

R-Generalized Fuzzy Closed Sets with Respect to an Ideal in Fuzzy Topological Spaces

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Abstract

The work in this paper is a generalization The concept of r-generalized fuzzy closed sets in fuzzy topological spaces was introduced by Kim. In this paper, we introduce and study the concept of r-generalized fuzzy closed sets with respect to an ideal in an ideal fuzzy topological space in Sostak sense.

Keywords: R-generalized fuzzy closed sets; Rv-generalized fuzzy closed sets with respect to an ideal in an ideal fuzzy topological space in Sostak sense

Introduction

Sostak introduce the fundamental concept of fuzzy topological structure as an extension of both crisp topology and Chang's fuzzy topology [1], in the sense that not only the object was fuzzified, but also the axiomatic. Chattopdhyay et al. [2,3] have redefined the similar concept. In El-Naschie [4-14] and Kim and Ko [15] gave a similar definition namely "Smooth fuzzy topology". We must point out that [16-19]; the concept of fuzzy topological spaces has been a significant concept in string theory and E-infinity theory pertaining to quantum particular physics ever since El-Naschie ([4-14]). After that several authors [20,21] have introduced the smooth definition and studied smooth fuzzy idea topological spaces being unaware of Sostak works.

Throughout this paper, let X be a nonempty set $I=[0;1]$ and $I_0=(0;1]$: For $\alpha \in I$; $\bar{\alpha}(x) = \alpha$ for all $x \in X$: The family of all fuzzy sets on X denoted by I^X : For two fuzzy sets we write λ_{qu} to mean that is quasi-coincident (q-coincident, for short) with μ , i.e., there exists at least one point $x \in X$ such that $\lambda(x) + \mu(x) > 1$: Negation of such a statement is denoted as $\lambda_{\bar{q}\mu}$:

Definition 1.1

A mapping $\tau: I^X \rightarrow I$ is called a fuzzy topology on X if it satisfies the following conditions [17]:

$$\tau(\bar{0}) = \tau(\bar{1}) = \bar{1}$$

$$\tau(\bigvee_{i \in I} \mu_i) \geq \bigwedge_{i \in I} \tau(\mu_i), \text{ for any } \{\mu_i\}_{i \in I} \in I^X$$

$$\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2) \text{ for any } \mu_1, \mu_2 \in I^X$$

The pair $(X; \tau)$ is called a fuzzy topological space (for short, fts).

Definition 1.2

Let (X, τ) be a fts, $\lambda, \mu \in I^X$ and $r \in I_0$.

A fuzzy set λ is called r-generalized fuzzy closed (for short, r-gfc) if $C_\gamma(\lambda; \gamma)$ whenever $\lambda \leq \mu$ and $\tau(\mu) \geq \gamma$

A fuzzy set λ is called r-generalized fuzzy closed (for short, r-gfc) if $I_r(\lambda; \gamma) \geq \mu$ whenever $\lambda \geq \mu$ and $\tau(\bar{1} - \mu) \geq \gamma$

Definition 1.3

A mapping $I: I^X \rightarrow I$ is called fuzzy ideal on X if:

$$(I_1) I(0)=1; I(1)=0:$$

$$(I_2) \text{ If } \lambda \leq \mu; \text{ then } I(\lambda) \geq I(\mu); \text{ for each } \lambda \in I^X:$$

$$(I_3) \text{ For each } \lambda; \mu \in I^X; I(\lambda \vee \mu) \geq I(\lambda) \wedge I(\mu) [\text{finite additivity}].$$

Lemma 1.1.

Let (X, τ, I) be a fts. The simplest fuzzy ideal on X are $I^0, I^1: I^X \rightarrow I$ where

$$I^0(\lambda) = \begin{cases} 1, & \text{if } \lambda = \bar{0} \\ 0, & \text{otherwise,} \end{cases} \quad I^1(\lambda) = \begin{cases} 0, & \text{if } \lambda = \bar{1} \\ 1, & \text{otherwise} \end{cases}$$

If we take $I=I^0$, for each $A \in I^X$ we have $A^* = C_r(A, r)$.

If we take $I=I^1$, for each $A \in \Theta'$ we have $A^* = \bar{0}$, where, $\bar{1} \notin \Theta'$ be a subset of I^X [4-14].

