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Scalar Field Theory for Mass Determination

Panicaud B*

Department of Physics, University of Technology of Troyes, France

Abstract

Matter is actually under numerous investigations because of our misunderstanding on some observed phenomena, especially at the astronomic and cosmological scales. In the present article, a scalar field theory is investigated to explain the mass of particles from a global point of view. An universal mechanism is developed and a general relation is eventually proposed that enables to make some accurate predictions for the mass of composite particles. Numerical values are provided including predictions for existing particles, with discrete energy spectrum in relation to large-scale phenomena.

Keywords: Mass determination; Mass prediction; Dark energy; Quantum oscillator; Baryons masses

Introduction

Nowadays, the mass concept seems to be well understood and is mainly associated to particles of matter. However, several problems still remain unexplained. First, astrophysical observations [1,2] lead to the conclusion that some dark matter could exist to explain, for example, the rotation curve of galaxies as proposed on galaxy clusters by Zwicky [3] and proved on several observed galaxies by Rubin [4]. This dark matter has several explanations either baryonic or non-baryonic, but none is definitively accepted, meaning that matter is eventually not well known. In the same way, the concept of dark energy has no more convincing explanation. Second, mass can be explained by different theories that will be reminded in the present article. However, few of them lead to predictions that can be experimentally tested. Some mechanisms are nowadays identified and experimentally verified, such as the mass acquisition of massive bosons Z and W through the Higgs process [5,6]. Last but not least, mass is still the last physical unit based on a physical standard bulk material subjected to inherent difficulties [7,8]. We can definitively wonder what mass means. In the present article, the possible origins of the mass of physical systems are reminded with different explanations according to the different physical theories. Too exotic theories are not considered (such as negative mass). Therefore, the proposed list is not exhaustive but shows the main explanations, through some examples. Leading to a no-way road, we suggest and build another theory. This paper is thus an attempt to calculate the masses of both fundamental and composite particles from a master formula containing several parameters. The developments are based on a Lagrangian approach with a variational principle. It is consistent with the developments in physics over the last 50 years. Indeed, field theory is the based of actual physics, for which the present theory is derived using the classical tools of differential geometry. This theory is based on arguments already considered but not in the same way as presently. We base our theory on few simple assumptions. That theory for mass calculation is then described and more specifically the assumed interaction of matter with a real scalar field. The parameters of this model are then investigated. The use of that model on fundamental and composite particles enables to quantify explicitly these parameters. Relations between those parameters lead to predictions of particles masses. The investigation of one of those parameters according to the energy level of different families of particles leads to some observable energy density, which is eventually interpreted.

Review of Possible Mass Explanations

Mass is mainly associated to matter. Viewing at the classification of particles, we can see that fermions and bosons may have a mass. It is worth noting that all fermions have a mass, including neutrinos, whereas only some bosons have. For physical body, mass is usually separated in active gravitational mass, passive gravitational mass and inertial mass [9]. Reciprocity of the gravitational action leads to the equality between the active and passive masses [9]. The strong equivalence principle leads to the equality between the gravitational and inertial masses according to the Einstein's theory of General Relativity [10,11]. In addition, Einstein proposes the equivalence between the total mass and total energy in the framework of Special Relativity [12]. This last equivalence will be systematically used further. Besides, mass definition is clear for closed systems; whereas for open systems where energetic interactions take place with the surrounding environment through boundaries, it is always more difficult to define it clearly.

Global energy balance

Mass may be defined through the balance of total energy of a physical system (according to the equivalence between the total mass and total energy). It means a global definition of the corresponding mass. There are several examples. First, let us consider the relativistic scattering of particles. When energy thresholds are reached, particles may be created with specific masses. These thresholds and masses are defined according to the global balance of energy [11]. This "mechanism" applies during collisions in labs, stars or during the Big Bang. The last one may be considered as the fabric of particles defining their masses once forever in specific conditions that are no more accessible. In such examples, each system is constituted of several particles without interaction (or weakly interacting) with the outside of the system. Second, during nuclear or chemical reactions, the separated and thus non-interacting constituents have a different energy to an interacting system when constituents are closer. Global balance of energy between both configurations (interacting and non-interacting) enables to calculate the bounding

***Corresponding author:** Panicaud B, Professor, Department of Physics, University of Technology of Troyes, France, Tel: 33 3 25 71 76 00; E-mail: benoit.panicaud@utt.fr

