  $c_1$  and  $c_2$  are cognitive and social parameters, respectively and  $rand_1$  and  $rand_2$  are random values in  $[0, 1]$ .

The position of the corresponding particle is updated as follows:

$$x_{id}^{(k+1)} = x_{id}^{(k)} + v_{id}^{(k+1)} \quad (17)$$

Generally, the solution quality of the PSO method for optimization problems is sensitive to the calculation of the velocity of particles. Therefore, there have been several improvements on the calculation of velocity of particles to enhance its search ability and solution quality. Clerc and Kennedy have proposed an improvement of velocity calculation for particles with added constriction factor [26] which is to insure the stable convergence of the PSO algorithm. The modified velocity of particles with constriction factor  $C$  is calculated as follows:

$$v_{id}^{(k+1)} = C \times \left[ v_{id}^{(k)} + c_1 \times rand_1 \times (pbest_{id}^{(k)} - x_{id}^{(k)}) + c_2 \times rand_2 \times (gbest_i^{(k)} - x_{id}^{(k)}) \right] \quad (18)$$

$$C = \frac{2}{2 - \varphi + \sqrt{\varphi^2 - 4\varphi}}; \text{ where } \varphi = c_1 + c_2, \varphi > 4 \quad (19)$$

In this improvement, the factor  $\phi$  has an impact on the convergence characteristic of the method and must be greater than 4.0 for convergence stability. In the contrary, if the value of  $\phi$  is high, the constriction  $C$  will be small, leading diversification and slower response. Therefore, the best typical value of  $\phi$  suggested by Lim, Montakhab and Nouri [27] is 4.1 (i.e.  $c_1=c_2=2.05$ ).

### Pseudo-gradient concept

The pseudo-gradient is usually used for determining the maximum rate of change direction of non-differentiable functions where the conventional gradient is not applicable. Therefore, it is appropriate for using in population based search methods to enhance their search ability. This concept has been used in population based methods such as genetic algorithm [28] and evolutionary programming [29].

For a non-differentiable objective function  $f(x)$  where  $x=[x_1, x_2, \dots, x_n]$  in a  $n$ -dimension optimization problem, a pseudo-gradient  $g_p(x)$  for the objective function at a certain point  $x_k=[x_{k1}, x_{k2}, \dots, x_{kn}]$  in the search space of the problem moving to another one  $x_l$  is defined for the two cases as follows [29]:

i)  $f(x_l) < f(x_k)$ : the direction from point  $x_k$  to point  $x_l$  is defined as the positive direction. The pseudo-gradient at point  $x_l$  is determined by:

$$g_p(x_l) = [\delta(x_{l1}), \delta(x_{l2}), \dots, \delta(x_{ln})]^T \quad (20)$$

where  $\delta(x_{li})$  is the direction indicator for element  $x_l$  moving from point  $k$  to point  $l$  defined by:

$$\delta(x_{li}) = \begin{cases} 1 & \text{if } x_{li} > x_{ki} \\ 0 & \text{if } x_{li} = x_{ki} \\ -1 & \text{if } x_{li} < x_{ki} \end{cases} \quad (21)$$

ii)  $f(x_l) \geq f(x_k)$ : the direction from point  $x_k$  to point  $x_l$  is defined as the negative direction. The pseudo-gradient at point  $x_l$  is determined by:

$$g_p(x_l) = 0 \quad (22)$$

As shown in the definition, if the value of the pseudo-gradient  $g_p(x_l) \neq 0$ , a better solution for the problem could be found in the next step based on the direction of the pseudo-gradient  $g_p(x_l)$  at point  $l$ . On

the contrary, the search direction at this point may not appropriate due to no improvement can be found for the problem based on this direction.

### Pseudo-gradient based particle swarm optimization

In this paper, the proposed PG-PSOCF is the PSO with constriction factor guided by pseudo-gradient to form a new improved PSO method.

