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Qualitative Analysis of Nonlinear Evolution Equations

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A great number of processes of the applied sciences can be modeled by means of evolution equations or systems. Nonlinear partial differential equations and systems exhibit a number of properties which are absent from the linear theories; these nonlinear properties are often related to important features of the real world phenomena which the mathematical model is supposed to describe; at the same time these new properties are closely connected with essential new difficulties of the mathematical treatment. The study of nonlinear processes has been a continuous source of new problems and it has motivated the introduction of new methods in the areas of mathematical analysis, partial differential equations and other disciplines, becoming a most active area of mathematical research.

The success of the new methods of modern analysis has enabled mathematicians to give rigorous answers to important questions of the nonlinear world. One of the most remarkable properties that distinguish nonlinear evolution problems from the linear ones is the possibility of eventual occurrence of singularities starting from perfectly smooth data, more specifically, from classes of data for which a theory of existence, uniqueness and continuous dependence can be established for small time intervals, so called well posedness in the small. While singularities can arise in linear problems, this happens through the singularities contained in the coefficients or data or the problem (called fixed singularities). On the contrary, in nonlinear systems they may arise from the nonlinear mechanisms of the problem and their time and location are to be determined by the mathematical analysis (called moving singularities).

The qualitative theory of evolution equations has been developing extensively in the last few decades, and a lot of important, interesting and beautiful results have been obtained concerning the dynamics of solutions and qualitative description of steady states. There are a number of other popular models of evolution problems involving nonlinear parabolic equations, possibly degenerate, such as the porous medium and p-Laplace and other degenerate quasilinear equations. The basic questions on the analysis of the main questions were raised in the study of blowup for reaction-diffusion equations. This list can be suitably adapted to other singularity formation problems, which includes the questions of when, where and how. Here an expanded list of six items is proposed: (1) Does blow-up occur? (2) When? (3) Where? (4) How? (5) What happens later? (6) How to compute it numerically. We refer the readers to [1] for more details for every item. There exist some good texts which display many of the results which are known and contain extended lists of references, like the book by Quittner and Souplet [2], and the surveys paper by Galaktionov and Vazquez [1]. However, this is a very active field where there are many new developments almost everyday.

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