

Rheology for the Twenty-First Century

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Editorial

During the 19th century, several renowned scientists introduced viscous forces in mechanics giving the bases of real fluids mechanics. Among them, Henri Navier and Georges Gabriel Stokes built the famous Navier-Stokes equation describing, from Newton's second law, the motion of viscous fluids and (while studying friction in liquids) Maurice Couette invented a well-known viscometer which is always in use today. The French mathematician, Henri Poincaré, in a famous paper entitled "On partial differential equations of mathematical physics" [1], emphasize upon the important work performed by Maurice Couette showing importance of the link between fluid mechanics and mathematics.

From Maurice Couette's work, dynamic viscosity measurement devices emerged and batch system fluid mechanics (mixing) knowledge started in the simple geometry made of two coaxial cylinders (that is, the Taylor-Couette flow).

For continuous flow systems, the work of Jean-Léonard-Marie Poiseuille (both an engineer and a doctor) gave the well-known Hagen-Poiseuille law giving the relationship between pressure drop and flow rate for the laminar flow of a Newtonian liquid in a pipe of circular cross section shape. This important experimental result obtained by Poiseuille stemmed from his efforts to understand blood circulation in human body. The famous viscosity unit "Poise" was created in honour of Poiseuille's work and another type of viscometer called capillary appeared in physics laboratories. Then, at the beginning of 20th century, fluids viscosity was better understood and measuring devices were available.

In the year 1905, a then unknown physicist, Albert Einstein developed a theory of Brownian motion involving viscosity. Using Stokes' results on the friction force caused by viscosity on a falling sphere in a viscous liquid, he established the famous Stokes-Einstein equation of Brownian motion theory. It is then clear that, at the beginning of 20th century, just before the revolution of relativity, Einstein was perfectly aware of the importance of viscosity in physical phenomena.

In his intensive work to understand the deep nature of gravity, Einstein needed all important developments of differential geometry and particularly the powerful tensorial calculation methods developed by two Italian mathematicians, Tullio Levi-Cevita and Ricci Curbastro. In the year 1915, Einstein published his General Relativity theory explaining that space-time continuum curvature expressed through Ricci tensor is proportional to a stress energy tensor, proportionality constant being the famous quantity often called " $\chi=8\pi G/c^4$ ". Nobody now, excepting perhaps Einstein himself, could see in this equation a link with rheology.

After World War II, industrial development of polymers marked a new step in research and technologies in the field of fluid mechanics. The mechanical behaviour of those synthetic liquids does not follow the previous ones and they were named non-Newtonian in opposition with the classical liquids called Newtonian. For Newtonian liquids, viscosity only depends on temperature and the solute concentration. In the case of a non-Newtonian liquid, viscosity also depends on the mechanical treatment applied. The term "Rheology" (in Greek: "Study of Flow") was

introduced by Eugene Bingham in USA) in the year 1928 to name the study of both plasticity (describing materials that permanently deform after a sufficient applied stress) and non-Newtonian fluids behaviour.

From a mathematical point of view, rheology makes an extensive use of tensors and differential geometry. Newtonian fluids undergo strain rates proportional to applied shear stress. Using tensorial formulation, it means that stress tensor is proportional to strain rate tensor, the proportionality constant being the fluid dynamic Newtonian viscosity. In the case of non-Newtonian liquids, proportionality is not respected and the relationship between the two tensors is much more complex. Among the wide variety of non-Newtonian fluids behaviours, shear thinning is certainly the most common, in which case the often called "apparent viscosity" (ratio of shear stress and shear rate by analogy with Newtonian viscosity) decreases when the shear rate increases. The large variety of complex fluids found in industrial applications (chemistry, metallurgy, food industry, and so on) and in nature (geology, physiology, and so on) and viscometers development allowed a huge quantity of fluids characterizations in laboratories and extensive research work to be performed throughout the world. But an important challenge was remaining as to how to be able to use all this knowledge about rheological properties established in laboratories in real situations, that is, in complex industrial installations and in nature.