For implementation of the pseudo-gradient in PSOCF, the two considered points for calculation of the pseudo-gradient include the particle's position at iterations  $k$  and  $k+1$  those are  $x^{(k)}$  and  $x^{(k+1)}$ , respectively. Therefore, the updated position for particles in (17) can be rewritten as:

$$x_{id}^{(k+1)} = \begin{cases} x_{id}^{(k)} + \delta(x_{id}^{(k+1)}) \times |v_{id}^{(k+1)}| & \text{if } g_p(x_{id}^{(k+1)}) \neq 0 \\ x_{id}^{(k)} + v_{id}^{(k+1)} & \text{otherwise} \end{cases} \quad (23)$$

As observed in (23), if the value of the pseudo-gradient is non-zero, the particle is moving on the right direction to the optimal solution in the search space of the problem with the enhanced velocity. Otherwise, the particle's position is normally updated as in (17). With the implementation of the pseudo-gradient in PSOCF, the new improved PG-PSOCF can be more effective than the conventional PSO in solving optimization problems due to the enhanced search ability.

### Implementation of PG-PSOCF for the MOOPF

For implementation of the proposed PG-PSOCF to the MOOPF problem, each particle position representing a vector of control variables is defined as follows:

$$x_d = [P_{g2d}, \dots, P_{gNd}, V_{g1d}, \dots, V_{gNd}, Q_{c1d}, \dots, Q_{cNd}, T_{1d}, \dots, T_{Nd}]^T \quad (24)$$

$$d=1, \dots, N_p$$

The upper and lower boundaries of the position of particles  $x_d$  are also the upper and lower limits of the variables contained in the vector. The upper and lower limits for the velocity of each particle are determined based on their lower and upper bounds of position:

$$v_{d,max} = R \times (x_{d,max} - x_{d,min}) \quad (25)$$

$$v_{d,min} = -v_{d,max} \quad (26)$$

where  $R$  is the limit factor for velocity of particles.

The positions and velocities of particles are randomly initialized within their limits as follows:

$$x_d^{(0)} = x_{d,min} + rand_3 \times (x_{d,max} - x_{d,min}) \quad (27)$$

$$v_d^{(0)} = v_{d,min} + rand_4 \times (v_{d,max} - v_{d,min}) \quad (28)$$

where  $rand_3$  and  $rand_4$  are random values in  $[0, 1]$ .

During the iterative process, the positions and velocities of particles are always adjusted satisfying their limits after each iteration as follows:

$$v_d^{new} = \min \{ v_{d,max}, \max \{ v_{d,min}, v_d \} \} \quad (29)$$

$$x_d^{new} = \min \{ x_{d,max}, \max \{ x_{d,min}, x_d \} \} \quad (30)$$

The fitness function of the problem is defined based on the problem objective functions and the dependent variables including real power output at reference bus, reactive power outputs at generation buses, load bus voltages, and power flow in transmission lines. The fitness function of the problem is represented as follows:

$$FT = \omega * F_1(u, x) + (1 - \omega) * F_2(u, x) + K_p (P_{g1} - P_{g1}^{lim})^2 + K_q \sum_{i=1}^{N_g} (Q_{gi} - Q_{gi}^{lim})^2 + K_v \sum_{i=1}^{N_l} (V_{li} - V_{li}^{lim})^2 + K_s \sum_{l=1}^{N_t} (P_l - P_{l,max})^2 \quad (31)$$

where  $\omega$  is the weight factor for objectives;  $K_p$ ,  $K_q$ ,  $K_v$ , and  $K_s$  are penalty factors for real power at reference bus, reactive power at generation buses, load bus voltages, and power flow in transmission lines, respectively.

The limits of the state variables in (31) are determined based on their calculated values as follows:

$$x^{lim} = \begin{cases} x_{max} & \text{if } x > x_{max} \\ x_{min} & \text{if } x < x_{min} \\ x & \text{otherwise} \end{cases} \quad (32)$$

where  $x$  and  $x^{lim}$  respectively represent the calculated values and limits of  $P_{g1}$ ,  $Q_{gi}$ ,  $V_{li}$ , or  $P_{l,max}$ .

The overall procedure of the proposed PG-PSOCF for solving the OPF problem is addressed as follows:

Step 1: Select the controlling parameters for PG-PSOCF including number of particles  $N_p$ , maximum number of iterations  $It_{max}$ , cognitive and social acceleration factors  $c_1$  and  $c_2$ , limit factor for maximum velocity  $R$ , and penalty factors for constraints in fitness function (31). Set the pseudo-gradient to zeros.

