

Representations of Six-dimensional Mubarakzhanov Lie algebras

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Abstract

A corrected list of the Mubarakzhanov algebras, that is, six-dimensional indecomposable solvable Lie algebras for which the nilradical is five-dimensional, is presented. For each class of algebra a matrix representation and a system of vector fields representation is given. As a consequence it follows that the dimension of the minimal matrix representation of any six-dimensional Lie algebra is at most six.

Keywords: Lie algebra; Lie group; Mubarakzhanov algebra; Minimal matrix representation; Vector field representation MSC Classification: 17B99

Introduction

As long ago as 1963 Mubarakzhanov classified real six-dimensional indecomposable solvable Lie algebras for which the nilradical is five-dimensional [1]. This paper is frequently referred to but suffers from a number of defects. Recently Shabanskaya and Thompson [2] reconsidered Mubarakzhanov's paper and made a number of improvements. Many errors, both substantive and typographical were corrected, some algebras were shown to be redundant, others missing; finally even in those algebras that were essentially correct, in many classes of algebras that depend on parameters, the parameters could be simplified and some removed entirely.

In the present article, given that Mubarakzhanov's list has now been corrected, we turn to the issue of finding representations. We know of course that finding representations for solvable algebras is a completely different enterprise from the semi-simple case. In most cases the algebra will have a trivial center and so the adjoint representation furnishes a faithful representation. However, using the adjoint representation is not always satisfactory. In the first place many algebras have non-trivial centers; secondly, it frequently happens that in a class of algebras that depend on parameters the center will be non-trivial except for certain special values of the parameters. Therefore, one has to use a different representation in the exceptional case, which is an unsatisfactory state of affairs. In the representations given below we have endeavored to the extent possible to avoid such split cases.

We give two kinds of representations for the solvable algebras. First of all we consider matrix representations, in most cases as subalgebras of $gl(6, \mathbb{R})$ or occasionally $gl(5, \mathbb{R})$. However, there is something of a logistical problem in that it is wasteful of space to present a list of six matrices for each class of algebras. Alternatively, one could write a single matrix, M say, depending on some variables such that the span of the matrices engenders the given algebra. In this paper we go one step further and simply convert the given 5×5 or 6×6 into a single 25 or 36 row-vector, that we call the *DNA vector*, by using the rows of M .

In the second kind of representation we give a system of six vector fields on \mathbb{R}^6 using local coordinates (p, q, x, y, z, w) , which furnish a basis for the algebra under Lie bracket. Actually, in our opinion the best way to represent a solvable algebra is to give the matrix Lie group that is engendered by exponentiating the space of matrices spanned by the matrix M referred to in the preceding paragraph. It is then a

simple matter to construct the matrix Lie algebra by differentiating and evaluating at the identity. One can also easily construct a system of left or right-invariant vector fields starting from the group matrix. All of these quantities can be obtained in principle from the DNA-vector, which explains the name. We hope eventually to publish such a list of matrices in a suitable venue but again space is an issue.

There is another interesting consequence of our list of matrix representations. Following Burde [3,4] we define the invariant $\mu(g)$ to be the minimum value of p such that a Lie algebra $_{p \in \mathbb{R}}$ has a representation as a subalgebra of $gl(p, \mathbb{R})$. Then we can assert that $\mu \leq 6$ for all of the Mubarakzhanov algebras. Ado's Theorem states that there exists a finite-dimensional faithful representation as a subalgebra of $gl(p, \mathbb{R})$ for some $p \in \mathbb{N}$. The theorem does not give much information about the value of p but leads one to believe that p may be very large in relation to the size of \mathfrak{n} where n is the dimension of g . A little care must be exercised because there may well be inequivalent representations for which this minimum value is attained. Of course if g has a trivial center then the adjoint representation furnishes a faithful representation of g and in the notation used above, $\mu \leq n$. Nonetheless many algebras have non-trivial centers, nilpotent algebras for example, and then the adjoint representation is not faithful. Even if the center is trivial it could well be the case that $\mu(g) < n$.

It has been shown in Ghanam et al. [5] that for all five-dimensional algebras $\mu(g) \leq 5$. If we refer Ghanam et al. and Rawashdeh and Thompson [6,7] then we can assert quite generally that for all six-dimensional algebras $\mu(g) \leq 6$.

Of course it is interesting to ascertain the value of μ from a theoretical point of view. However, an important practical reason is that calculations involving symbolic programs such as Maple and Mathematica use up lots of memory when storing matrices; accordingly, calculations are likely to be faster if one can represent matrix Lie algebras using matrices

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of a small size. In this paper the many calculations were performed with the help of Maple.

This article is organized as follows. In Table 2, we discuss the classification of the low-dimensional Lie Algebras. In Table 3, we list all the possible five-dimensional nilpotent algebras that occur as nilradicals in the Mubarakzhanov algebras. In Table 4, we reproduce mutatis mutandis the updated list of Mubarakzhanov algebras that appeared in Shabanskaya and Thompson [2]. In Table 5, we provide matrix representations for all the Mubarakzhanov algebras using the device of DNA-vectors introduced above. Finally in Table 6, we supply the list of vector field representations. In these last two Sections in the interests of efficiency we are content simply to provide the lists without any accompanying theory.

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Classification of Low-Dimensional Lie Algebras

Many authors have considered the classification problem for the low-dimensional Lie algebras. The classification of Lie algebras in two and three dimensions has been known for almost a century and dimensions four, five and six were carried out by two Russian mathematicians Morozov and Mubarakzhanov amongst others [1,8,9]. For a much more recent account the reader may also consult Basarab et al. and Campoamor [10,11].

An important inequality for a solvable Lie algebra g asserts that

$$\dim(\text{nil}(g)) \geq \frac{1}{2} \dim(g)$$

where $\text{nil}(g)$ denotes the nilradical of g [8]. In dimension six there are four possible values for $\dim(\text{nil}(g))$, that is; three, four, five or six. Case three does not occur because such a Lie algebra is decomposable. Case six comprises six-dimensional nilpotent Lie algebras that were classified by Morozov [8] and in the latter reference representations were supplied [12]. As regards the classification of the five-dimensional nilradical algebras they were studied by Mubarakzhanov [1], who found ninety nine classes of such Lie algebra; his list has been updated in Shabanskaya and Thompson [2] and this class of algebras is the concern of the present article. In the last case, that is the dimension of the nilradical is four, the algebras were classified by Turkowski [13]. A preliminary list of representations for the Turkowski algebras has been carried out by Rawashdeh and Thompson [7]. More details concerning

the classification of the low-dimensional Lie algebras may be found in Turkowski [14].

The Nilradical

The nilradical $\text{nil}(g)$ of an indecomposable six-dimensional solvable Lie algebra g for which $\text{nil}(g)$ is five-dimensional can be one of nine types. In his paper Mubarakzhanov devotes a separate paragraph corresponding to each of these nine possible nilradicals.

- algebras 1 to 12 have abelian nilradical R^5 (Table 1)
- algebras 13 to 38 have nilradical isomorphic to $\mathbb{R}^2 \oplus H$ (Table 2)
- algebras 39 to 53 have nilradical isomorphic to $\mathbb{R} \oplus A_{4,1}$ (Table 3)
- algebras 54 to 70 have nilradical isomorphic to $A_{5,1}$ (Table 4)
- algebras 71 to 75 have nilradical isomorphic to $A_{5,2}$ (Table 5)
- algebras 76 to 81 have nilradical isomorphic to $A_{5,3}$ (Table 6)
- algebras 82 to 93 have nilradical isomorphic to $A_{5,4}$ (Table 7)
- algebras 94 to 98 have nilradical isomorphic to $A_{5,5}$ (Table 8)
- algebra 99 has nilradical isomorphic to $A_{5,6}$ (Table 9).

Here R^n denotes the n -dimensional abelian Lie algebra, H denotes the three-dimensional Heisenberg algebra, $A_{4,1}$ and the algebras $A_{5,n}$ are taken from [12] and denote nilpotent algebras, 4 and 5, respectively, being the dimensions of the algebra.

Classification of the Mubarakzhanov Algebras

In this Section we give an amended version of Mubarakzhanov's list of algebras. We have simplified the notation using for the parameters a, b, c, d . Also ϵ can only assume the values of 0 or ± 1 and δ can only be 0 or 1 and α can only be 0 or 2 except in $g_{6,8,8}$. We do not consider δ and ϵ to be parameters since they only assume discrete values. At the start of each table we supply the brackets for the nilradical and in the table the brackets of the basis vectors with e_i . We do not highlight algebras where we merely improve the range of values of the parameters.

- Algebras below that have no asterisks are considered to be essentially correct and differ from Mubarakzhanov's algebras mutatis mutandis, that is, only by small notational differences
- Algebras that have one asterisk denote algebras in which the parameters in Mubarakzhanov's algebra can be simplified or an entry can be removed completely

Algebra	$[e_1, e_2]$	$[e_2, e_3]$	$[e_3, e_4]$	$[e_4, e_5]$	$[e_5, e_6]$	Conditions on parameters
$\dagger g_{6,1}$	e_1	ae_2	be_3	ce_4	de_5	$0 < d \leq c \leq b \leq a \leq 1$
$\dagger g_{6,2}$	ae_1	$e_1 + ae_2$	e_3	be_4	ce_5	$0 < c \leq b \leq 1$
$\dagger g_{6,3}$	ae_1	$e_1 + ae_2$	$e_2 + ae_3$	e_4	be_5	$0 < b \leq 1$
$\dagger g_{6,4}$	ae_1	$e_1 + ae_2$	$e_2 + ae_3$	$e_3 + ae_4$	e_5	
$g_{6,5}$	e_1	$e_1 + e_2$	$e_2 + e_3$	$e_3 + e_4$	$e_4 + e_5$	
$\dagger g_{6,6}$	e_1	ae_2	$e_2 + ae_3$	be_4	$e_4 + be_5$	$a \leq b$
$g_{6,7}$	ae_1	$e_1 + ae_2$	$e_2 + ae_3$	be_4	$e_4 + be_5$	$a = 0, b = 1$ or $a = 1$
$\dagger g_{6,8}$	ae_1	be_2	ce_3	$de_4 + e_5$	$-e_4 + de_5$	$0 < c \leq b \leq a $
$\dagger g_{6,9}$	ae_1	be_2	$e_2 + be_3$	$ce_4 - e_5$	$e_4 + ce_5$	$a \neq 0$
$g_{6,10}$	ae_1	$e_1 + ae_2$	$e_2 + ae_3$	$be_4 - e_5$	$e_4 + be_5$	
$\dagger g_{6,11}$	e_1	$ae_2 - be_3$	$be_2 + ae_3$	$ce_4 - de_5$	$de_4 + ce_5$	$bd \neq 0$
$\dagger g_{6,12}$	ae_1	$be_2 - e_3$	$e_2 + be_3$	$e_2 + be_4 - e_5$	$e_3 + e_4 + be_5$	$a \neq 0$

Table 1: Lie algebras having nilradical isomorphic to R^5 .