Definition 1.4

Let (X, τ, I) be a fuzzy ideal topological space [16]. Let $\mu, \lambda \in I^X$, the r-fuzzy open local function μ_r^* of μ is the union of all fuzzy points x_i such that if $\rho \in Q(x, \gamma)$ and $I(\lambda) \geq r$ then there is at least one $y \in X$ for which $\rho(y) + \mu(y) - 1 > \lambda(y)$.

Theorem 1.1

Let (X, τ) be a fts. Then for each $r \in I_0$, $\lambda \in I^X$ we define an operator $C_r: I^X \times I_0 \rightarrow I^X$ as follows:

$$C_r(\lambda, \gamma) = \bigwedge \{ \mu \in I^X : \lambda \leq \mu, \tau(\bar{1} - \mu) \geq \gamma \}$$

For $\lambda, \mu \in I^X$ and $r, s \in I_0$, the operator C_r satisfies the following conditions:

$$C_r(\bar{0}, \gamma) = \bar{0}$$

$$\lambda \leq C_r(\lambda, \gamma)$$

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$$\begin{aligned} C\tau(\lambda, \gamma) \vee C\tau(\mu, \gamma) &= C\tau(\lambda \vee \mu, \gamma) \\ C\tau(\lambda, \gamma) \vee C\tau(\lambda, s) &\text{ if } \gamma \leq s \\ C\tau(C\tau(\lambda, \gamma), \gamma) &= C\tau(\lambda, \gamma) \end{aligned}$$

Theorem 1.2

Let (X, τ) be a fts. Then for each $r \in I_0, \lambda \in I^X$ we define an operator $I_\tau : I^X \times I_0 \rightarrow I^X$ as follows [18]:

$$I_\tau((\lambda, \gamma)) = \vee \{ \mu \in I^X : \lambda \geq \mu, \tau(\mu) \geq \gamma \}$$

For $\lambda, \mu \in I^X$ and $r, s \in I_0$, the operator I_τ satisfies the following conditions:

$$\begin{aligned} I_\tau((\bar{1} - \lambda, \gamma)) &= \bar{1} - C_\tau(\lambda, \gamma) \text{ and } C_\tau(\bar{1} - \lambda, \gamma) = \bar{1} - I_\tau(\lambda, \gamma) \\ I_\tau(\bar{1}, \gamma) &= \bar{1} \\ \lambda &\geq I_\tau(\lambda, \gamma) \\ I_\tau(\lambda, \gamma) \wedge I_\tau\{\mu, \gamma\} &= I_\tau(\lambda \wedge \mu, \gamma) \\ I_\tau(\lambda, \gamma) \wedge I_\tau\{\lambda, s\} &\text{ if } \gamma \geq s. \\ I_\tau(I_\tau(\lambda, \gamma), \gamma) &= I_\tau\{\lambda, \gamma\} \end{aligned}$$

r-generalized fuzzy closed sets with respect to an ideal

Definition 2.1

Let (X, τ, I) be fuzzy ideal topological space, $\mu \in I^X$ and $r \in I_0$. A fuzzy set μ is called r-generalized fuzzy closed with respect to an ideal I (briefly, r-gffc) if $I(C_\tau(\mu, \gamma) \setminus \lambda) \geq \gamma$, whenever $\mu \leq \lambda$ and $\tau(\lambda) \geq r$.

Lemma 2.2

Every r-gfc set is r-gffc.

Proof

Let $\mu \leq \lambda$ and $\tau(\lambda) \geq r$. Since μ is r-gfc set, then $C\tau(\mu, \gamma) \leq \lambda$, this implies that $C\tau(\mu, \gamma) \bar{q} \bar{1} - \lambda$, implies $C_\tau(\mu, r)(x) + (1 - \lambda)(x) \leq 1$, then $C_\tau(\mu, \gamma)(x) - \lambda(x) \leq 0$. Thus, $I(C_\tau(\mu, r) \setminus \lambda) \geq \gamma$ [16-19].