Step 2: Initialize the initial position  $x_{id}$  and velocity  $v_{id}$  of  $N_p$  particles within in their limits.

Step 3: For each particle, calculate value of the state variables based on the power flow solution using Newton-Raphson and evaluate the fitness function  $F_{pbstd}$  in (31). Determine the best particle with the lowest value of fitness function  $F_{gbstd} = \min(F_{pbstd}, d=1, \dots, N_p)$ .

Step 4: Set the best particle's position of each particle  $pbstd_{id}$  to  $x_{id}$ ,  $d=1, \dots, N_p$  and the best particle in the population  $gbstd_1$  to the position of the particle corresponding to  $F_{pbstd}$  in Step 3. Set iteration counter  $k=1$ .

Step 5: Calculate new velocity  $v_{id}^{(k)}$  using (18) and update position  $x_{id}^{(k)}$  using (23) for each particle. Note that the obtained position and velocity of particles should be satisfied their lower and upper bounds given by (29) and (30).

Step 6: Solve power flow problem using Newton-Raphson based on the newly obtained position of particles.

Step 7: Evaluate fitness function  $FT_d$  in (31) for each particle with the newly obtained power flow solution. Compare the calculated values of  $FT_d$  to the previous best  $F^{(k-1)}_{pbstd}$  for each particle to obtain the best fitness function up to the current iteration  $F^{(k)}_{pbstd}$ .

Step 8: Select the best position  $pbstd_{id}^{(k)}$  corresponding to  $F^{(k)}_{pbstd}$  for each particle and determine the new global best fitness function  $F^{(k)}_{gbstd}$  and the corresponding position  $gbstd_i^{(k)}$ .

Step 9: Calculate the value of the pseudo-gradient indicators at the current point.

Step 10: If  $k < It_{max}$ ,  $k=k+1$  and return to Step 5. Otherwise, stop.

### Fuzzy based mechanism for best compromise solution

In the multiobjective optimization problems, there is always a conflict and trade-off among the objectives which provides decision maker (DM) several options for decision making. One of the methods

to find the best compromise solution from the Pareto-optimal front of a multiobjective optimization problem is fuzzy satisfying method [30]. This method determines the distance from the value of each objective in the obtained solutions to its maximum value using a linear membership function. A solution is considered the best if the sum of the distances from all objectives in that solution is greater than the sums of the distances from any other solutions.

The fuzzy goal is represented in linear membership function as follows [31]:

$$\mu_j = \begin{cases} 1 & \text{if } F_j \leq F_j^{min} \\ \frac{F_j^{max} - F_j}{F_j^{max} - F_j^{min}} & \text{if } F_j^{min} < F_j < F_j^{max} \\ 0 & \text{if } F_j \geq F_j^{max} \end{cases} \quad (33)$$

where  $\mu_j$  is membership value of objective  $j$ , and  $F_j^{max}$  and  $F_j^{min}$  are maximum and minimum values of objective  $j$ , respectively.

For each non-dominated solution, the membership function is normalized as follows [32]:

$$\mu^k = \frac{\sum_{j=1}^{N_{obj}} \mu_j}{\sum_{k=1}^{N_p} \sum_{j=1}^{N_{obj}} \mu_j} \quad (34)$$

where  $\mu^k$  is membership function of non-dominated solution  $k$ ;  $N_{obj}$  is the number of objective functions; and  $N_p$  is the number of Pareto-optimal solutions.

The solution with maximum membership function  $\mu^k$  can be chosen as the best compromise solution for the problem.

### Numerical Results

The proposed PG-PSOCF has been tested on the IEEE 30-bus with two objectives including total operation cost and emission. The test system has 41 transmission lines, six generators at buses 2, 5, 8, 11, and 13, and four transformers at lines 6-9, 6-10, 4-12 and 27-28. The total load demand of the system is 283.4 MW and 126.2 MVar. The data for the system can be found in [1,33]. The data for total cost, emission and transmission line limits is given in Table 1 and power flow limits of transmission lines are given in Table 2.