Algebra	$[e_1, e_6]$	$[e_2, e_6]$	$[e_3, e_6]$	$[e_4, e_6]$	$[e_5, e_6]$	Remarks
$g_{6,13}$	$(a+b)e_1$	ae_2	be_3	e_4	ce_5	$a+2b \neq 0, a \leq b , 0 < c \leq 1$
$g_{6,14}$	$(a+b)e_1$	ae_2	be_3	e_4	$e_1 + (a+b)e_5$	$ a \leq b $
$g_{6,15}$	$(a+1)e_1$	$e_2 + e_4$	$ae_3 + e_5$	e_4	ae_5	$ a \leq 1$
$g_{6,16}$	e_1	$e_2 + e_4$	e_5	$e_1 + e_4$	0	
$tg_{6,17,1}$	ae_1	ae_2	e_4	δe_1	e_5	$(\delta = 0, 1)$ together $g_{6,17}$ in [1]
$g_{6,17,2}^{***}$	e_1	e_2	e_4	0	$e_1 + e_5$	
$tg_{6,18}$	$(a+1)e_1$	ae_2	$e_3 + e_4$	e_4	be_5	$b \neq 0$
$g_{6,19}$	$(a+1)e_1$	ae_2	$e_3 + e_4$	e_4	$e_1 + (a+1)e_5$	
$tg_{6,20}$	e_1	0	$e_3 + e_4$	$e_1 + e_4$	be_5	$b \neq 0$
$tg_{6,21}$	$2ae_1$	$ae_2 + e_3$	ae_3	e_4	be_5	$0 < b \leq 1$
$g_{6,22}$	$2ae_1$	$ae_2 + e_3$	ae_3	e_4	$e_1 + 2ae_5$	
$g_{6,23}^*$	$2\delta e_1$	$\delta e_2 + e_3$	$\delta e_3 + e_4$	δe_4	$\epsilon e_1 + (2\delta + a)e_5$	$(\delta, \epsilon, a) = (1, 0, \neq -2)$ or $(0, 0, 1)$ (6.23.1) (suspensions) or $(1, 1, 0)$ (6.23.2)
$tg_{6,24}^*$	0	e_3	e_4	e_1	e_5	
$g_{6,25}$	$(a+1)e_1$	e_2	ae_3	$be_4 + e_5$	be_5	$ a \leq 1$
$g_{6,26}$	$(a+1)e_1$	e_2	ae_3	$(a+1)e_4 + e_5$	$e_1 + (a+1)e_5$	$ a \leq 1$
$g_{6,27}^*$	$(a+b)e_1$	ae_2	$be_3 + e_4$	$be_4 + e_5$	$\delta e_1 + be_5$	$a = 1, \delta = 0$ or $a = 0, b = 1, \delta = 0, 1$
$g_{6,28}$	$2e_1$	$e_2 + e_3$	e_3	$be_4 + e_5$	be_5	
$g_{6,29}$	$2e_1$	$e_2 + e_3$	e_3	$2e_4 + e_5$	$e_1 + 2e_5$	
$g_{6,30}$	0	e_3	0	$e_4 + e_5$	e_5	
$g_{6,31}$	$2e_1$	$e_2 + e_3$	$e_3 + e_4$	$e_4 + e_5$	e_5	
$g_{6,32}^{****}$	$2ae_1$	$ae_2 + e_3$	$ae_3 - e_2$	$\delta e_1 + (2a + b)e_4$	ce_5	$(\delta, b) = (0, \neq -2a)$ or $(1, 0), c \neq 0, 2a + b \geq c$
$tg'_{6,32}$	$2ae_1$	$ae_2 + e_3$	$ae_3 - e_2$	be_4	ce_5	$a \geq 0, 0 < b \leq c $
$g_{6,33}^{****}$	$2ae_1$	$ae_2 + e_3$	$ae_3 - e_2$	be_4	$e_1 + 2ae_5$	$b \neq 0, b \leq 2a$
$tg'_{6,33}$	$2ae_1$	$ae_2 + e_3$	$ae_3 - e_2$	$e_1 + 2ae_4$	be_5	$a \geq 0, b \neq 0$
$g_{6,34}^*$	$2ae_1$	$ae_2 + e_3$	$ae_3 - e_2$	$(2a+b)e_4 + e_5$	$\delta e_1 + (2a + b)e_5$	$a \geq 0, \delta = 0$ or $\delta = 1, b = 0$
$g_{6,35}$	$(a+b)e_1$	ae_2	be_3	$ce_4 + e_5$	$ce_5 - e_4$	$a+2b \neq 0, a \leq b , c \geq 0$
$g_{6,36}$	$2ae_1$	$ae_2 + e_3$	ae_3	$be_4 + e_5$	$be_5 - e_4$	$b \geq 0$
$g_{6,37}$	$2ae_1$	$ae_2 + e_3$	$ae_3 - e_2$	$be_4 + ce_5$	$be_5 - ce_4$	$a \geq 0, c > 0$
$g_{6,38}^*$	$2ae_1$	$ae_2 + e_3$	$ae_3 - e_2 + e_4$	$ae_4 + e_5$	$ae_5 - e_4$	$a \geq 0$

Table 2: Lie algebras having nilradical isomorphic to $H \oplus \mathbb{R}^2: [e_2, e_3] = e_1$.

Algebra	$[e_1, e_6]$	$[e_2, e_6]$	$[e_3, e_6]$	$[e_4, e_6]$	$[e_5, e_6]$	Remarks
$tg_{6,39}$	$(b+1)e_1$	$(b+2)e_2$	ae_3	be_4	e_5	$a \neq 0$
$g_{6,40}$	$(a+1)e_1$	$(a+2)e_2$	$e_2 + (a+2)e_3$	ae_4	e_5	
$g_{6,41}$	$(a+1)e_1$	$(a+2)e_2$	ae_3	$e_3 + ae_4$	e_5	
$g_{6,42}$	$(a+1)e_1$	$(a+2)e_2$	e_3	ae_4	$e_3 + e_5$	
$g_{6,43}$	0	e_2	$e_2 + e_3$	$-e_4$	$e_3 + e_5$	
$tg_{6,44}$	$2e_1$	$3e_2$	ae_3	e_4	$e_4 + e_5$	$a \neq 0$
$g_{6,45}$	$2e_1$	$3e_2$	$e_2 + 3e_3$	e_4	$e_4 + e_5$	
$g_{6,46}$	$2e_1$	$3e_2$	e_3	$e_3 + e_4$	$e_4 + e_5$	
$tg_{6,47}$	e_1	e_2	ae_3	$\epsilon e_2 + e_4$	0	$a \neq 0, \epsilon = 0, \pm 1$
$g_{6,48}$	e_1	e_2	$e_2 + e_3$	e_4	0	
$g_{6,49}$	e_1	e_2	0	$\epsilon e_2 + e_4$	e_3	$\epsilon = 0, \pm 1$
$g_{6,50}$	e_1	e_2	$\epsilon e_2 + e_3$	$e_3 + e_4$	0	$\epsilon = 0, \pm 1$
$tg_{6,51}$	0	0	e_3	ϵe_2	0	$\epsilon = \pm 1$
$tg_{6,52}$	0	0	e_3	ϵe_2	e_4	$\epsilon = 0, \pm 1$

Table 3: Lie algebras having nilradical isomorphic to $\mathbb{R} \oplus A_{4,1}: [e_1, e_5] = e_2, [e_4, e_6] = e_1$.

- Algebras that have two asterisks denote algebras in which there is a serious computational or merely typographical mistake
- Algebras that have three asterisks denote algebras which do not appear at all in Mubarakzhanov's list
- Algebras that have four asterisks denote algebras which are redundant

Algebra	$[e_1, e_6]$	$[e_2, e_6]$	$[e_3, e_6]$	$[e_4, e_6]$	$[e_5, e_6]$	Remarks
$g_{6,53}$	0	0	e_3	e_4	$-e_5$	
$g_{6,54}$	e_1	ae_2	$(1-b)e_3$	$(a-b)e_4$	be_5	$ a \leq 1$
$g_{6,55}$	e_1	$(1+a)e_2$	$(1-a)e_3$	$e_1 + e_4$	ae_5	
$g_{6,56}^*$	e_1	0	e_2	$-e_4$	e_5	
$g_{6,57}$	e_1	$2ae_2$	$(1-a)e_3$	ae_4	$e_4 + ae_5$	
$g_{6,58}$	$3e_1$	$2e_2$	$e_2 + 2e_3$	e_4	$e_4 + e_5$	
$g_{6,59}^*$	e_1	0	e_3	e_2	δe_4	$\delta = 0, 1$
$g_{6,60}^*$	e_1	$2e_2$	0	$e_1 + e_4$	$e_4 + e_5$	
$g_{6,61}^*$	$2e_1$	0	e_3	$-e_4$	$e_3 + e_5$	
$g_{6,62}^*$	$2e_1$	e_2	$e_2 + e_3$	0	$e_3 + e_5$	
$g_{6,63}$	e_1	ae_2	e_3	$e_2 + ae_4$	0	
$g_{6,64}^*$	e_1	e_2	$\epsilon e_2 + e_3$	$e_1 + e_4$	0	$\epsilon = \pm 1$
$g_{6,65}^{**}$	$ae_1 + e_2$	ae_2	$(a-b)e_3 + e_4$	$(a-b)e_4 + be_5$	$a = 1$ or $b = 1$	
$g_{6,66}$	$2e_1 + e_2$	$2e_2$	$e_3 + e_4$	e_4	$e_3 + e_5$	
$g_{6,67}^{****}$	$2e_1 + e_2$	$2e_2$	$e_3 + e_4$	e_4	$ae_4 + e_5$	equivalent to $g_{6,65}(a = 2, b = 1)$
$g_{6,68}^*$	$e_1 + e_2$	e_2	$e_3 + e_4$	$e_1 + e_4$	0	
$g_{6,69}^{****}$	$e_1 + e_2$	e_2	$e_3 + e_4$	$e_2 + e_4$	0	equivalent to $g_{6,65}(b = 0)$
$g_{6,70}^{***}$	$ae_1 + e_2$	$-e_1 + ae_2$	$\delta e_2 + (a-b)e_3 + e_4$	$-e_1 + (a-b)e_4$	be_5	$\delta = 0$ ($g_{6,70}$ in [1]) or $\delta = 1$ and $b = 0$ (new algebra)

Table 4: Lie algebras having nilradical isomorphic to $A_{5,1}$: $[e_3, e_6] = e_1$, $[e_4, e_6] = e_2$. In $g_{6,54} - g_{6,65}^*$, a and b are used in place of λ and γ [1].

Algebra	$[e_1, e_6]$	$[e_2, e_6]$	$[e_3, e_6]$	$[e_4, e_6]$	$[e_5, e_6]$	Remarks
$g_{6,71}$	$(a+3)e_1$	$(a+2)e_2$	$(a+1)e_3$	ae_4	e_5	
$g_{6,72}$	$4e_1$	$3e_2$	$2e_3$	e_4	$e_4 + e_5$	
$g_{6,73}^{***}$	e_1	e_2	$\epsilon e_1 + e_3$	$ae_1 + \epsilon e_2 + e_4$	0	$\epsilon = \pm 1$, (if $a = 0$, then $g_{6,73}$ [1])
$g_{6,74}$	e_1	e_2	e_3	e_4	0	
$g_{6,75}^6$	e_1	e_2	e_3	$e_1 + e_4$	0	

Table 5: Lie algebras having nilradical isomorphic to $A_{5,2}$: $[e_2, e_6] = e_1$, $[e_3, e_6] = e_2$, $[e_4, e_6] = e_3$.

