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Pt-Symmetric Nonlinear Directional Fiber Couplers with Gain and Loss for Ultrashort Optical Pulses

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Abstract

In this paper, we study the higher-order nonlinear Schrdinger (HNLS) equations for a directional fiber optic coupler with the third-order dispersion (TOD), self-steeping (SS) and stimulated Raman scattering (SRS) effects and gain in one waveguide and loss in the other. We have reduced the system equations to an uncoupled HNLS equation which has bright and dark soliton solutions. It is shown that the Hamiltonian for this equation has a PT-symmetric form. Furthermore the equilibrium points are found and the discussion about their stability is presented.

Keywords: Fibers; Nonlinear optics; Directional coupler; Photonic; Ultrashort optical; Pulses; PT-symmetry

Introduction

The study of physical phenomena by means of mathematical models is an essential element in both theoretical and experimental fields [1-4]. In this paper, there exists a special class of nonlinear wave equations that support soliton solutions in nonlinear physical systems. Propagating solitons through nonlinear directional fiber couplers is an interesting field in optical communication systems [5-8]. This should not be surprising because the soliton pulses are used as the information carriers (elementary bits) to transmit digital signals over long distances [9].

Recently some analytical solutions and analytical studies for higher-order nonlinear problems in optical fibers have been investigated [10,11]. Signal propagation through the nonlinear coupler can be affected by higher-order effects such as third-order dispersion (TOD), self-steeping (SS) and stimulated Raman scattering (SRS), which describes the propagation of ultrashort pulses. The TOD can make the asymmetrical temporal broadening [9], the SS is responsible for asymmetrical spectral broadening [12], and the SRS can account for the self-frequency shift in the femtosecond regime [5]. Furthermore directional couplers composed of waveguides with gain and loss regions can be used to realize PT-symmetric optical structures [13-17]. In this paper, our study focuses on the HNLS equations for such couplers. We assume that gain in one waveguide and loss in the other have the same absolute value. We will derive the Hamiltonian of the system and investigate its PT-symmetric properties, also we derive the equilibrium points and discuss about stability.

Theoretical Equation

The PT-symmetric coupler with gain in one waveguide and loss in the other one has been studied theoretically and experimentally [16]. Since nonlinear directional couplers have two identical cores with gain in one waveguide and loss in the other, by considering the ultrashort pulse condition which causes the existence of three higher-order effects, the coupled equations for propagating pulses can be extended to the following higher-order nonlinear coupling equations:

$$iu_z+\alpha_1 u_{\tau\tau}+\alpha_2|u|^2 u+i\epsilon[\alpha_3 u_{\tau\tau\tau}+\alpha_4 u\,(|u|^2)u_\tau+\alpha_5(|u|^2)_\tau]=-v-i\gamma u$$

$$iv_z+\alpha_1 v_{\tau\tau}+\alpha_2|v|^2 v+i\epsilon[\alpha_3 v_{\tau\tau\tau}+\alpha_4 v(|v|^2)v_\tau+\alpha_5 v(|v|^2)_\tau]=-u-i\gamma v \qquad (1)$$

Where u and v represent the slowly varying envelops, z and τ are variables for propagation direction and retarded time respectively, ϵ is the ratio of the width of spectra to the center frequency. α1, α2, α3, α4 and α5 are group velocity dispersion (GVD), self-phase modulation (SPM), TOD, SS, and SRS coefficients respectively, which are real parameters. For analysis of the coupled equations, an assumption of the following form is considered:

$$u(z,\tau)=\exp^{i(\Omega z-\theta)}U(z,\tau),\ v(z,\tau)=\exp^{i(\Omega z)}v(z,\tau) \qquad (2)$$

Where θ is a constant angle satisfying:

$$\sin\theta=\gamma \qquad (3)$$

and Ω is a real parameter.