For obtaining the power flow solution of the system, the Matpower toolbox [34] is used. Since the bus voltage limits have a great effect on the final results. Therefore, in this research two kinds of bus voltage limit at buses are considered in the range [0.95, 1.05] and [0.95, 1.10]. The tap changer limit of transformers is set to [0.9, 1.1] for all cases. The two capacitor banks are installed at buses 10 and 24.

The proposed PG-PSOCF is coded in the Matlab platform and run on a 3.2 GHz PC. The control parameters of the proposed PG-PSOCF method for all cases of the test system are simply selected as follows:  $N_p=10$ ,  $c_1=c_2=2.05$ ,  $R=0.15$ ,  $It_{max}=200$ . For each test case, the proposed method is performed 20 independent runs.

### Cost objective function

In this case, there is only the total cost objective function is considered. The results obtained by the PG-PSOCF method including min total cost, average total cost, max total cost, standard deviation and average computational time for two kinds of bus voltage limits

	$G_1$ (bus 1)	$G_2$ (bus 2)	$G_3$ (bus 5)	$G_4$ (bus 8)	$G_5$ (bus 11)	$G_6$ (bus 13)
Cost coefficients						
$a_i$ (\$/h)	0	0	0	0	0	0
$b_i$ (\$/MWh)	2	1.75	1	3.25	3	3
$c_i$ (\$/MW <sup>2</sup> h)	0.00375	0.0175	0.0625	0.00834	0.025	0.025
Emission coefficients						
$\alpha_i$ (ton/h)	0.04091	0.02543	0.04258	0.05326	0.04258	0.06131
$\beta_i$ (ton/MWh)	-20.05554	-0.06047	-0.05094	-0.03550	-0.05094	-0.05555
$\gamma_i$ (ton/MW <sup>2</sup> h)	0.06490	0.05638	0.04586	0.03380	0.04586	0.05151
$\xi_i$ (ton/h)	0.0002	0.0005	0.000001	0.002	0.000001	0.00001
$\lambda_i$ (1/MW)	2.857	3.333	8.000	2.000	8.000	6.667

Table 1: Cost and emission coefficients for generators.

Line	1-2	1-3	2-4	3-4	2-5	2-6	4-6	5-7	6-7	6-8	6-9
$P_{l,max}$ (MW)	130	130	65	130	130	65	90	70	130	32	65
Line	6-10	9-10	9-11	4-12	12-13	12-14	12-15	12-16	14-15	15-18	16-17
$P_{l,max}$ (MW)	32	65	65	65	65	32	32	32	16	16	16
Line	18-19	19-20	10-20	10-17	10-21	10-22	21-22	15-23	22-24	23-24	24-25
$P_{l,max}$ (MW)	16	32	32	32	32	32	32	16	16	16	16
Line	25-26	25-27	28-27	27-29	27-30	29-30	8-28	6-28			
$P_{l,max}$ (MW)	16	16	65	16	16	16	32	32			

Table 2: Limits of transmission lines.

	$V_{busmax} = 1.05 \text{ pu}$	$V_{busmax} = 1.10 \text{ pu}$
Min total cost (\$/h)	802.2801	799.1994
Average total cost (\$/h)	802.7527	799.9818
Max total cost (\$/h)	805.4520	804.4023
Standard deviation (\$/h)	0.9124	1.2758
Average CPU time (s)	15.335	15.248
Total emission (ton/h)	0.3631	0.3666
Power losses (MW)	9.4364	8.6699

Table 3: Results by PG-PSOCF for cost dispatch case with different bus voltage limits.

	PSO [17]	IPSO [17]	PG-PSOCF
$P_{g1}$ (MW)	178.4646	177.0431	177.2254
$P_{g2}$ (MW)	46.274	49.209	48.6302
$P_{g3}$ (MW)	21.4596	21.5135	21.3220
$P_{g4}$ (MW)	21.446	22.648	21.0422
$P_{g5}$ (MW)	13.207	10.4146	11.8500
$P_{g6}$ (MW)	12.0134	12	12.0000
Cost (\$/h)	802.205	801.978	799.1994

Table 5: Result comparison for cost dispatch case with bus voltage limit of 1.1 pu.