Algebra	$[e_1, e_6]$	$[e_2, e_6]$	$[e_3, e_6]$	$[e_4, e_6]$	$[e_5, e_6]$	Remarks
$g_{6,76}$	$(2a+1)e_1$	$(a+1)e_2$	$(a+2)e_3$	e_4	ae_5	$ a \leq 1$
$g_{6,77}$	e_1	e_2	$2e_3$	$\epsilon e_1 + e_4$	0	$\epsilon = \pm 1$
$g_{6,78}$	$-e_1$	0	e_3	$e_3 + e_4$	$-e_5$	
$g_{6,79}$	$3e_1$	$2e_2$	$e_1 + 3e_3$	$e_4 + e_5$	e_5	
$g'_{6,80}^{***}$	$3ae_1 + e_3$	$2ae_2$	$3ae_3 - e_1$	$ae_4 - e_5$	$e_4 + ae_5$	
$g'_{6,81}^{***}$	e_3	0	$-e_1$	$\epsilon e_1 - e_5$	e_4	$\epsilon = \pm 1$
$g_{6,80}^{****}$	$2e_1$	e_2	e_3	0	e_5	equivalent to $g_{6,76}(a = 0)$
$g_{6,81}^{****}$	$2e_1$	e_2	e_3	0	$\epsilon e_3 + e_5$	$\epsilon = \pm 1$ equivalent to $g_{6,77}$

Table 6: Lie algebras having nilradical isomorphic to $A_{5,3}$: $[e_2, e_6] = e_3$, $[e_2, e_6] = e_1$, $[e_4, e_6] = e_2$.

Algebra	$[e_1, e_6]$	$[e_2, e_6]$	$[e_3, e_6]$	$[e_4, e_6]$	$[e_5, e_6]$	Remarks
$g_{6,82} \lambda = a, \lambda_1 = b$	$2\delta e_1$	$(\delta + a)e_2$	$(\delta + b)e_3$	$(\delta - a)e_4$	$(\delta - b)e_5$	$\delta = 1, 0 \leq a \leq b$ or $\delta = 0, 0 \leq a \leq 1, b = 1$
$g_{6,83} \lambda = a$	$2\delta e_1$	$(\delta + a)e_2 + e_3$	$(\delta + a)e_3$	$(\delta - a)e_4$	$(\delta - a)e_5 - e_4$	$\delta = 1, 0 \leq a$ or $\delta = 0, a = 1$
$g_{6,84}$	0	e_2	0	$-e_4$	e_3	
$g_{6,85} \lambda = a$	$2e_1$	$(a+1)e_2$	e_3	$(1-a)e_4$	$e_3 + e_5$	$a \geq 0$
$g_{6,86}^{****}$	$2e_1$	$e_2 + e_3$	e_3	e_4	e_5	equivalent to $g_{6,83}$ for $\alpha = 2, \lambda = 0$
$g_{6,87}^{**}$	$2e_1$	$e_2 + e_3$	$e_3 + e_4$	e_4	$e_3 + e_5$	
$g_{6,88}^* \mu_0 = a, \nu_0 = 1$	ae_1	$(\frac{\alpha}{2} + a)e_2 + e_3$	$(\frac{\alpha}{2} + a)e_3 - e_2$	$(\frac{\alpha}{2} - a)e_4 + e_5$	$(\frac{\alpha}{2} - a)e_5 - e_4$	$a \geq 0$
$g_{6,89}^* s = a, \nu_0 = b$	$2\delta e_1$	$(\delta + a)e_2$	$\delta e_3 + be_5$	$(\delta - a)e_4$	$\delta e_5 - be_3$	$\delta = 1, a \geq 0$ and $b > 0$ or $\delta = 0$ and $a = 1, b > 0$ or $a = 0, b = 1$
$g_{6,90}^{**} \nu_0 = a$	$2\delta e_1$	δe_2	$\delta e_3 + ae_5$	$e_2 + \delta e_4$	$\delta e_5 - ae_3$	$\delta = 1, a \neq 0$ or $\delta = 0, a = \pm 1$
$g_{6,91}^{*****}$	0	0	e_5	e_2	$-e_3$	included in $g_{6,90}$ as $\delta = 0, a = 1$
$g'_{6,92}^{****} s = a, \nu_0 = b$	$2\delta e_1$	$\delta e_2 + ae_4$	$\delta e_3 + be_5$	$\delta e_4 - ae_2$	$\delta e_5 - be_3$	$\delta = 1, 0 < a \leq b $ or $\delta = 0, a = 1, 0 < b \leq 1$ are new algebras
$g_{6,92}^{****} \nu_0 = a, \mu_0 = b$	$2\delta e_1$	$\delta e_2 + ae_3$	$\delta e_3 - be_2$	$\delta e_4 + be_5$	$\delta e_5 - ae_4$	equivalent to cases of $g_{6,82}, g_{6,83}$ or $g_{6,88}$
$g_{6,93}^{**} \nu_0 = a$	$2\delta e_1$	$\delta e_2 + e_4 + ae_5$	$\delta e_3 + ae_4$	$\delta e_4 - ae_3$	$\delta e_5 - ae_2 - e_3$	$\delta = 1, a \geq 0$ or $\delta = 0, a = 1$

Table 7: Lie algebras having nilradical isomorphic to $A_{5,4}$: $[e_2, e_6] = e_1$, $[e_3, e_6] = e_1$.

Algebra	$[e_1, e_6]$	$[e_2, e_6]$	$[e_3, e_6]$	$[e_4, e_6]$	$[e_5, e_6]$	Remarks
$g_{6,94}$	$(a+2)e_1$	$(a+1)e_2$	ae_3	$2e_4$	e_5	
$g_{6,94}$	$(a+2)e_1$	$(a+1)e_2$	ae_3	$2e_4$	e_5	
$g_{6,94}$	$(a+2)e_1$	$(a+1)e_2$	ae_3	$2e_4$	e_5	
$g_{6,97}$	$4e_1$	$3e_2$	$2e_3 + e_4$	$2e_4$	e_5	
$g_{6,98}$	$*e_1$	$\delta e_1 + e_2$	e_3	0	δe_4	$\delta = 0, 1$

Table 8: Lie algebras having nilradical isomorphic to A5,5: $[e_3, e_4] = e_1, [e_2, e_5] = e_1, [e_3, e_5] = e_2$.

Algebra	$[e_3, e_4]$	$[e_2, e_5]$	$[e_3, e_5]$	$[e_4, e_5]$	$[e_1, e_6]$	$[e_2, e_6]$	$[e_3, e_6]$	$[e_4, e_6]$	$[e_5, e_6]$	Remarks
$g_{6,99}$	e_1	e_1	e_2	e_3	$5e_1$	$4e_2$	$3e_3$	$2e_4$	e_5	

Table 9: Lie algebras having nilradical isomorphic to A5,6.