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energy [13]. This "mechanism" is much more important (10^6 times) for nuclear interactions than for chemical interactions, because of the amplitude and characteristic length of nuclear interactions compared to the electromagnetic ones. Third, Komar made an attempt to define mass in the context of General Relativity, especially when exists a gravity field [14]. Other mass definitions exist such as Bondi or ADM masses [15,16]. This approach can be interpreted as a generalization of the equivalence between total mass and total energy in the context of General Relativity, by using the global energy balance with the momentum-energy tensor of contravariant component $T_{\mu\nu}$ and the metric tensor of contravariant component $g_{\mu\nu}$. The Komar mass MK is a definition of the total mass of the system based on its total energetic content, according to the relation [14,10]

$$M_K = \frac{1}{c^4}\int\int\int_V \sqrt{g_{44}}\left(2T^{\mu\nu} - g^{\mu\nu}\left(T^{\alpha\beta}g_{\alpha\beta}\right)\right)u_\mu u_\nu dV \quad (1)$$

where c is the speed of light, V is the space volume, u_μ is the covariant component of the quadrivector speed of particles such that $u_\mu u_\mu = c_2$, and g_{44} is the covariant "time" component of the metric tensor. Greek subscript or superscript runs from 1 to 4 in the present article.

Total energy decompositions

There is another way to define the mass of physical systems: by decomposing their different parts due to different physical effects. In such a decomposition, it is assumed that the different parts are independent and can be associated additively. There are several examples. In Special Relativity, the link with the total mass is given by the equivalence $E=mc^2$, where E is the total energy and m is the total mass.

First, at non-relativistic limit, the total energy of a single and isolated particle without internal degrees of freedom is the sum of its energy at rest m_0c^2 and its kinetic energy. The total mass of this particle for an observer is thus the sum of its mass at rest m_0 plus a mass depending on the speed of the particle to the observer (supposed to be inertial/Galilean) (see eqn. 3). This decomposition is the first-order expansion of the general equivalence between mass and energy. It can be also interpreted as a consequence of the fourth-component (on time direction) of the relation between the quadrivector momentum-energy of con-travariant component p^μ and the quadrivector speed of contravariant component u^μ:

$$E/c = p^4 = m_0u^4 = m_0\gamma c = mc \quad (2)$$

$$\Rightarrow m = m_0\gamma = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

$$\approx m_0 + \frac{1}{2}m_0v^2/c^2 \quad (3)$$

where γ is the Lorentz factor [10] depending on the spatial norm of the speed v^2. Second, when taking into account for internal degrees of freedom of a single and isolated particle, it is necessary to add an internal energy, for which a statistic assumption is often performed. We consider that internal thermalization is reached. Each internal degrees of freedom n_d has thus the same equi probable energy [16]. It enables to decompose the total mass as the sum of a constant part m_1 (different throughout this article), plus a contribution depending on temperature T:

$$m \approx m_0 = m_1 + \frac{n_d k_B T}{2c^2} \quad (4)$$

where k_B is the Boltzmann constant. A similar relation was proposed by De Broglie in 1955 to explain the mass from a domain at temperature T.

It is quite easy to extend eqn. 4 for Special Relativity by using the Lorentz factor γ, provided that temperature is correctly coupled to γ [17]. It is more difficult to extend it for General Relativity. For example, a heated and massive gas leads to an increase of its mass. However, its numerical value depends on the chosen equation for mass definition (Komar or others). Moreover, when the system is dynamic (gas exploding...), these mass definitions are quite useless because the calculations are easy only for stationary metric tensor.

Third, when extending to open systems with external weak interactions, any physical effect can contribute to the total mass using a similar way. For example, if we consider the same single particle as previously with an internal structure and specific magnetic properties, then interaction with an external magnetic vector field of component B^k_{ext} can contribute to the total mass, such as:

$$m \approx m_0 = m_1 + \frac{n_d k_B T}{2c^2} + \frac{B^2_{ext}V}{2\mu c^2} \quad (5)$$

where V is the volume of the particle and μ is its magnetic permeability. Any other energy may contribute to the total mass. This last example ought to be placed in section 2.4 as a case of mass due to an interaction mechanism. We see that the contribution depends on the square of the inverse of the speed of light. Consequently, the numerical contributions of those effects to the total mass are often very small.

Last, another decomposition is possible when considering a composite system, based on the additivity of energy when independent parts of this system are assumed. Indeed, let us consider a complex system composed of n_p parts.

If the different parts do not interact, or weakly such as it can be neglected, then the total mass can me written as:

$$m = \sum_{i=1}^{n_p} m_i \approx \sum_{i=1}^{n_p} m_{0i} \quad (6)$$

Last approximation corresponds to the non-relativistic limit. For continuous media, the sums in eqn. 6 are replaced by integrals.