	NLP [1]	EP [14]	TS [13]	IEP [15]	MDE-OPF [16]	MSLFA [19]	PG-PSOCF
$P_{g1}$ (MW)	176.26	173.848	176.04	176.2358	175.974	179.1929	176.0340
$P_{g2}$ (MW)	48.84	49.998	48.76	49.0093	48.884	48.9804	48.8786
$P_{g3}$ (MW)	21.51	21.386	21.56	21.5023	21.51	20.4517	21.5350
$P_{g4}$ (MW)	22.15	22.63	22.05	21.8115	22.24	20.9264	22.1439
$P_{g5}$ (MW)	12.14	12.928	12.44	12.3387	12.251	11.5897	12.2448
$P_{g6}$ (MW)	12	12	12	12.0129	12	11.9579	12.0000
Cost (\$/h)	802.4	802.62	802.29	802.465	802.376	802.287	802.2801

Table 4: Result comparison for cost dispatch case with bus voltage limit of 1.05 pu.

are given in Table 3. As observed from the table, the total cost for the case with bus voltage limit of 1.05 pu is higher than that for the case with bus voltage limit of 1.1 pu while the total emission for the two cases are nearly the same. For the both cases, the standard deviation is very small which indicates that the proposed method can obtain high quality solution for this case.

The best results by the proposed PG-PSOCF for the two cases have been compared to those from other methods as shown in Tables 4 and 5. For the both cases, the proposed method can obtain better total cost than the others. Therefore, the proposed PG-PSO is very effective for solving the OPF problem.

### Emission objective function

In this case, there is only the emission objective function is

	$V_{busmax} = 1.05 \text{ pu}$	$V_{busmax} = 1.10 \text{ pu}$
Min emission (ton/h)	0.2049	0.2048
Average emission (ton/h)	0.2092	0.2063
Max emission (ton/h)	0.2398	0.2195
Standard deviation (\$/h)	0.0087	0.0032
Average CPU time (s)	15.137	15.241
Total cost (\$/h)	944.7824	943.7578
Power losses (MW)	3.3514	3.0357

Table 6: Results by PG-PSOCF for emission dispatch case with different bus voltage limits.

considered. The obtained results by the proposed method for the two cases of bus voltage limits including min emission, average emission, max emission, standard deviation, and average CPU time are given in Table 6. The total emission for the both cases of bus voltage limits is not different. Moreover, the standard deviation of the proposed method for the both cases is also very small.

The result comparisons from the proposed method and other methods for this case with two bus voltage limits are given in Tables 7 and 8. As shown in the tables, the total emission from the proposed method is less than that from the others. Therefore, the proposed PG-PSOCF is also very effective for this case.

### Multiobjective function

In this case, both total cost and emission are simultaneously

	GA [19]	PSO [19]	SLFA [19]	MSLFA [19]	PG-PSOCF
P <sub>g1</sub> (MW)	78.2885	59.8075	64.4840	65.7798	63.9471
P <sub>g2</sub> (MW)	68.1602	80	71.3870	68.2688	67.4886
P <sub>g3</sub> (MW)	46.7848	50	49.8573	50	50.0000
P <sub>g4</sub> (MW)	33.4909	35	35	34.9999	35.0000
P <sub>g5</sub> (MW)	30	27.1398	30	29.9982	30.0000
P <sub>g6</sub> (MW)	36.3713	40	39.9729	39.9970	40.0000
Emission (ton/h)	0.21170	2.096	2.063	0.2056	0.2049

Table 7: Best result comparison for emission dispatch case with bus voltage limit of 1.05 pu.

	GA [17]	PSO [17]	IPSO [17]	PG-PSOCF
P <sub>g1</sub> (MW)	69.7300	67.1300	67.0400	63.9471
P <sub>g2</sub> (MW)	67.8400	68.9400	68.1400	67.4886
P <sub>g3</sub> (MW)	49.7300	49.8600	50.0000	50.0000
P <sub>g4</sub> (MW)	34.4200	34.8900	35.0000	35.0000
P <sub>g5</sub> (MW)	29.1500	29.6700	30.0000	30.0000
P <sub>g6</sub> (MW)	39.2900	39.9400	40.0000	40.0000
Emission (ton/h)	0.2072	0.2063	0.2058	0.2048

Table 8: Best result comparison for emission dispatch case with bus voltage limit of 1.1 pu.