Algebra	DNA vector
$g_{6,1}$	$-s_6, 0, 0, 0, 0, s_1, 0, -as_6, 0, 0, 0, s_2, 0, 0, -bs_6, 0, 0, s_3, 0, 0, 0, -cs_6, 0, s_4, 0, 0, 0, 0, -ds_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,2}$	$as_6, -s_6, 0, 0, 0, s_1, 0, -as_6, 0, 0, 0, s_2, 0, 0, -s_6, 0, 0, s_3, 0, 0, 0, -cs_6, 0, s_4, 0, 0, 0, 0, -ds_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,3}$	$-as_6, -s_6, 0, 0, 0, s_1, 0, -as_6, -s_6, 0, 0, s_2, 0, 0, -as_6, 0, 0, s_3, 0, 0, 0, -s_6, 0, s_4, 0, 0, 0, 0, -ds_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,4}$	$-as_6, -s_6, 0, 0, 0, s_1, 0, -as_6, -s_6, 0, 0, s_2, 0, 0, -as_6, -s_6, 0, 0, s_3, 0, 0, 0, -as_6, 0, s_4, 0, 0, 0, 0, -bs_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,5}$	$-s_6, -s_6, 0, 0, 0, s_1, 0, -s_6, -s_6, 0, 0, s_2, 0, 0, -s_6, -s_6, 0, 0, s_3, 0, 0, 0, -s_6, -s_6, s_4, 0, 0, 0, 0, -s_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,6}$	$-as_6, -s_6, 0, 0, 0, s_1, 0, -as_6, 0, 0, 0, s_2, 0, 0, -bs_6, -s_6, 0, 0, s_3, 0, 0, 0, -bs_6, 0, s_4, 0, 0, 0, 0, -s_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,7}$	$-bs_6, -s_6, 0, 0, 0, s_1, 0, -bs_6, 0, 0, 0, -s_4, 0, 0, -as_6, -s_6, 0, 0, s_1, 0, 0, 0, -as_6, -s_6, s_2, 0, 0, 0, 0, -as_6, s_3, 0, 0, 0, 0, 0, 0$
$g_{6,8}$	$-ds_6, -s_6, 0, 0, 0, s_1, 0, -ds_6, 0, 0, 0, s_4, 0, 0, -as_6, 0, 0, s_2, 0, 0, 0, -bs_6, 0, s_3, 0, 0, 0, 0, -cs_6, s_1, 0, 0, 0, 0, 0, 0$
$g_{6,9}$	$-cs_6, -s_6, 0, 0, 0, s_1, 0, -cs_6, 0, 0, 0, s_5, 0, 0, -bs_6, -s_6, 0, 0, s_2, 0, 0, 0, -bs_6, 0, s_3, 0, 0, 0, 0, -as_6, s_1, 0, 0, 0, 0, 0, 0$
$g_{6,10}$	$-as_6, -s_6, 0, 0, 0, s_1, 0, -as_6, -s_6, 0, 0, s_2, 0, 0, -as_6, 0, 0, s_3, 0, 0, 0, -bs_6, s_6, -s_4, 0, 0, 0, 0, -s_6, -bs_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,11}$	$-as_6, 0, 0, 0, 0, s_1, 0, -bs_6, -s_6, 0, 0, s_2, 0, s_6, -bs_6, 0, 0, s_3, 0, 0, 0, -cs_6, -ds_6, s_5, 0, 0, 0, 0, ds_6, -cs_6, -s_4, 0, 0, 0, 0, 0, 0$
$g_{6,12}$	$-as_6, 0, 0, 0, 0, s_1, 0, -bs_6, -s_6, -s_6, 0, s_3, 0, s_6, -bs_6, 0, -s_6, -s_2, 0, 0, 0, -bs_6, -s_6, s_5, 0, 0, 0, 0, s_6, -bs_6, -s_4, 0, 0, 0, 0, 0, 0$
$g_{6,13}$	$-(a+b)s_6, -s_6, -s_2, 0, 0, 2s_1, 0, -as_6, 0, 0, 0, s_2, 0, 0, -bs_6, 0, 0, s_3, 0, 0, 0, -s_6, 0, s_4, 0, 0, 0, 0, -cs_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,14}$	$-(a+b)s_6, -s_2, s_3, -2s_6, 0, -2s_1, 0, -as_6, 0, 0, 0, s_3, 0, 0, -bs_6, 0, 0, s_2, 0, 0, 0, -(a+b)s_6, 0, s_5, 0, 0, 0, 0, -s_6, s_4, 0, 0, 0, 0, 0, 0$
$g_{6,15}$	$-(a+1)s_6, -s_2, s_3, 0, 0, -s_1, 0, -s_6, 0, 0, 0, s_3, 0, 0, -as_6, 0, 0, as_2, 0, -s_6, 0, -s_6, 0, s_3 + s_6, 0, 0, -s_6, 0, -as_6, s_2 + as_4, 0, 0, 0, 0, 0, 0$
$g_{6,16}$	$0, 0, 0, 0, -s_6, -1$ $2s_5, 0, -s_6, -s_6, -1$ $2s_3, -s_2, s_1, 0, 0, -s_6, -s_6, 0, s_4, 0, 0, 0, -s_6, 0, s_2, 0, 0, 0, 0, 0, -1$ $2s_3, 0, 0, 0, 0, 0$
$g_{6,17,1}$	$-s_6, 0, 0, 0, 0, s_1, 0, -as_6, 0, -s_6, -s_3, s_1, 0, 0, -as_6, 0, 0, -s_3, 0, 0, 0, 0, -s_6, s_4, 0, 0, 0, 0, 0, s_2, 0, 0, 0, 0, 0, 0$
$g_{6,17,2}$	$0, s_3, 0, 0, 0, 0, -s_4, 0, 0, 0, 0, 0, -s_6, 0, 0, -s_6, s_3, -s_6, -s_1, 0, 0, 0, -s_6, 0, s_2, 0, 0, 0, 0, -s_6, -s_6, 0, 0, 0, 0, 0, 0$
$g_{6,18}$	$-bs_6, 0, 0, 0, 0, s_1, 0, -s_6, 0, -s_6, 0, s_4, 0, 0, -(a+1)s_6, 0, s_3, -s_1, 0, 0, 0, -s_6, 0, s_3, 0, 0, 0, 0, -as_6, s_2, 0, 0, 0, 0, 0, 0$
$g_{6,19}$	$-(a+1)s_6, 0, -s_3, s_2, -s_6, 2s_1, 0, -s_6, 0, -s_6, 0, s_4, 0, 0, -as_6, 0, 0, s_2, 0, 0, 0, -s_6, 0, s_3, 0, 0, 0, 0, -(a+1)s_6, 2s_5, 0, 0, 0, 0, 0, 0$
$g_{6,20}$	$-bs_6, 0, 0, 0, 0, s_1, 0, -s_6, -s_6, -s_6, s_2, -s_3, s_1, 0, 0, -s_6, -s_6, 0, s_4, 0, 0, 0, -s_6, 0, s_3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$
$g_{6,21}$	$-2as_6, as_2, s_2, -s_3, 0, 0, 2s_1, 0, -as_6, -s_6, 0, 0, s_3, 0, 0, -as_6, 0, 0, as_2, 0, 0, 0, -s_6, 0, s_5, 0, 0, 0, 0, -bs_6, s_4, 0, 0, 0, 0, 0, 0$
$g_{6,22}$	$-s_6, 0, 0, 0, 0, s_1, 0, -2as_6, s_2, -s_3, 2cs_6, 2s_1, 0, 0, -as_6, -as_6, 0, s_3, 0, 0, 0, -as_6, 0, s_2, 0, 0, 0, 0, -2as_6, -s_5, 0, 0, 0, 0, 0, 0$
$g_{6,23,1}$	$-2s_6, 0, s_2, -s_3, 2s_6, 2s_1, 0, -s_6, -s_6, 0, 0, s_3 + s_4, 0, 0, -s_6, -s_6, 0, s_2 + s_3, 0, 0, 0, -s_6, 0, s_2, 0, 0, 0, 0, -2s_6, -s_5, 0, 0, 0, 0, 0, 0$
$g_{6,23,2}$	$-(2\delta + a)s_6, 0, 0, 0, 0, s_1, 0, -2\delta s_6, 0, s_2, s_2 - s_3, s_1, 0, 0, -\delta s_6, -s_6, 0, s_4, 0, 0, 0, -\delta s_6, -s_6, s_3, 0, 0, 0, 0, -\delta s_6, s_2, 0, 0, 0, 0, 0, 0$
$g_{6,24}$	$-s_6, 0, 0, 0, 0, s_1, 0, -2s_6, s_2, -s_3, -2s_1, 0, 0, 0, -s_6, 0, -s_4, 0, 0, 0, 0, -s_6, -s_3, 0, 0, 0, 0, -s_2, 0, 0, 0, 0, 0, 0, 0$
$g_{6,25}$	$-bs_6, s_4, 0, 0, 0, -s_5, 0, 0, 0, 0, 0, -s_6, 0, 0, -(a+1)s_6, 0, s_3, -s_1, 0, 0, 0, -as_6, 0, s_3, 0, 0, 0, 0, -s_6, s_2, 0, 0, 0, 0, 0, 0$
$g_{6,26}$	$-as_6, -s_6, 0, 0, s_2, s_1, 0, -as_6, -s_6, 0, 0, s_5, 0, 0, -as_6, 0, 0, s_4, 0, 0, 0, -s_6, 0, s_2, 0, 0, 0, 0, (1-a)s_6, s_3, 0, 0, 0, 0, 0, 0$
$g_{6,27}$	$-(a+b)s_6, \delta s_6, 0, -s_2, -s_1, 0, -bs_6, -s_6, 0, s_5, 0, 0, -bs_6, -s_6, s_4, 0, 0, 0, -bs_6, s_3, 0, 0, 0, 0, 0, 0$
$g_{6,28}$	$-2s_6, s_2, -s_3, 0, 0, 2s_1, 0, -s_6, -s_6, 0, 0, s_3, 0, 0, -s_6, 0, 0, s_2, 0, 0, 0, -bs_6, -s_6, s_5, 0, 0, 0, 0, -bs_6, s_4, 0, 0, 0, 0, 0, 0$
$g_{6,29}$	$-2s_6, s_2, -1$ $2s_5, -s_6, -s_6, 0, s_1, 0, -s_6, -s_6, 0, 0, 1$ $2s_3, 0, 0, -s_6, 0, 0, s_2, 0, 0, 0, -2s_6, -s_6, s_5, 0, 0, 0, 0, -2s_6, s_4, 0, 0, 0, 0, 0, 0$
$g_{6,30}$	$-s_6, 0, 0, s_1, -s_5, 0, 0, s_2, 0, -s_1, 0, 0, 0, s_2, -s_3, 0, 0, 0, 0, -s_6, 0, 0, 0, 0, 0, 0$
$g_{6,31}$	$-2s_6, 0, 0, 3$ $2s_2, -3$ $2s_3, -3s_6, 9$ $2s_1, 0, -s_6, -s_6, 0, 0, 3$ $2s_5, 0, 0, -s_6, -s_6, 0, 3$ $2s_4, 0, 0, 0, -s_6, -s_6, 3$ $2s_3, 0, 0, 0, 0, -s_6, 3$ $2s_2, 0, 0, 0, 0, 0$
$g_{6,32}$	$-cs_6, 0, 0, 0, 0, s_1, 0, -bs_6, 0, 0, 0, s_4, 0, 0, -2as_6, -s_2, s_3, -2s_1, 0, 0, 0, -as_6, -s_6, s_3, 0, 0, 0, s_6, -as_6, s_2, 0, 0, 0, 0, 0, 0$
$g_{6,33}$	$-bs_6, 0, 0, 0, 0, s_1, 0, -2as_6, -s_3, s_2, -s_6, 2s_1, 0, 0, -as_6, s_6, 0, s_2, 0, 0, -s_6, -as_6, 0, s_3, 0, 0, 0, 0, -2as_6, 2s_4, 0, 0, 0, 0, 0, 0$