Quantum effects and interpretation

Quantum mechanics leads to some specific interpretation of the mass due to the duality between particles and waves. Indeed, the energy of a particle is related to the angular frequency of its wave; whereas the group speed vector of contravariant component is related to the variation of this energy E to the wavenumber vector of covariant component k_i. It leads to a quantum explanation of the rest mass of the wave group, according to (at non-relativistic limit) [18]:

$$m_0 = \frac{p^i}{v^i_g} = \frac{\hbar^2 k^i}{\frac{\partial E(k_l)}{\partial k_i}} \quad (7)$$

where $\hbar$ is the reduced Planck constant. This "mechanism" can be used, for example, to calculate the mass contribution in a lattice with electrical charges moving within. Indeed, we can calculate in a more general way the effective second-order mass tensor of particles useful in solid-state physics [19,20]:

$$m^{ij}_0 = \hbar^2\left(\frac{\partial^2 E(k_l)}{\partial k_i \partial k_j}\right)^{-1} \quad (8)$$

Let us consider the case of a cubic lattice of lattice spacing a with an energy $E(k_l) = E_0 + \sum_{l=1}^{3}(Aa^2(k_l)^2)$, where A is the amplitude of the energy of particles moving on this lattice. With equations 7 or 8, we obtain the same result for the isotropic effective mass of the particles:

these relations can also be applied to the quantum tunnelling or to the cyclotron motion. It is also possible to have quantum contribution due to degenerate states. For example, we can mention the case of degenerate fermions gas and degenerate bosons gas for which quantum effects lead to a specific relation for the effective mass of those "almost ideal" gases [21]. For example, the degenerate Fermi gas with repulsive interaction of amplitude U_a and interaction characteristic length

$$L_a = \frac{m_1 U_a}{4\pi\hbar^2}$$

$$m_0 = m_1 + 0.59 m_1 L_a^2 \left(\frac{3\zeta}{\pi}\right)^{2/3} \quad (9)$$

where ζ is the volumic concentration of particles. Quantum effects are required to explain the masses of subatomic particles and more precisely at a scale below the Compton wavelength. Some attempts to explain the light hadrons masses can be found [22]. The quantum theory offers an interesting framework for quantification of masses. Formulas based on the resonance theory of elementary particles as de Broglie waves have been proposed, such as for example $m_{0,n} \approx 137\ n\ m_e$ where n is the mode and m_e is the electron mass [23]. At the opposite, the masses of leptons (electron, muon and tau) may be described with an empirical relation, known as Koide formula [24]:

$$m_e + m_\mu + m_\tau = \frac{2}{3}\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2 \quad (10)$$

Some authors have tried to applied it to the prediction of the neutrinos masses or extended it for other particles [25-27]. Eventually, we have also to focus on the seesaw mechanism as a possible explanation for masses of neutrinos [28]. Indeed, because of the quantum probabilities, each neutrino can transform into each other. It is known as neutrinos oscillation [29,30]. Their mass could be then explained thanks to a super-partner particle for which the mass is big enough to correlate with the small observed masses of neutrinos [28,30,31]. More explanations and details for the neutrino masses can be found [32].

Mechanisms of interaction

From the previous explanations, some cases correspond to local effect due to interactions with the different fields. There are several examples for such open systems. First, such an effect is not limited to microscopic scale and can be observed at macroscopic scale. We can mention the hydrodynamic interaction of a sphere of finite volume V accelerating in a fluid of mass density ρ [33]. The local interaction of such a finite solid with the environing fluid leads to a contribution of the mass as:

$$m \approx m_0 = m_1 + \frac{\rho V}{2} \quad (11)$$

Second, at microscopic field, similar mechanisms may be considered. For example, the mass of massive bosons Z and W can be explained through the local interaction with a complex scalar field [5,6]. This mechanism leads to the acquisition of particle mass thanks to the Higgs boson, through the broken symmetry in the scalar field [5,6]. Such a mechanism may be generalized to any particles (Goldstone model [34]). However it presents different limitations, such as the number of necessary coupling constants as numerous as the existing particles.

Third, there exists different possible couplings with scalar field. The present article focuses especially with a specific coupling between matter and a real scalar field (see in section 3).