Example

The converse Lemma 2.2 is not true. Let $X = \{a, b\}$ be a set.

$$\mu_1(a) = 0.4, \mu_1(b) = 0.5; \mu_2(a) = 0.4, \mu_2(b) = 0.6; \mu_3(a) = 0.3, \mu_3(b) = 0.5.$$

We define fuzzy topology and fuzzy ideal $\tau, I : I^X \rightarrow I$ as follows

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{1}, \underline{0} \\ \frac{1}{2}, & \text{if } \lambda = \mu_1, \\ \frac{1}{2}, & \text{if } \lambda = \mu_2 \\ 0, & \text{otherwise} \end{cases} \quad \mathbb{J}(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{0} \\ \frac{1}{2}, & \text{if } \lambda = \underline{0.5} \\ \frac{1}{2}, & \text{if } \underline{0} < \lambda < \underline{0.5} \\ 0, & \text{otherwise} \end{cases}$$

Then μ is r-gffc set because,

$$\mu \leq \mu_1, \tau(\mu_1) \geq \frac{1}{2} C_\tau\left(\mu, \frac{1}{2}\right) = \underline{1} - \mu_1 \setminus \mu_1 = a_{0.3}.$$

Theorem 2.1

Let (X, τ, I) be a fuzzy ideal topological space, $\mu, \lambda \in I^X$ and $r \in I_0$. If μ and λ are r-gffc sets, then $\mu \vee \lambda$ is r-gffc.

Proof

Suppose μ and λ are r-gffc sets. If $\mu \vee \lambda \leq \rho$ and $\tau(\rho) \geq \gamma$, then $\mu \leq$

ρ and $\lambda \leq \rho$. By assumption, $I(C_\tau(\mu, \gamma) \setminus \rho) \geq \gamma$ and $I(C_\tau(\lambda, \gamma) \setminus \rho) \geq \gamma$ and hence

$$I(C_\tau(\mu \vee \lambda, \gamma) \setminus \rho) = C\tau(\mu, \gamma) \setminus \rho \vee C\tau(\lambda, \gamma) \setminus \rho \geq \gamma.$$

Therefore, $\mu \vee \lambda$ is r-gffc.

Remark

The intersection of two r-gffc sets need not be an r-gffc set as shown by the following example.

Example

The converse Lemma 2.2 is not true. Let $X = \{a, b\}$ be a set.

$$\mu_1(a) = 0.4, \mu_1(b) = 0.5; \mu_2(a) = 0.4, \mu_2(b) = 0.6; \mu_3(a) = 0.3, \mu_3(b) = 0.5.$$

We define fuzzy topology and fuzzy ideal $\tau, I : I^X \rightarrow I$ as follows:

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{1}, \underline{0} \\ \frac{1}{2}, & \text{if } \lambda = \mu_1, \\ \frac{1}{2}, & \text{if } \lambda = \mu_2 \\ 0, & \text{otherwise} \end{cases} \quad \mathbb{J}(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{0} \\ \frac{1}{2}, & \text{if } \lambda = \underline{0.5} \\ \frac{1}{2}, & \text{if } \underline{0} < \lambda < \underline{0.5} \\ 0, & \text{otherwise} \end{cases}$$

Then μ is r-gffc set because,

$$\mu \leq \mu_1, \tau(\mu_1) \geq \frac{1}{2} C_\tau\left(\mu, \frac{1}{2}\right) = \underline{1} - \mu_1 \setminus \mu_1 = a_{0.3}$$

Therefore,

$$I\left(C_\tau\left(\mu, \frac{1}{2}\right) \setminus \mu_1\right) \geq \frac{1}{2}.$$

But μ is not r-gfc set because

$$\mu \leq \mu_1, \tau(\mu_1) \geq \frac{1}{2} C_\tau\left(\mu, \frac{1}{2}\right) = \underline{1} - \mu_1 \not\leq \mu_1$$

Theorem 2.2

Let (X, τ, I) be a fuzzy ideal topological space, $\mu, \lambda \in I^X$ and $\gamma \in I_0$. If μ is r-gffc set and $\mu \leq \lambda \leq C_\tau(\mu, \gamma)$, then λ are r-gffc.