By applying $U=V\equiv\phi$ eqn. 1 reduce into:

$$i\phi_z+(\alpha_1-\epsilon\alpha_3\Omega)\phi_{\tau\tau}+(\alpha_2-\epsilon\alpha_4\Omega)|\phi|^2\phi+i\epsilon[\alpha_3\phi_{\tau\tau\tau}+\alpha 4|\phi|\,\phi\tau+\alpha_5\phi(|\phi|^2)_\tau]$$
$$+[-\alpha_1\Omega_2+\epsilon\alpha_3\Omega^3+\Omega-a^2]\phi=0 \qquad (4)$$

Where a2=Ω−cosθ. In order to make eqn.4 look like Kodama and Hasegawa derived general HNLS equation [18,19], we obtain:

$\eta 1=(\alpha 1-\epsilon\alpha 3\Omega)$

$\eta 2=(\alpha 2-\epsilon\alpha 4\Omega)$

$\eta 3=\alpha 3$

$\eta 4=\alpha 4$

$\eta 5=\alpha 5$

$(-\alpha 1\Omega 2+\epsilon\alpha 3\Omega 3+\Omega-a2)=0$

By substituting these changes eqn.4 is reduced into

$$i\phi_z+\eta_1\phi_{\tau\tau}+\eta_2|\phi|^2\phi+i\epsilon[\eta_3\phi_{\tau\tau\tau}+\eta_4|\eta|^2\phi_{\tau+}\eta_5 u(|\phi|^2)_\tau]=0 \qquad (5)$$

By using the traveling wave transformation

$$\phi(z,\tau)=q(\lambda)\exp(i\theta),\ \lambda=b(\tau-cz),\ \theta=k\tau-\omega z \qquad (6)$$

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Where b, c, m and ω are real constants and $\phi(\lambda)$ is a real value function of λ, eqn. 5 is transfered into the following form:

$$i[b^3\epsilon\eta_3 q''' + b\epsilon(\eta_4+2\eta 5)q^2q' - (bc-2bk\eta_1+3bk^2\epsilon\eta_3)q'] + [(b^2\eta_1-3b^2k\epsilon\eta_3)q'' + (\omega-k^2\eta_1+k^3\epsilon\eta_3)q + (\eta 3-k\epsilon\eta_4)q^3] = 0 \quad (7).$$

The prime means the differentiation with respects to λ. Now we divide eqn.7 into real and imaginary parts and obtain the following equations:

$$b^3\epsilon\eta_3 q''' + b\epsilon(\eta_4+2\eta 5)q^2q' - (bc-2bk\eta_1+3bk^2\epsilon\eta_3)q' = 0 \quad (8)$$

$$(b^2\eta 1-3b^2k\epsilon\eta_3)q'' + (\omega-k2\eta_1+k^3\epsilon\eta_3)q + (\eta_3-k\epsilon\eta_4)q^3 = 0 \quad (9)$$

Integrating eqn.8 and taking integration constant as zeros gives:

$$b^3\epsilon\eta_3 q'' + b(2k\eta_1-3k^3\epsilon\eta_3-c)q + 1/3b\epsilon(\eta_4+2\eta_5)q^3 = 0 \quad (10)$$

As we want to have a solution for eqn. 9 and eqn. 10 simultaneously, we should have the following relations for k and ω:

$$k = \frac{\eta_1\eta_4 - 3\eta_2\eta_3 + 2\eta_1\eta_5}{6\epsilon\eta_3\eta_5} \quad (11)$$

$$\omega = \frac{2k(\eta_1)^2 - \eta_1(c + 8k^2\epsilon\eta_3)}{\epsilon\eta_3} + k(3c + 8k^2\epsilon\eta_3 \quad (12)$$

Then the system equations are reduced into the ordinary differential equations as follow:

with

$$\beta_1 q'' + \beta_2 q + \beta_3 q^3 = 0 \quad (13)$$

$$\beta_1 = \frac{b^2(3\eta_2\eta_3 - \eta_1\eta_4)}{2\eta_5} \quad (14)$$

$$\beta 2 = -(3\eta_2\eta_3 - \eta_1\eta_4) \times \frac{(3\eta_2\eta_3 - \eta_1\eta_4)^2 + 4c\eta_5^2(3\epsilon\eta_3 - (\eta_1)^2)}{24\epsilon^2\eta_3^2\eta_5^3} \quad (15)$$

Introduce M ≡ q and N ≡ qλ, substitute into eqn.13, two-dimensional independent systems are obtained:

$$M'=N,\ N' = \frac{\beta 2}{\beta 1}M - \frac{\beta 3}{\beta 1}M^3 \quad (16)$$

reach the Hamiltonian function as:

$$H(M, N) = \frac{1}{2}N^2 + \frac{\beta_2}{2\beta_1}M^2 + \frac{\beta 3}{4\beta 1}M^4. \quad (17)$$

PT-Symmetric Hamiltonian

Directional couplers with gain in one waveguide and loss in the other can be used to realize PT-symmetric optical structures [20,21].