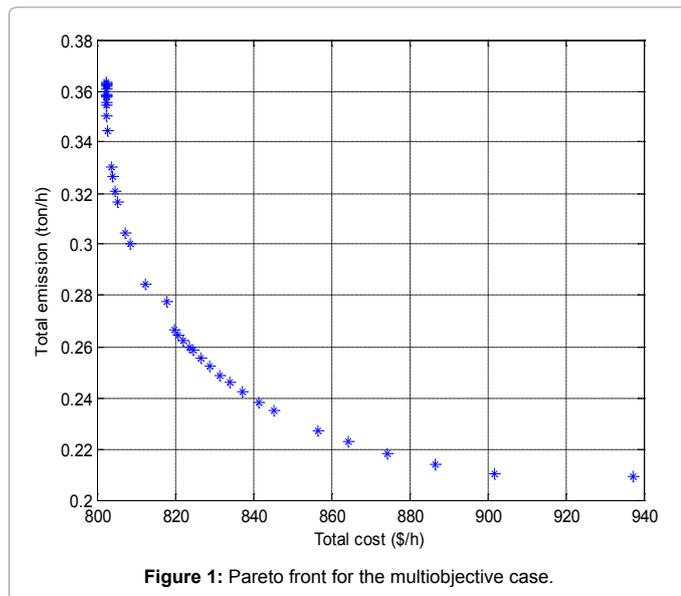


Figure 1: Pareto front for the multiobjective case.

	GA [19]	PSO [19]	SLFA [19]	MSLFA [19]	PG-PSOCF
P <sub>g1</sub> (MW)	96.1251	97.8588	98.9772	97.55027	95.0194
P <sub>g2</sub> (MW)	68.5168	61.9419	58.6832	60.42367	61.4059
P <sub>g3</sub> (MW)	26.7031	31.1310	35.0661	31.6343	31.9402
P <sub>g4</sub> (MW)	35	34.4808	31.7585	35	35.0000
P <sub>g5</sub> (MW)	30	29.7100	29.9182	30	30.0000
P <sub>g6</sub> (MW)	34.7555	36.0884	35.8174	35.21483	35.1872
Total cost (\$/h)	872.9601	872.8731	872.8533	867.713	866.0267
Emission (ton/h)	0.2270	0.2253	0.2249	0.2247	0.2229

Table 9: Best result comparison for multiobjective dispatch case with bus voltage limit of 1.05 pu.

considered in the problem. Since there is not much different total cost and emission between the bus voltage limits, only the case with bus voltage limit of 1.05 pu is considered for the multiobjective function. For obtaining the Pareto front for this case, multiple solutions are determined by changing the value of weight factor  $\omega$  from 0 to 1.

Figure 1 depicts the Pareto front obtained by the proposed method for different bus voltage limits.

Based on the obtained solution for the Pareto front, the fuzzy based mechanism is used for obtaining the best compromise solution for the problem. The best compromise solution obtained by the proposed method is 866.0267 (\$/h) and 0.2229 (ton/h) which is better than other methods as shown in Table 9. Therefore, the proposed PG-PSOCF is also very effective for the multiobjective case of the problem.

## Conclusion

In this paper, the proposed PG-PSOCF method has been effectively and efficiently implemented for solving the MOOPF problem. The PG-PSOCF is the conventional PSO method with constriction factor guided by pseudo-gradient for enhancement its search ability and solution quality. The proposed can properly deal with the MOOPF problem using the fuzzy based mechanism for best compromise solution. The test results for the IEEE 30 bus system with different bus voltage limits have indicated that the proposed method can obtain better solution quality than many other methods. Moreover, the proposed method can be also extended for dealing with more complex and larger scale OPF problems. Therefore, the proposed PG-PSOCF could be a powerful and favorable method for solving the MOOPF problem.

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