Proof

Let μ is r-gffc set and $\mu \leq \lambda \leq C_\tau(\mu, \gamma)$. Suppose $\lambda \leq \rho$ and $\tau(\rho) \geq \gamma$. Then $\mu \leq \rho$. Since μ is r-gffc, we have $I(C_\tau(\mu, \gamma) \setminus \rho) \geq \gamma$. Now $\lambda \leq C_\tau(\mu, \gamma)$ implies that

$$C_\tau(\lambda, \gamma) \setminus \rho \leq C_\tau(\mu, \gamma) \setminus \rho,$$

and hence, $I(C_\tau(\lambda, \gamma) \setminus \rho) \geq r$. Therefore, λ is r-gffc set [20,21].

Definition 2.2

Let (X, τ, I) be fuzzy ideal topological space, $\mu \in I^X$ and $\gamma \in I_0$. A fuzzy set μ is called r-fuzzy generalized open with respect to an ideal I (briefly, r-gfIo) if $\underline{1} - \mu$ is r-gffc set.

Theorem 2.3

Let (X, τ, I) be a fuzzy ideal topological space, $\mu, \lambda, \rho \in I^X$ and $\gamma \in I_0$. If μ is r-gfIo sets if and only if $\lambda \setminus \rho \leq \text{Int}\tau(\mu, r)$ for some $I(\rho) \geq r$, whenever $\lambda \leq \mu$ and $\tau(\underline{1} - \lambda) \geq \gamma$.

Proof

Suppose that μ is r-gfIo sets. Suppose $\lambda \leq \mu$ and $\tau(\underline{1} - \lambda) \geq \gamma$. We have $\underline{1} - \lambda \geq \underline{1} - \mu$. By assumption,

$$C_r(\underline{1} - \mu, \gamma) \leq \underline{1} \vee \rho$$

For some $I(\rho) \geq \gamma$. This implies

$$\underline{1} - ((\underline{1} - \lambda) \vee \rho) \leq \underline{1} - C_r(\underline{1} - \mu)$$

and hence, $\lambda \vee \rho \leq \text{Int}_r(\mu, \gamma)$.

Conversely, assume that $\lambda \leq \mu$ and $\tau(\underline{1} - \lambda) \geq \gamma$ imply $\lambda \vee \rho \leq \text{Int}_r(\mu, \gamma)$ for some $I(\rho) \geq \gamma$. Consider $\tau(\omega) \geq \gamma$ such that $\underline{1} - \mu \leq \omega$. Then $\underline{1} - \omega \leq \mu$. By assumption,

$$\underline{1} - \omega \vee \rho \leq \text{Int}_r(\mu, \gamma) = \underline{1} - C_r(\underline{1} - \mu, \gamma)$$

for some $I(\rho) \geq \gamma$. This gives that $\underline{1} - (\omega \vee \rho) \leq \underline{1} - C_r(\underline{1} - \mu, \gamma)$.

Therefore, $C_r(\underline{1} - \mu, \gamma) \leq \omega \vee \rho$ for some $I(\rho) \geq \gamma$. This show that

$I(C_r(\underline{1} - \mu, \gamma) \setminus \omega) \geq \gamma$. Hence, $\underline{1} - \mu$ is r-gfIc set.

Recall that the sets μ and λ are fuzzy separated if $C_r(\mu, \gamma) \bar{q} \lambda$ and $\mu \bar{q} C_r(\lambda, \gamma)$.

Theorem 2.4

Let (X, τ, I) be a fuzzy ideal topological space, $\mu, \lambda \in I^X$ and $r \in I_0$. If μ and λ are fuzzy separated and r-gfIc sets, then $\mu \vee \lambda$ is r-gfIc.

Proof

Suppose μ and λ are fuzzy separated and r-gfIc sets and $\rho \leq \mu \vee \lambda$, and $\tau(\underline{1} - \rho) \geq \gamma$. Then $\rho \wedge C_r(\mu, \gamma) \leq \mu$ and $\rho \wedge C_r(\lambda, \gamma) \leq \lambda$. By assumption,

$$\rho \wedge C_r(\mu, \gamma) \setminus v_1 \leq \text{Int}_r(\mu, \gamma), \rho \wedge C_r(\lambda, \gamma) \setminus v_2 \leq \text{Int}_r(\lambda, \gamma),$$

for some $I(v_1, v_2) \geq \gamma$. This means $I(\rho \wedge C_r(\mu, \gamma) \setminus \text{Int}_r(\mu, \gamma)) \geq \gamma$, and $I(\rho \wedge C_r(\lambda, \gamma) \setminus \text{Int}_r(\lambda, \gamma)) \geq \gamma$. Thus, $I(\rho \wedge C_r(\mu, \gamma) \setminus \text{Int}_r(\mu, \gamma) \vee (\rho \wedge C_r(\lambda, \gamma) \setminus \text{Int}_r(\lambda, \gamma))) \geq \gamma$.