$g_{6,34}$	$-2as_6-s_3s_2-\delta s_6, 0, 2s_1, 0, -as_6, s_6, 0, 0, s_2, 0, -s_6-as_6, 0, 0, s_3, 0, 0, 0, -(2a+b)s_6-s_6, 2s_2, 0, 0, 0, 0, -(2a+b)s_6, 2s_4, 0, 0, 0, 0, 0, 0$
$g_{6,35}$	$-cs_6, s_6, 0, 0, 0, s_4-s_6-cs_6, 0, 0, 0, s_5, 0, 0, -(a+b)s_6-s_3, s_2, 2s_1, 0, 0, 0, -as_6, 0, s_2, 0, 0, 0, 0, -bs_6, s_3, 0, 0, 0, 0, 0, 0$
$g_{6,36}$	$-cs_6, s_6, 0, 0, 0, s_4-s_6-cs_6, 0, 0, 0, s_5, 0, 0, -2a_{s_6}-s_2, s_3, -2s_1, 0, 0, 0, -as_6-s_6, s_3, 0, 0, 0, 0, -as_6, s_2, 0, 0, 0, 0, 0, 0$
$g_{6,37}$	$-a2s_6-s_2s_3, 0, 0, -2s_1, 0, -as_6-s_6, 0, 0, s_3, 0, s_6-as_6, 0, 0, s_2, 0, 0, 0, -bs_6-cs_6, s_3, 0, 0, 0, cs_6-bs_6, s_4, 0, 0, 0, 0, 0, 0$
$g_{6,38}$	$-2as_6, 0, 0, -s_3-s_2-2s_1, 0, -as_6-s_6-s_6, 0, -s_3+2s_2, s_3, -as_6, 0, -s_6, 2s_4, 0, 0, 0, -as_6-s_6-s_2, 0, 0, 0, s_6-as_6, s_3, 0, 0, 0, 0, 0, 0$
$g_{6,39}$	$(b-a+2)s_6, 0, 0, 0, s_3, 0, 0, s_5, 0, s_2, 0, 0, s_6, s_5-s_1, 0, 0, 0, 2s_6, s_4, 0, 0, 0, 0, (b+2)s_6$
$g_{6,40}$	$0, s_5, 0, s_3, s_2, 0, s_6, s_5, 0, -s_1, 0, 0, 2s_6, 0, s_4, 0, 0, 0, (a+2)s_6, s_6, 0, 0, 0, 0, (a+2)s_6$
$g_{6,41}$	$-(2+a)s_6, s_5, 0, 0, s_1, s_2, 0, -(a+1)s_6, 0, s_5, -s_4, -s_1, 0, 0, -as_6, s_6, 0, -s_3, 0, 0, 0, -as_6, 0, s_4, 0, 0, 0, 0, -s_6, 0, 0, 0, 0, 0, 0$
$g_{6,42}$	$-s_6, 0, 0, s_6, 0, -s_3, 0, -(a+2)s_6, 1$ $4s_5, -1$ $4s_1, 0, -s_2, 0, 0, -(a+1)s_6, s_4, -2s_5, 3s_1, 0, 0, 0, -s_6, 0, s_5, 0, 0, 0, 0, -as_6, s_4, 0, 0, 0, 0, 0, 0$
$g_{6,43}$	$-s_6, -s_5, -s_6, 0, s_1, s_2, 0, 0, 0, -s_5, s_4, 0, 0, 0, -s_6, 0, -s_6, s_3, 0, 0, 0, s_6, 0, -s_4, 0, 0, 0, 0, -s_6, s_5, 0, 0, 0, 0, 0, -s_6, -s_5, -s_6, 0, s_1, s_2, 0, 0, 0, -s_5, s_4, 0, 0, 0, 0, -s_6, 0, -s_6, s_3, 0, 0, 0, s_6, 0, -s_4, 0, 0, 0, 0, -s_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,44}$	$3s_6, -s_5, 0, 0, s_1, 3s_2, 0, 2s_6, 0, -s_5, s_4-s_5, 2s_1, 0, 0, as_6, 0, 0, s_3, 0, 0, 0, s_6, s_6, s_4, 0, 0, 0, 0, s_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,45}$	$3s_6, -s_5, s_6, 0, s_1, 3s_2, 0, 2s_6, 0, -s_5, s_4-s_5, 2s_1, 0, 0, 3s_6, 0, 0, 3s_3, 0, 0, 0, s_6, s_6, s_4, 0, 0, 0, 0, s_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,46}$	$3s_6, -s_5, 0, 0, s_1, 3s_2, 0, 2s_6, 0, -s_5, s_4-s_5, 2s_1, 0, 0, s_6, s_6, 0, 0, s_3, 0, 0, 0, s_6, s_6, s_4, 0, 0, 0, 0, s_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,47}$	$-s_6, s_5, 0, -\epsilon s_6, -s_1, s_2, 0, -s_6, 0, s_5, s_4, -s_1, 0, 0, -as_6, 0, 0, s_3, 0, 0, 0, -s_6, 0, s_4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$
$g_{6,48}$	$-s_6, s_5, -s_6, 0, -s_1, s_2, 0, -s_6, 0, s_5, s_4, -s_1, 0, 0, -s_6, 0, 0, s_3, 0, 0, 0, -s_6, 0, s_4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$
$g_{6,49}$	$0, 0, 0, -s_6, 0, s_3, 0, -s_6, -s_5, s_1, -s_6, 2s_2, 0, 0, -s_6, s_4, 0, s_1, 0, 0, 0, 0, 0, s_5, 0, 0, 0, 0, -s_6, 2\epsilon s_4, 0, 0, 0, 0, 0, 0$
$g_{6,50}$	$-s_6, s_5, -\epsilon s_6, 0, -s_1, s_2, 0, -s_6, 0, s_5, s_4, -s_1, 0, 0, -s_6, -s_6, 0, s_3, 0, 0, 0, -s_6, 0, s_4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$
$g_{6,51}$	$-s_6, 0, 0, 0, 0, s_3, 0, 0, s_5, 0, s_4, s_2, 0, 0, 0, s_5, 0, -s_1, 0, 0, 0, 0, 0, s_4, 0, 0, 0, 0, 0, \epsilon s_6, 0, 0, 0, 0, 0, 0$
$g_{6,52}$	$-\epsilon s_6, 0, 0, 0, 0, s_3, 0, 0, 2s_5, -s_6, s_4, 2s_2, 0, 0, 0, s_5, 0, -s_1, 0, 0, 0, 0, s_5, s_4, 0, 0, 0, 0, 0, s_6, 0, 0, 0, 0, 0, 0$
$g_{6,53}$	$s_6, 0, s_5, 0, -s_2, 0, s_6, 0, s_5, -s_1, 0, 0, 0, 0, s_4, 0, 0, 0, 0, s_3, 0, 0, 0, 0, s_6$
$g_{6,54}$	$-as_6, 0, s_5, 0, -s_2, 0, -s_6, 0, s_5, -s_1, 0, 0, (b-a)s_6, 0, s_4, 0, 0, 0, (b-1)s_6, s_3, 0, 0, 0, 0, 0$
$g_{6,55}$	$-(a+1)s_6, 0, 0, 0, s_5, -1$ $2s_2, 0, -s_6, 1$ $2s_3, s_5, s_6, -1$ $2s_1, 0, 0, -as_6, 0, 0, s_5, 0, 0, 0, (a-1)s_6, 0, s_3, 0, 0, 0, 0, -s_6, 1$ $2s_4, 0, 0, 0, 0, 0, 0$
$g_{6,56}$	$-s_6, 0, s_5, 0, -s_1, 0, 0, -s_6, s_5, s_2, 0, 0, 0, 0, s_3, 0, 0, 0, s_6, -s_4, 0, 0, 0, 0, 0$
$g_{6,57}$	$-s_6, s_5, 0, 0, 0, -s_1, 0, (a-1)s_6, 0, 0, 0, s_3, 0, 0, -2as_6, s_5, 0, s_2, 0, 0, 0, -as_6, s_5, -s_4, 0, 0, 0, 0, -s_6, 0, 0, 0, 0, 0, 0$
$g_{6,58}$	$-3s_6, 0, -s_5, 0, 1$ $2s_3, 3$ $2s_1, 0, -2s_6, -s_6, -s_5, s_4-s_5, 2s_2, 0, 0, -2s_6, 0, 0, s_3, 0, 0, 0, -s_6, -s_6, s_4, 0, 0, 0, 0, -s_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,59}$	$-s_6, s_5, 0, 0, 0, -s_1, 0, -s_6, 0, 0, 0, s_3, 0, 0, 0, -s_5-s_6, 0, -s_2, 0, 0, 0, 0, -s_5-s_6, -s_4, 0, 0, 0, 0, 0, -\delta s_5, 0, 0, 0, 0, 0, 0$
$g_{6,60}$	$-s_6, 0, -s_5, -s_6, s_3, s_1, 0, -2s_6, 0, -s_5, s_4+s_5, 2s_2, 0, 0, 0, 0, 0, 0, 0, 0, -s_6, -s_6, s_4, 0, 0, 0, 0, -s_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,61}$	$0, 0, 0, 0, s_4, s_2, 0, s_6, 0, 0, 0, s_4, 0, 0, -2s_6, -s_5, s_3, 2s_1, 0, 0, 0, -s_6, -s_6, s_3, 0, 0, 0, 0, -s_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,62}$	$2s_6, 0, -s_5, s_3-1$ $2s_5, 2s_1, 0, s_6, s_6, 1$ $4s_4, 3s_2, 0, 0, s_6, -1$ $2s_6, s_3, 0, 0, 0, s_6, s_5, 0, 0, 0, 0, 0$
$g_{6,63}$	$-as_6, -s_5-s_6, 0, 0, 0, s_2, 0, -as_6, 0, 0, 0, s_4, 0, 0, -s_6, s_5, s_3, -s_1, 0, 0, 0, -s_6, 0, s_3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$
$g_{6,64}$	$-s_6, 0, s_5, -s_6, -s_3, s_1, 0, -s_6, -\epsilon s_6, s_5, s_4, -s_2, 0, 0, -s_6, 0, 0, -s_3, 0, 0, 0, -s_6, 0, s_4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$
$g_{6,65}$	$-as_6, 0, s_6, 0, -s_4, -s_2, 0, (b-a)s_6, 0, s_6, 0, -s_4, 0, 0, -as_6, 0, s_3, s_1, 0, 0, 0, (b-a)s_6, 0, s_3, 0, 0, 0, 0, -bs_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,66}$	$-2s_6, -s_6, -s_5, 0, -s_3+s_4+s_5, -s_1+2s_2, 0, -2s_6, 0, -s_5, s_3-s_5, 2s_1, 0, 0, -s_6, -s_6, 0, s_4, 0, 0, 0, -s_6, -s_6, s_3, 0, 0, 0, 0, -s_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,68}$	$-s_6, -s_6, -s_5, 0, -s_3+s_4, s_2, 0, -s_6, -s_6, -s_5, s_3, s_1, 0, 0, -s_6, -s_6, 0, s_4, 0, 0, 0, -s_6, 0, s_3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$
$g_{6,70}$	$-as_6, s_6, s_5, 0, -s_3, as_1-s_2, -s_6, -as_6, \delta s_6, s_5, -s_4, s_1+as_2+\delta s_3, 0, 0, (b-a)s_6, s_6, 0, (b-a)s_3+s_4, 0, 0, -s_6, (b-a)s_6, 0, -s_3+(b-a)s_4, 0, 0, 0, 0, -bs_6, -bs_5, 0, 0, 0, 0, 0, 0$
$g_{6,71}$	$-(a+3)s_6, s_5, 0, 0, -s_1, 0, -(a+2)s_6, s_5, 0, s_2, 0, 0, -(a+1)s_6, s_5, -s_3, 0, 0, 0, -as_6, s_4, 0, 0, 0, 0, 0$
$g_{6,72}$	$-4s_6, -s_5, 0, 0, s_2, 4s_1, 0, -3s_6, -s_5, 0, s_3, 3s_2, 0, 0, -2s_6, -s_5, s_4-s_5, 2s_3, 0, 0, 0, -s_6, -s_6, s_4, 0, 0, 0, 0, -s_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,73}$	$-s_6, -s_5, -\epsilon s_6, -as_6, s_2-\epsilon s_4, s_1, 0, -s_6, -s_5, -\epsilon s_6, s_3, s_2, 0, 0, -s_6, -s_5, s_4, s_3, 0, 0, 0, -s_6, 0, s_4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$
$g_{6,74}$	$-s_6, s_5, 0, 0, -s_2, s_1, 0, -s_6, s_5, 0, s_3, -s_2, 0, 0, -s_6, s_5, -s_4, s_3, 0, 0, 0, -s_6, 0, -s_4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$
$g_{6,75}$	$-s_6, -s_5, 0, -s_6, s_2, s_1, 0, -s_6, -s_5, 0, s_3, s_2, 0, 0, -s_6, -s_5, s_4, s_3, 0, 0, 0, -s_6, 0, s_4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$
$g_{6,76}$	$-(a+2)s_6, 0, 1$ $2s_4, 1$ $2s_2, -s_3, 0, -(2a+1)s_6, s_5, 0, -s_1, 0, 0, -(a+1)s_6, s_5, s_2, 0, 0, 0, -s_6, -s_4, 0, 0, 0, 0, 0, 0$
$g_{6,77}$	$-s_6, 0, s_5, -\epsilon s_6, -s_2, s_1, 0, -2s_6, -s_4, -s_2, 0, -2s_3, 0, 0, -s_6, s_5, s_4, -s_2, 0, 0, 0, -s_6, 0, s_4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$
$g_{6,78}$	$s_6, 0, -s_5, 0, s_2, -s_1, 0, -s_6, -s_4, s_2-s_6, 0, s_3, 0, 0, 0, -s_5, s_4, 0, 0, 0, 0, -s_6, 0, s_4, 0, 0, 0, 0, s_6, -s_5, 0, 0, 0, 0, 0, 0$
$g_{6,79}$	$-3s_6, -s_6, s_4-s_5, s_2, 0, 3s_1-2s_3, 0, -3s_6, -s_4, 0, s_2, 3s_3, 0, 0, -2s_6, s_4, s_4-s_5, 2s_2, 0, 0, 0, -s_6, -s_6, s_5, 0, 0, 0, 0, -s_6, s_4, 0, 0, 0, 0, 0, 0$
$g_{6,81}$	$0, s_6, -s_4, -\epsilon s_6, s_2, s_1, -s_6, 0, s_5, s_2, 0, s_3, 0, 0, 0, -s_4, -s_5, 0, 0, 0, 0, -s_6, s_4, 0, 0, 0, s_6, 0, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,82}$	$-2\delta s_6, -s_4, s_2, 0, 0, s_1, 0, -(\delta+a)s_6, 0, 0, 0, s_3, 0, 0, 0, -(\delta+b)s_6, 0, 0, s_5, 0, 0, 0, 0, -(\delta+a)s_6, 0, -s_4, 0, 0, 0, 0, (b-\delta)s_6, s_2, 0, 0, 0, 0, 0, 0$
$g_{6,83}$	$-2\delta s_6, s_5, -s_4, 0, 0, s_1, 0, -(\delta+a)s_6, s_6, 0, 0, -s_3, 0, 0, -(\delta+a)s_6, 0, 0, s_2, 0, 0, 0, (a-\delta)s_6, -s_6, -s_4, 0, 0, 0, 0, (a-\delta)s_6, s_5, 0, 0, 0, 0, 0, 0$
$g_{6,84}$	$0, s_5, 0, s_4, s_1, 0, 0, s_5, 0, -s_3, 0, 0, 0, 0, -s_6, 0, 0, 0, 0, -s_6, -s_2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$
$g_{6,85}$	$-2s_6, -s_4, -s_5, -s_2, -s_3, -2s_1, 0, -(a+1)s_6, 0, 0, 0, -s_2, 0, 0, -s_6, 0, s_6, -s_3, 0, 0, 0, (a-1)s_6, 0, s_4, 0, 0, 0, 0, -s_6, s_5, 0, 0, 0, 0, 0, 0$