This problem was solved by a Canadian scientist, Metzner, who published two essential papers in 1955 and 1957 [2,3]. For continuous flow systems of circular geometry, that is, for pipes or cylindrical ducts of circular cross section, Metzner and Reed [2] proposed to define the wall shear rate and generalized Reynolds number in order to obtain the same friction curve (correlation between friction factor and the Reynolds number) for Newtonian and non-Newtonian liquids in the laminar flow regime. For batch systems often called mixing systems, complex geometry of flow gives 3D flow patterns and calculation of a representative shear rate was impossible. Metzner and Otto [3] proposed a representative shear rate proportional to turbine rotational speed, the famous Metzner-Otto proportionality constant called "Ks" being representative of the global geometry of the mixing system.

A huge amount of experimental work was then performed by using the methods of Metzner et al. and rheology applications were developed significantly. In some of these applications, scientists and engineers had to face difficulties coming from complex geometries. How to use the Metzner-Reed method in ducts of complex cross-section shape? Moreover, and even for Newtonian liquids, it remains very difficult to solve the Navier-Stokes equation analytically for ducts of non-circular

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cross-section. Few authors [4,5] proposed analytical methods to describe the flow of complex fluids in complex geometries, which are successfully tested experimentally and numerically.

Applications of rheology in complex geometries showed how important was the mathematical description of shapes involved in the flow or more accurately in the velocity field created by energy density introduction in the fluid by the use of pumping or mixing devices. It was then clear that this problem requires the use of differential geometry and particularly the use of curvature definition introduced by Einstein in General Relativity theory which has been discussed above. At the end of 20th century, important progresses made in rheology and understanding of vector fields theories associated with mathematical developments in both analytical differential geometry and numerical methods gave a favourable environment for explanations and solutions to the main problems of fundamental physics.

In the year 2011, the Indian cosmologist, Padmanabhan [6] showed that Navier-Stokes and General Relativity equations could be seen to be very similar and the theory of gravitation could be hydrodynamic. Around the same time, Delplace [7] proposed to introduce curvature of flow velocity field in the Reynolds number definition and to modify Einstein's constant χ in order to obtain the rheological relationship between shear rate and shear stress tensors reported above. The main consequence of this approach was the introduction of dynamic viscosity and kinematic viscosity in Einstein's gravitational field theory and then the new vision of space-time as a fluid was widely discussed in the most recent works in theoretical physics.

Finally, viscosity has a major role in thermodynamics as the cause of irreversibility of physical phenomena. As expressed by Goff [8], "To understand physical phenomena being the cause of irreversibility, it is absolutely necessary to consider that they are always friction phenomena". As discovered by Ludwig Boltzmann in the 19th century, entropy is the thermodynamical quantity measuring irreversibility of physical phenomena and viscosity is the physical property of matter explaining why entropy always increases and time is always flowing in the same direction.

To conclude this editorial, we have tried to show how rheology developments during the past two centuries led to an important

contribution in both industrial applications and theoretical physics by trying to explain the world's physical phenomena. Today, the story is continuing and developments of nano-technologies or nuclear fusion reactors used for energy production are good examples of industrial applications using viscous fluids theories and rheology of non-Newtonian fluids. In theoretical physics, recent results seem to indicate that the famous quantum gravity theory, which is able to reconcile small and large scale phenomena, could arise from introduction of viscosity in fields equations [9].

The closed-form solvability or integral-ability of some of the problems in this subject does not seem to be easy. For example, the quasi-elliptic partial differential equation describing the flow of a power-law liquid in a pipe of arbitrary cross section is presumably not solved as yet and, remarkably, even if we know from experiments that a solution should exist. Finally, these great perspectives will certainly make rheology a major discipline of the 21st century in honour of its illustrious founders of the 19th century.

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