Therefore,

$$I(\rho \wedge (C_r(\mu, \gamma) \vee C_r(\lambda, \gamma)) \setminus (\text{Int}_r(\mu, \gamma) \vee \text{Int}_r(\lambda, \gamma))) \geq \gamma$$

But $\rho = \rho \wedge (\mu \vee \lambda) \leq \rho \wedge (C_r(\mu \vee \lambda, \gamma))$, and we have

$$I(\rho \setminus \text{Int}_r(\mu \vee \lambda, \gamma)) \leq I(\rho \wedge C_r(\mu \vee \lambda, \gamma) \setminus \text{Int}_r(\mu \vee \lambda, \gamma)) \leq I(\rho \wedge C_r(\mu \vee \lambda, \gamma) \setminus (\text{Int}_r(\mu, \gamma) \vee \text{Int}_r(\lambda, \gamma))) \geq \gamma.$$

Hence, $\rho \setminus v \leq \text{Int}_r(\mu \vee \lambda, \gamma)$ for some $I(v) \geq \gamma$. This proves that $\mu \vee \lambda$ is r-gfIc.

Corollary 1.1

Let (X, τ, I) be a fuzzy ideal topological space, $\mu, \lambda \in I^X$ and $r \in I_0$. If μ and λ are r-gfIc sets, $\underline{1} - \mu$ and $\underline{1} - \lambda$ are fuzzy separated. Then $\mu \wedge \lambda$ is r-gfIc.

Proof

Obvious.

Corollary 1.2

Let (X, τ, I) be a fuzzy ideal topological space, $\mu, \lambda \in I^X$ and $r \in I_0$. If μ and λ are r-gfIc sets, then $\mu \wedge \lambda$ is r-gfIc.

Proof:

Obvious.

Theorem 2.5

Let (X, τ, I) be a fuzzy ideal topological space, $\mu, \lambda \in I^X$ and $r \in I_0$. If

and $\mu \leq \lambda$, and μ r-gfIc relative to λ and λ is r-gfIc relative to X , then μ r-gfIc relative to X .

Proof

Suppose that $\mu \leq \lambda$, μ is r-gfIc relative to λ and λ is r-gfIc relative to X . Let $\rho \leq \mu$ and $\tau(\underline{1} - \rho) \geq \gamma$. Since μ is r-gfIc relative to λ . By Theorem 2.5. $\rho \setminus v_1 \leq \text{Int}_r(\mu, \gamma)$ for some $I_\lambda(v_1) \geq r$. This implies that there exists $\tau(\omega_1) \geq \gamma$ such that

$$\rho \setminus v_1 \leq \omega_1 \wedge \lambda \leq \mu,$$

for some $I_\lambda(v_1) \geq \lambda$. Let $\rho \leq \lambda$ and $\tau(\underline{1} - \rho) \geq \gamma$. Since λ is r-gfIc, we have

$$\rho \setminus v_2 \leq \text{Int}_r(\lambda, \gamma)$$

for some $I(v_2) \geq \gamma$. This implies that there exists $\tau(\omega_2) \geq r$ such that

$$\rho \setminus v_2 \leq \omega_2 \leq \lambda,$$

for some $I(v_2) \geq \gamma$. Now

$$\rho \setminus (v_1 \vee v_2) = (\rho \setminus v_1) \wedge (\rho \setminus v_2) \leq \omega_1 \wedge \omega_2 \leq \omega_1 \wedge \lambda \leq \mu.$$

This implies that $\rho \setminus (v_1 \vee v_2) \leq \text{Int}_r(\mu, \gamma)$ for some $I(v_1 \vee v_2) \geq \gamma$.

Thus, μ r-gfIc relative to X .

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