$g_{6,87}$	$-2s6, s2, s2 -s5, s2 + s3 -s5, s2 + s3 -s4 -s5, 2s1, 0, -s6, -s6, 0, 0, s4, 0, 0, -s6, -s6, 0, s3, 0, 0, 0, -s6, -s6, s5, 0, 0, 0, 0, -s6, s2, 0, 0, 0, 0, 0$
$g_{6,88}$	$-2as6, -s2, -s3, 0, 0, -s1, 0, -(a+a)s6, bs6, 0, 0, s4, 0, -bs6, -(a+a)s6, 0, 0, s5, 0, 0, -(a+a)s6, bs6, 0, 0, 0, 0, -bs6, -(a+a)s6, 0, 0, 0, 0, 0, 0g_{6,88}$ $-2as6, -s2, -s3, 0, 0, -s1, 0, -(a+a)s6, bs6, 0, 0, s4, 0, -bs6, -(a+a)s6, 0, 0, s5, 0, 0, -(a+a)s6, bs6, 0, 0, 0, 0, -bs6, -(a+a)s6, 0, 0, 0, 0, 0$
$g_{6,89}$	$-2\delta s6, 0, -2s2, -s5, -s3, -2s1, 0, -(\delta + a)s6, 0, 0, 0, -2s2, 0, 0, -(\delta + a)s6, 0, 0, s4, 0, 0, 0, -\delta s6, -bs6, -s3, 0, 0, 0, bs6, -\delta s6, s5, 0, 0, 0, 0, 0$
$g_{6,90}$	$-2\delta s6, s5, -s3, -s4, s2, -2s1, 0, -\delta s6, as6, 0, 0, -s3, 0, -as6, -\delta s6, 0, 0, -s5, 0, 0, -\delta s6, -s6, -s2, 0, 0, 0, -\delta s6, -s4, 0, 0, 0, 0, 0$
$g_{6,92}$	$-2\delta s6, -s5, s3, -s4, s2, 2s1, 0, -\delta s6, as6, 0, 0, s3, 0, -as6, -\delta s6, 0, 0, s5, 0, 0, -\delta s6, bs6, s2, 0, 0, 0, -bs6, -\delta s6, s4, 0, 0, 0, 0, 0$
$g_{6,93}$	$-2\delta s6, -s2, -s5, s4, -s3, -2s1, 0, -\delta s6, as6, -s6, 0, s4, 0, -as6, -\delta s6, 0, -s6, -s3, 0, 0, 0, -\delta s6, as6, s2, 0, 0, 0, -as6, -\delta s6, s5, 0, 0, 0, 0, 0$
$g_{6,94}$	$-(a+2)s6, s5, 0, s3, s1, 0, -(a+1)s6, s5, 0, -s2, 0, 0, -as6, 0, s3, 0, 0, 0, -2s6, s4, 0, 0, 0, 0, 0$
$g_{6,95}$	$-2s6, -s5, s4, s3 -s6, -s2, -2s1, 0, -s6, -s5, 0, -s3, -s2, 0, 0, 0, 0, 0, 0, 0, -2s6, 0, -2s4, 0, 0, 0, 0, -s6, s5, 0, 0, 0, 0, 0$
$g_{6,96}$	$-3s6, -s5, -s4, s3 -s5, s2 -1$ $2s4, 3s1, 0, -2s6, -s5, -s6, s3, 2s2, 0, 0, -s6, 0, -s6, s3, 0, 0, 0, -2s6, 0, 2s4, 0, 0, 0, 0, -s6, s5, 0, 0, 0, 0, 0$
$g_{6,97}$	$-4s6, -s5, s3, 1$ $2s3 -s4, s2, 4s1, 0, -3s6, 0, -s5, s3, 3s2, 0, 0, -2s6, -s6, 0, 2s4, 0, 0, 0, -2s6, 0, 2s3, 0, 0, 0, -s6, s5, 0, 0, 0, 0, 0$
$g_{6,98}$	$-s6, -s5 -\delta s6, -s4, s3, -s2, -s1, 0, -s6, -s5, 0, s3, -s2, 0, 0, -s6, 0, 0, s3, 0, 0, 0, -\delta s6, -s5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$
$g_{6,99}$	$-5s6, -s5, -s4, s3, s2, 5s1, 0, -4s6, -s5, 0, s3, 4s2, 0, 0, -3s6, -s5, s4, 3s3, 0, 0, 0, -2s6, 0, 2s4, 0, 0, 0, 0, -s6, s5, 0, 0, 0, 0, 0$

Table 10: Matrix Representations for Mubarakzayanov Algebras.