In quantum mechanics theory the requirement for the Hamiltonian to be PT-symmetric is that to commute with PT operators: [H, PT]=0.

Since the Hamiltonian in eqn. 17 can be written in the usual form of in quantum mechanics [14,22,23]:

$$H = 1/2p^2 + \nu(x) \quad (18)$$

where the p operator is $p = -i\frac{d}{dx}$.

Any Hamiltonian in this form is real and symmetric [13,24,25] and the condition of PT-transmission is satisfied by this Hamiltonian. In general, for optical structures a necessary condition for a Hamiltonian to be PT-symmetric is that the complex potential satisfies: $n(x; y)=n^*(-x; y)$ [13]. It means that the absolute value of gain is the same as the absolute value of loss, and gain/loss regions should have mirror configurations with respect to the central symmetry point [16], which is happened in this Hamiltonian. Clearly, Hamiltonian in eqn.17 can be realized as a PT-symmetric Hamiltonian.

Stability Analysis

According to the Hamiltonian eqn.17 and the introduced parameters M, N in eqn.16, we derive the equilibrium points and investigate their stabilities. We have to use the following Jacobian matrix of the system:

$$J = \begin{pmatrix} 0 & 1 \\ \frac{-\beta_2 + 3\beta_3 M^2}{\beta_1} & a_{22} \end{pmatrix} \quad (19)$$

The corresponding characteristic equation is derived as:

$$\rho 2 + \frac{\beta 2 + 3\beta_3 M^2}{\beta_1} = 0 \quad (20)$$

Simple calculations show that the eigenvalues are:

$$\rho_1, 2 = \pm\sqrt{\frac{-(\beta_2 + 3\beta_3 M_2)}{\beta_1}} \quad (21)$$

There are two cases for deriving the equilibrium points which are related to the sign of the value under radical in eqn. 21.

In the first case if $\frac{\beta_2}{\beta_3} \geq 0$, only one equilibrium point (0, 0) is obtained and the related eigenvalue is:

$\pm\sqrt{-\frac{\beta_2}{\beta_3}}$. If $\frac{\beta_2}{\beta_3} < 0$ it is an unstable point and If $\frac{num}{\beta_2}\beta_1 > 0$, it is a center point.

In the second case if $\frac{\beta_2}{\beta_3} < 0$, three equilibrium points (0, 0), $(\sqrt{-\frac{\beta_2}{\beta_3}}, 0)$ and $(-\sqrt{-\frac{\beta_2}{\beta_3}}, 0)$ are obtained and the corresponding eigenvalues are $\pm\sqrt{\frac{-\beta_2}{\beta_3}}$, $\pm\sqrt{\frac{2\beta_2}{\beta_1}}$ and $\pm\sqrt{\frac{2\beta_2}{\beta_1}}$. As it can be seen, the second and third points have the same eigenvalue, so they are degenerate.

If $\frac{num}{\beta_2}\beta_1 > 0$, (0, 0) point is an unstable point where as two other points are both centers. If $\frac{num}{\beta_2}\beta_1 > 0$ (0,0) is a center point and can be stable or unstable while two others points both are unstable ones.

Conclusion

In this paper, the stability of an ultrashort pulse in a higher-order nonlinear fiber coupler with gain in one fiber waveguide and loss in the other one have been studied. Three effects, the third-order dispersion (TOD), self-steeping (SS) and stimulated Raman scattering (SRS) are considered in the equations. Furthermore we reduced two coupled equations to the one nonlinear equation, then obtain the Hamiltonian of system. We have shown the Hamiltonian is PT -symmetric.

After that we have obtained the equilibrium points as: (0,0), $(\sqrt{-\frac{\beta_2}{\beta_3}}, 0)$ and $(-\sqrt{-\frac{\beta_2}{\beta_3}}, 0)$. The second and third points have the same eigenvalue which means that they are degenerate. Therefor these points have been investigated under two the conditions, first $\frac{\beta_2}{\beta_1} \geq 0$, there exists an unstable point and two centers. In the second condition $\frac{\beta_2}{\beta_1} < 0$ there exist one center point and two unstable points.

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