Algebra	Basis of Vector Fields
$g_{6,1}$	$D_{p'}, D_{q'}, D_{x'}, D_{y'}, D_{z'}, D_{w'} + pD_{p'} + aqD_{q'} + bxD_{x'} + cyD_{y'} + dzD_{z'}$
$g_{6,2}$	$D_{p'}, D_{q'}, D_{x'}, D_{y'}, D_{z'}, D_{w'} + (ap+q)D_{p'} + aqD_{q'} + xD_{x'} + byD_{y'} + czD_{z'}$
$g_{6,3}$	$D_{p'}, D_{q'}, D_{x'}, D_{y'}, D_{z'}, D_{w'} + (ap+q)D_{p'} + (aq+x)D_{q'} + axD_{x'} + yD_{y'} + bzD_{z'} + D_{w'}$
$g_{6,4}$	$D_{p'}, D_{q'}, D_{x'}, D_{y'}, D_{z'}, D_{w'} + (ap+q)D_{p'} + (aq+x)D_{q'} + (ax+y)D_{x'} + ayD_{y'} + zD_{z'} + D_{w'}$
$g_{6,5}$	$D_{p'}, D_{q'}, D_{x'}, D_{y'}, D_{z'}, D_{w'} + (p+q)D_{p'} + (q+x)D_{q'} + (x+y)D_{x'} + (y+z)D_{y'} + zD_{z'}$
$g_{6,6}$	$D_{p'}, D_{q'}, D_{x'}, D_{y'}, D_{z'}, D_{w'} + (bp+q)D_{p'} + bD_{q'} + (ax+y)D_{x'} + ayD_{y'} + zD_{z'}$
$g_{6,7}$	$D_{p'}, D_{q'}, D_{x'}, D_{y'}, D_{z'}, D_{w'} + (bp+q)D_{p'} + bqD_{q'} + (ax+y)D_{x'} + ayD_{y'} + zD_{z'}$
$g_{6,8}$	$D_{p'}, D_{q'}, D_{x'}, D_{y'}, D_{z'}, D_{w'} + bpD_{p'} + cqD_{q'} + (dx+y)D_{x'} + (dy-x)D_{y'} + azD_{z'}$
$g_{6,9}$	$D_{p'}, D_{q'}, D_{x'}, D_{y'}, D_{z'}, D_{w'} + (bp+q)D_{p'} + bqD_{q'} + (ax+y)D_{x'} + (ay-x)D_{y'} + azD_{z'} + D_{w'}$
$g_{6,10}$	$D_{p'}, D_{q'}, D_{x'}, D_{y'}, D_{z'}, D_{w'} + (ap+z)D_{p'} + bqD_{q'} + (bx+y)D_{x'} + (by-x)D_{y'} + azD_{z'}$
$g_{6,11}$	$D_{p'}, D_{q'}, D_{x'}, D_{y'}, D_{z'}, D_{w'} + (cp+q)D_{p'} + (cq-p)D_{q'} + (ax+y)D_{x'} + (by-x)D_{y'} + zD_{z'} + D_{w'}$
$g_{6,12}$	$D_{p'}, D_{q'}, D_{x'}, D_{y'}, D_{z'}, D_{w'} + (bp+q)D_{p'} + (bq-p)D_{q'} + (bx+p+y)D_{x'} + (by+q-x)D_{y'} + azD_{z'} + D_{w'}$
$g_{6,13}$	$2D_{p'}D_{x'} + yD_{z'} - D_{y'} + xD_{z'}, D_{q'}, D_{x'}, D_{w'} + (a+b)pD_{p'} + cqD_{q'} + axD_{x'} + byD_{y'} + (a+b)zD_{z'}$
$g_{6,14}$	$2D_{p'}D_{x'} + yD_{z'} - D_{y'} + xD_{z'}, D_{q'}, D_{y'}, D_{w'} + (a+b)pD_{p'} + qD_{q'} + axD_{x'} + byD_{y'} + ((a+b)z + 2p)D_{z'}$
$g_{6,15}$	$2D_{p'}D_{p'} + qD_{z'} - D_{q'} + pD_{z'}, D_{x'}, D_{y'}, D_{w'} + pD_{p'} + aqD_{q'} + (p+x)D_{x'} + (ay+1)qD_{y'} + (a+1)zD_{z'}$
$g_{6,16}$	$D_{p'}D_{x'} - yD_{p'}, 1$ $2(D_{y'} + xD_{p'}), D_{q'}, 1$ $2D_{z'}D_{w'} + (p+q)D_{p'} + (x+q)D_{q'} + xD_{x'} + yD_{z'}$
$g_{6,17,1}$	$D_{q'}D_{z'} + zD_{q'} + D_{y'}D_{y'} + pD_{p'} + (y+qa)D_{q'} + xaD_{x'} + zD_{y'}$
$g_{6,17,2}$	$D_{z'}D_{y'} + wD_{z'} + D_{y'}D_{y'} - D_{p'}D_{p'} + D_{q'}qD_{q'} + yD_{y'} + (q+z)D_{z'}$
$g_{6,18}$	$D_{p'}D_{x'} + yD_{p'} - D_{y'} + xD_{p'} - D_{q'}D_{z'} + D_{w'} + (a+1)pD_{p'} + (y+q)D_{q'} + axD_{x'} + yD_{y'} + bzD_{z'}$
$g_{6,19}$	$D_{p'}D_{x'} + yD_{p'} - D_{y'} + xD_{p'} - D_{q'}D_{z'} + D_{w'} + (z+(a+1)p)D_{p'} + (y+q)D_{q'} + axD_{x'} + yD_{y'} + (a+1)zD_{z'}$
$g_{6,20}$	$D_{x'} - D_{q'} - zD_{x'}, D_{z'}, D_{y'}, D_{p'}, D_{w'} + bpD_{p'} + (x+y)D_{x'} + (y+z)D_{y'} + zD_{z'}$
$g_{6,21}$	$D_{q'} - D_{x'} + yD_{q'} - D_{y'} - xD_{q'}D_{p'} - D_{z'}D_{z'} + pD_{p'} + 2aqD_{q'} + a(x+y)D_{x'} + ayD_{y'} + bzD_{z'}$
$g_{6,22}$	$D_{q'} - D_{x'} + yD_{q'} - D_{y'} - xD_{q'}D_{p'} - D_{z'}D_{z'} + pD_{p'} + (2aq-z)D_{q'} + a(x+y)D_{x'} + ayD_{y'} + 2azD_{z'}$
$g_{6,23,1}$	$-2D_{p'}D_{q'} + (x+q)D_{p'}D_{x'} - qD_{p'}D_{y'}D_{z'}, D_{w'} + 2(p-z)D_{p'} + qD_{q'} + (x+q)D_{x'} + (y+x)D_{y'} + 2D_{z'}$
$g_{6,23,2}$	$-2D_{q'}D_{z'} + (y+z)D_{q'}D_{y'} - zD_{q'}D_{x'}D_{p'}, (a+2\delta)pD_{p'} + 2\delta qD_{q'} + (y+\delta x)D_{x'} + (z+\delta y)D_{y'} + z\delta D_{z'} + D_{w'}$
$g_{6,24}$	$D_{q'}D_{z'} - yD_{q'}D_{y'} + zD_{q'}D_{x'}D_{p'} + pD_{p'} + 2xD_{q'} + yD_{z'} + zD_{y'}$ $g_{6,25} - D_{x'}D_{z'} + zD_{x'}D_{z'}D_{p'} + wD_{q'} - D_{q'}D_{y'} + bpD_{p'} + bqD_{q'} + (a+1)xD_{x'} + ayD_{y'} + zD_{z'}$
$g_{6,26}$	$D_{x'}D_{p'} + qD_{z'} - D_{q'}D_{z'}D_{y'} + pD_{p'} + qaD_{q'} + (y+(a+1)x)D_{x'} + (z+(a+1)y)D_{y'} + (a+1)zD_{z'}$
$g_{6,27}$	$D_{p'} - (D_{q'} + zD_{p'})D_{z'}, D_{y'}, D_{x'}, D_{w'} + ((a+b)p + \delta x)D_{p'} + (aq + \delta w)$ $2)D_{q'} + (y+bx)D_{z'} + (z+by)D_{y'} + bzD_{z'}$
$g_{6,28}$	$-2D_{x'} - (D_{z'} + yD_{x'}) - D_{y'} + zD_{x'}D_{q'}D_{p'} + D_{w'} + (bp+q)D_{p'} + bqD_{q'} + 2xD_{x'} + (y+z)D_{y'} + zD_{z'}$
$g_{6,29}$	$-2D_{p'} - (D_{q'} + (x+1)D_{p'}) - D_{x'} + qD_{z'} - 2D_{z'} - 2D_{y'}D_{w'} + (y+2p)D_{p'} + qD_{q'} + (1+x+q)D_{x'} + (2y+z)D_{y'} + 2zD_{z'}$
$g_{6,30}$	$D_{x'}D_{z'} + yD_{x'} + wD_{y'} - D_{y'} - D_{p'} - wD_{q'}, D_{q'}D_{w'} + pD_{p'} + qD_{q'}$
$g_{6,31}$	-1 $2D_{p'} - 1$ $2(D_{z'} + yD_{p'}), -1$ $2(D_{y'} - zD_{p'}), -1$ $2D_{x'} - 1$ $2D_{q'}D_{w'} + 2pD_{p'} + (q+x)D_{q'} + (z+y)D_{y'} + (x+y)D_{x'} + zD_{z'}$
$g_{6,32}$	$-D_{z'} - D_{x'} - D_{y'} + xD_{z'}D_{q'}D_{p'} + cpD_{p'} + bqD_{q'} + (ax+y)D_{x'} + (ay-x)D_{y'} + (12(x^2 - y^2) + 2az)D_{z'}$
$g_{6,33}$	$2D_{z'} - D_{x'} + yD_{z'}D_{y'} - xD_{z'}D_{q'}D_{w'} + bpD_{p'} + 2aqD_{q'} + (ax-y)D_{x'} + (x+ay)D_{y'} + (\epsilon q + 2az)D_{z'}$
$g_{6,34}$	$-2D_{z'}D_{x'} + yD_{z'}D_{y'} - xD_{z'}D_{q'}D_{p'}, (q+(2a+b)p)D_{p'} + (2a+b)qD_{q'} + (ax-y)D_{x'} + (x+ay)D_{y'} + (-2\delta p + 2az)D_{z'} - D_{w'}$
$g_{6,35}$	$-2D_{z'}D_{x'} + yD_{z'}D_{y'} - xD_{z'}D_{q'}D_{w'} + (cp-q)D_{p'} + (p+cq)D_{q'} + axD_{x'} + byD_{y'} + (a+b)zD_{z'}$

$g_{6.36}$	$2D_z - D_y + xD_x - D_x - yD_z, D_{p'}, D_{q'}, D_{w'} + (cp - q)D_p + (p + cq)D_q + (ax + y)D_x + ayD_y + 2azD_z$
$g_{6.37}$	$2D_z - D_x + yD_y - D_y + xD_x - D_x - yD_z, D_{p'}, D_{q'}, D_{w'} - (ap + q)D_p - (p - aq)D_q + (ax + y)D_x + (ay - x)D_y + 2azD_z + D_w$
$g_{6.38}$	$2D_z - D_p + qD_q - D_q + pD_x - D_x - yD_y, (ap + q)D_p + (aq - p)D_q + (p + xa + y)D_x + (q - x + ya)D_y + 2zaD_z + D_w$
$g_{6.39}$	$D_y, D_x, D_{p'}, D_{q'}, D_{w'} + yD_x + zD_y - D_w + apD_p + qD_q + (b + 2)xD_x + (b + 1)yD_y + bzD_z$
$g_{6.40}$	$D_y, D_x, D_{p'}, D_{q'}, D_{w'} + yD_x + zD_y - D_w + (a + 2)pD_p + qD_q + ((a + 2)x + p)D_x + (a + 1)yD_y + azD_z$
$g_{6.41}$	$D_{q'}, D_{p'}, -D_x, D_y, D_z + qD_p + yD_{q'}, D_w + (a + 2)pD_p + (a + 1)qD_q + (ax - y)D_x + ayD_y + zD_z$
$g_{6.42}$	$3D_z - x$ $4D_{q'} - D_{q'}, D_{p'}, D_y + xD_x - D_x - z$ $4D_q + 2yD_z - D_w + (p - x)D_p + (a + 2)qD_q + xD_x + ayD_y + (a + 1)zD_z$
$g_{6.43}$	$-D_{q'} - zD_{p'}, D_{p'}, D_{q'}, -D_y, D_z + yzD_p + yD_{q'}, D_w + (x + p)D_p + wD_q + (z + x)D_x - yD_y + zD_z$
$g_{6.44}$	$D_{p'}, 3D_{q'}, D_x, D_y, D_z + yD_p + 3pD_{q'}, D_w + (2p + 1$ $2z)D_q + (12z^3 + 3q)D_q + axD_x + (y + z)D_y + zD_z$
$g_{6.45}$	$-(2D_q + zD_p), 3D_{q'}, 3D_x, D_y + zD_{q'}, D_z - qD_p - yD_q + D_y, D_w + (x + 3p)D_p + 2qD_q + 3xD_x + (y + z)D_y + zD_z$
$g_{6.46}$	$-D_{p'}, 3D_{q'}, D_x, D_x + D_y, D_z - yD_p - 3pD_{q'}, D_w + (2p - 12z^2)D_p + (3q + 1$ $2z^3)D_q + (x + y + z)D_x + (y + z)D_y + zD_z$
$g_{6.47}$	$D_{q'}, D_{p'}, D_x, D_y, D_z + yD_q + qD_{p'}, D_w + qD_q + (p + \epsilon y)D_p + axD_x + yD_y$
$g_{6.48}$	$D_{q'}, D_{p'}, D_x, D_y, D_z + qD_p + yD_{q'}, D_w + (x + p)D_p + qD_q + xD_x + yD_y$
$g_{6.49}$	$D_z + xD_{q'}, 2D_{q'}, -D_{p'}, D_y + xD_z, zD_{q'} - D_x, xD_p + (q + 2\epsilon y)D_q + yD_y + zD_z + D_w$
$g_{6.50}$	$D_{p'}, D_{q'}, D_x, D_y, D_z + yD_p + pD_{q'}, D_w + pD_p + (q + \epsilon x)D_q + (x + y)D_x + yD_y$
$g_{6.51}$	$D_y, D_x, D_{p'}, D_q + wD_x, D_z + yD_x + qD_{p'}, -\epsilon D_w + pD_p$
$g_{6.52}$	$-D_x - D_{q'}, D_{p'}, -D_y + \epsilon_w$ $2D_{q'}, D_z + xD_q + yD_x + wD_y, D_w + pD_p + \epsilon_y$ $2D_q$
$g_{6.53}$	$D_x, D_{q'}, D_x, D_y, D_p + yD_q + zD_x, -D_w - pD_p + yD_y + zD_z$
$g_{6.54}$	$D_x, D_{p'}, D_x, D_y, yD_p + D_q + zD_x, D_w + apD_p + bqD_q + xD_x + (a - b)yD_y + (1 - b)zD_z$
$g_{6.55}$	$-D_x, D_{p'}, -2D_{q'} - zD_x, D_y, D_z + yD_p + qD_{p'}, -D_w + (a + 1)pD_p + (1 - a)qD_q + (x - y)D_x + yD_y + azD_z - D_w$
$g_{6.56}$	$D_{p'}, D_x, D_y, D_z, yD_p + D_q + zD_x, pD_p + qD_q + yD_y - zD_z + D_w$
$g_{6.57}$	$-D_{p'} - (2)D_x, D_{q'}, 2D_y, -(D_z + qD_p + yD_x + wD_y), D_w + pD_p + (1 - a)qD_q + 2axD_x + ayD_y + azD_z + D_w$
$g_{6.58}$	$-3D_{p'} - 2D_{q'} - 2D_x - zD_p - D_y - zD_{q'} - D_z + xD_p + yD_q - wD_y, D_w + 3pD_p + (x + 2q)D_q + 2xD_x + (y + z)D_y + zD_z$
$g_{6.59}$	$-D_{p'} - D_x, D_{q'} - D_y, -(D_z + qD_p - yD_x - hzD_y), -D_w + pD_p + qD_q + yD_x + hzD_y$
$g_{6.60}$	$D_{p'}, D_{q'}, D_x + zD_{p'}, D_p + zD_q + D_y, -D_z + yD_q + D_y, D_w + (y + p)D_p + 2qD_q - wD_x + (y - z)D_y + zD_z + D_w$
$g_{6.61}$	$-2D_{p'} - D_{p'}, zD_x + D_y, zD_p + D_{q'} - yD_x + D_z - qD_q + 2xD_x + (z + y)D_y + zD_z + D_w$
$g_{6.62}$	$D_{p'}, 1$ $2D_{q'}, 1$ $2(D_x + zD_p - D_q), 1$ $2(-D_x + zD_p), -D_x + xD_p - 1$ $2D_y, -D_w + 2pD_p + (x + q)D_q + (x - z$ $2)D_x + zD_z$
$g_{6.63}$	$-D_x, D_{p'}, D_y, D_{q'}, D_x + qD_p - yD_z, D_w + (q + ap)D_p + aqD_q + yD_y + zD_z$
$g_{6.64}$	$D_{p'}, D_{q'}, D_x, D_y, D_z + xD_p + yD_{q'}, D_w + (y + p)D_p + (\epsilon x + q)D_q + xD_x + yD_y$
$g_{6.65}$	$-D_y, D_p, D_x + zD_y, -D_q - zD_p, D_z - D_w + (ap - y)D_p + ((a - b)q - x)D_q + (a - b)xD_x + ayD_y + bzD_z$
$g_{6.66}$	$D_p - 2D_{q'} - 2D_x, D_x + zD_{q'} + D_y, zD_p + D_x, D_z - xD_p - yD_p + D_y,$ $D_w + (2p + q)D_p + 2qD_q + (x + y)D_x + (y + z)D_y + zD_z$
$g_{6.67}$	$-D_p - 2D_{q'} - 2D_x - D_y - zD_{q'} - D_x - D_x - zD_{p'} - D_z + xD_p + yD_q - aD_x,$ $D_w + (2p + q)D_p + 2qD_q + (x + y + az)D_x + yD_y + zD_z$
$g_{6.68}$	$D_q + D_{p'}, D_{p'}, D_x + D_y, D_x + cD_{q'}, D_z + xD_p + yD_{q'}, D_w + (p + q)D_p + (cx + q)D_q + (x + y)D_x + yD_y$
$g_{6.70}$	-1 $1 + a2D_{p'}, -1$ $1 + a2D_{q'}, \delta$ $q + a2D_p + a\delta$ $1 + a2D_{q'} - D_{q'}$ $-D_y, xD_p + yD_q + D_z, D_w + (pa - q)D_p + (p + aq + \delta x)D_q + ((a - b)x - y)D_x + (x + (a - b)y)D_y + bzD_z$
$g_{6.71}$	$D_{p'}, D_{q'}, D_x, D_y, -D_z + qD_p + xD_x + yD_y, D_w + (a + 3)pD_p + (a + 2)qD_q + (a + 1)xD_x + ayD_y + zD_z$
$g_{6.72}$	$-4D_{p'} - (3D_q + zD_p), -(2D_x + zD_q), -(D_y + zD_p), -D_z + qD_p + xD_q + yD_x - D_y,$ $D_w + 4pD_p + 3qD_q + 2xD_x + (y + z)D_y + zD_z$
$g_{6.73}$	$D_{p'}, D_{q'}, D_x, D_y, D_z + qD_p + xD_x + yD_y, D_w + (p + ay + \epsilon x)D_p + (q + \epsilon y)D_q + xD_x + yD_y$
$g_{6.74}$	$D_{p'}, D_{q'}, D_x, D_y, D_z + qD_p + xD_x + yD_y, D_w + pD_p + qD_q + xD_x + yD_y$
$g_{6.75}$	$D_{p'}, D_{q'}, D_x, D_y, D_z + qD_p + xD_x + yD_x + D_z, D_w + (y + p)D_p + qD_q + xD_x + yD_y$
$g_{6.76}$	$-2D_{p'}, 1$ $4zD_p + 8$ $5yD_q - 3D_x, -2D_{q'}, 2$ $5xD_q + 1$ $2zD_x + 1$ $2D_y,$ 1 $2xD_p - 4yD_x + 2D_z, D_w + (2a + 1)pD_p + (a + 2)qD_q + (a + 1)xD_x + yD_y + hzD_z$

$g_{6,77}$	$D_{p'}D_x + yD_{q'} - 2D_{q'}D_y - xD_{q'}D_z + xD_p + yD_{x'}D_w + (ay + p)D_p + 2qD_q + xD_x + yD_y$
$g_{6,78}$	$D_{p'}D_x - zD_p - yD_{q'} - D_{q'}D_y + yzD_{q'}D_z + yD_x + yzD_{p'}D_w - pD_p + (y + q)D_q + yD_y - zD_z$
$g_{6,79}$	$-3D_{p'} - (2D_x + yD_p + zD_q), -D_p - 3D_{q'} - D_z - D_x + xD_q - yD_{x'}$ $-D_y + xD_p + zD_{x'}D_w + (3p + q)D_p + 3qD_q + 2xD_x + (y + z)D_y + zD_z$
$g_{6,80}$	$-D_{p'}yD_p + xD_q + D_zD_{q'} - (D_x + yD_z + 1$ $2(x^2 + y^2)D_p),$ $D_y - 1$ $2(x^2 + y^2)D_{q'}D_w + (q + 3ap)D_p + (3aq - p)D_q + (ax - y)D_x + (x + ay)D_y + (2az + 1$ $2(x^2 - y^2)D_z$
$g_{6,81}$	$-D_{p'}yD_p + xD_q + D_zD_{q'} - (1$ $2(x^2 + y^2)D_p + D_x + yD_z), D_y - 1$ $2(x^2 + y^2)D_{q'}$ $D_w + (q + \epsilon x)D_p - pD_q - yD_x + xD_y + 1$ $2(x^2 - y^2)D_z$
$g_{6,82}$	$D_{p'}D_xD_yD_z + xD_{p'}D_q + yD_{p'}D_w + 2apD_p + (a - b)qD_q + (a + c)xD_x + (a + b)yD_y + (a - c)zD_z$
$g_{6,83}$	$D_{p'}D_x - D_{q'}x D_p + D_{q'} - qD_p - D_z, 2\delta pD_p + (-x + q\delta + qa)D_q + (\delta x + xa)D_x + (\delta y - ay + z)D_y + (z\delta - za)D_z + D_w$
$g_{6,84}$	$D_{x'} - D_q - pD_{x'}D_yD_z + yD_x + wD_{y'} - D_w - pD_p + qD_q$
$g_{6,85}$	$2D_{p'}D_q + yD_{p'}D_x + zD_{p'}D_y + qD_{p'}D_z + xD_{p'}D_w + 2pD_p + (a + 1)qD_q + (x - \epsilon z)D_x + (1 - a)yD_y + zD_z$
$g_{6,87}$	$-2D_{p'}D_q + D_z + yD_{p'}D_x + D_y + zD_{p'}D_y - qD_{p'}$ $D_z + D_x - xD_{p'}D_w + 2pD_p + qD_q + (x + z)D_x + (y + x)D_y + (z + q)D_z$
$g_{6,88}$	$D_zD_xD_y - D_p + xD_{z'} - D_q + yD_z,$ $D_w + ((\alpha$ 2 $-a)p - q)D_p + ((\alpha$ 2 $-a)q + p)D_q + ((\alpha$ $2 + a)x - y)D_x + ((\alpha$ $2 + a)y + x)D_y + \alpha$ $2zD_z$
$g_{6,89}$	1 $2D_{z'} - 1$ $2D_{y'} - 1$ $2(pD - D_{q'})D_x + yD_{z'} - 1$ $2(D_p + qD_z)D_w + (\delta p + bq)D_p + (\delta q - bp)D_q + (\delta - a)x D_x + (\delta + a)y D_y + 2\delta z D_z$
$g_{6,90}$	$-2D_{p'}D_z + qD_{p'}D_y + (ax - y)D_{p'}D_q - zD_{p'}D_x - (x + ay)D_{p'}$ $D_w + 2apD_p + qaD_q + (ax + by)D_x + (-bx + ay)D_y + (-q + az)D_z$
$g_{6,91}$	$g_{6,91} 4D_{p'} - 2xD_p + D_x - D_{y'} - 2$ 1 $2D_{q'} - 2xD_p + D_x + D_{y'} - 2$ 3 $2qD_p - 2$ 12 $D_zD_w + (q^2 - z^2)D_p + zD_q + xD_x - yD_y - qD_z$
$g_{6,92}$	$-2D_{z'} - D_x + xD_{z'} - D_y + pD_{z'}D_x + yD_{z'}D_y + qD_{z'}$ $D_w + (ap - bq)D_p + (\alpha q + bp)D_q + (\alpha x - ay)D_x + (ay + ax)D_y + 2azD_z$
$g_{6,93}$	$2D_{z'} - D_p + xD_{z'}D_y + qD_{z'}D_x - pD_{z'}D_y - D_q + yD_{z'}$ $D_w + (\delta p - aq)D_p + (\delta q + ap)D_q + (\delta x + p - ay)D_x + (\delta y + q + ax)D_y + 2\delta zD_z$
$g_{6,94}$	$D_{x'}D_yD_z + pD_{x'}D_q - D_{p'}D_q + yD_x + zD_{y'}D_w + 2pD_p + qD_q + (a + 2)x D_x + (a + 1)y D_y + az D_z$
$g_{6,95}$	$2D_{p'} - (D_q + zD_p), D_x - yD_p + zD_{q'}, 2D_{y'}D_z - qD_{p'}D_w + 2yD_y + zD_z + (y + 2p)D_p + qD_q$
$g_{6,96}$	$3D_{p'} - (2D_q + zD_p), D_x + yD_p + zD_{q'} - D_q - 2D_y + xD_{p'}$ $D_x + D_z - qD_p + (z - x)D_{q'}D_w + 3pD_p + (2q + y)D_q + (x + z)D_x + 2yD_y + zD_z$
$g_{6,97}$	$-4D_{p'} - 3D_q - zD_{p'} - (D_x + 2D_y + xD_p - zD_q), -2D_x + yD_{p'}$ $-D_z + qD_p + yD_{q'}D_w + 4pD_p + 3qD_q + (y + 2x)D_x + 2yD_y + zD_z$
$g_{6,98}$	$D_{p'}D_q - D_{p'}D_x + zD_{p'}D_y + xD_{p'}D_z + qD_p + xD_q - \delta wD_{y'}D_w + (\delta q + p)D_p + qD_q + xD_x$
$g_{6,99}$	$5D_{p'} - (4D_x + zD_q), 3D_x + yD_y + zD_{q'} - 2D_y + xD_p - zD_{q'}$ $D_z - qD_p - xD_q - yD_{x'}D_w + 5pD_p + 4qD_q + 3xD_x + 2yD_y + zD_z$

Table 11: Vector Field Representations for Mubarakzhanov Algebras.

• Algebras that are marked with † are suspensions in the sense of Table 3 in Shabanskaya and Thompson [2]

• Algebras that are marked with † supersede a Mubarakzhanov algebra that is problematic (Tables 10 and 11).

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