Last, we can discuss the case of interacting systems through gravitation. Boratav and Kerner [35] have presented an interpretation of mass as the interaction with the far distant universe. This mechanism is consistent with the Mach principle that has oriented Einstein for the construction of the General Relativity. Different assumptions are assumed: finite radius of the universe; constant mass density of the universe ρ_U at the observed scale; only gravitational interaction (without magneto-gravitational effect, but with radiative gravitational terms); expansion of the universe according to the Hubble law. It can be then proved that mass emerges from the radiative gravitational force of the far distant universe on a test particle. The inertial mass and the gravitational mass are equal provided that $\rho_U\ GH^{-2}=1$, where G is the Newtonian gravitational constant. Actually, the experimental evaluations of the Hubble constant $H \approx 73 km \cdot s^{-1} \cdot Mpc^{-1}$ and of the mass density $\rho_U \approx 10^{-26}$to $10^{-25}\ kg\ m^{-3}$ suggest that this relation could be numerically verified.

Mechanisms of self-interaction

Self-interaction is also a kind of interaction that could also explain the mass for open or closed systems. The most famous development has been made for electrodynamics. Several relations can be proposed based on different approximations: with or without relativistic assumption, with or without quantum assumption [36-39]. For some of those relations, the mass depends on the inverse of the length scale r:

$$m \approx m_0 = m_1 + Cste' \lim_{r\to 0}\left(\frac{m_0 L_T}{r}\right) \quad (12)$$

where L_T is the Thomson scattering length and $Cste'$ s a numerical parameter depending on the considered geometry. A strong divergence problem occurs for non-quantum approach (relativistic or not) [36,37] or for quantum and non- relativistic approach [38]. It is only with a simultaneous relativistic and quantum theory that the mass of particles can be renormalized with a logarithmic dependence on the length scale [39]:

$$m = \frac{m_1}{1 - \frac{3\alpha_e}{2\pi}\lim_{\Lambda r\to\infty}\log\left(\frac{\Lambda_r}{m_1}\right)} \quad (13)$$

This process of renormalisation is done in quantum electrodynamics and can be performed for all order of the expansion of interaction terms with Feynman diagrams [40]. This is possible because: the theory depends on only one parameter Λ_r related to the length scale; the electromagnetic energy is local; the coupling constant $\alpha_e \approx 0.0073$ is a small parameter to unity. Such a renormalisation explains mass as a result of the self-interaction, but does not enable to calculate a priori the observable mass.

Observable mass is the total mass. In the most general case, it corresponds to the Komar mass (eqn. 1), whose definition can also be applied to the calculation of the electromagnetic charge when taking into account of its local gravitational field; it leads at first order to the same relation as eqn. 12. We can wonder if such an explanation for mass by considering self-interaction could be experimentally checked. If we consider a macroscopic ball of electrical charge Q and finite radius r, the total mass is:

$$m \approx m_0 = m_1 + Cste'' \frac{Q^2 \mu_0}{4\pi r} \quad (14)$$

where $\mu_0 = (\epsilon c^2)^{-1} = 4\pi 10^7 USI$ is the vacuum permeability and $Cste''$ is a numerical parameter depending on the considered geometry $\in$ [1/2; 3/5]. For an electrical charge Q of some Coulomb, a radius r of some mm and for an uncharged mass m_1 around some mg, it should be possible to measure an effective increase of the charged mass m_0.

However, because of the additional electrostatic interaction of the charge with the surrounding environment (long-distance and strong interaction), it is more difficult to define clearly the mass of this open system and what is really weighted with a weighing machine. In other words, it is not sure that eqn. 14 is still valid. Accurate experimental tests would be required to separate the different effects. Eventually, the same developments can be proposed for gravitation and quantum chromodynamics. However, it has been demonstrated that gravitational interaction cannot be renormalized [10, 41].

Thermodynamics mechanism

Thermodynamics arguments may also be used to define mass. There are several examples. First, let us start with the proposition of Verlinde based on entropic considerations [42]. He has proposed that gravitation is a consequence of the holographic principle due to Bekenstein and would be an entropic force [43]. The gravitational mass "emerges in the part of space surrounded by a screen where the energy is evenly distributed over the occupied bits of information" [42]. He proves then that the inertial mass may be related to the entropy S of this information as:

$$m \approx m_0 = n^i \left(\nabla_i S\right) \frac{\hbar}{2\pi k_B c} \tag{15}$$

Where n^i is the contravariant component of a unitary space vector and ∇_i is the covariant component of the space differential operator. Second, we can also directly applied thermodynamics principles [16]. Let us propose a very simple but innovative example at microscopic scale for an atomic nucleus without detailing the inside nuclear interactions, using non-quantum and non-relativistic arguments. We suppose that a thermodynamic dissipation occurs at local scale because of the different particles inside the nucleus of global temperature T, where diffusion may occur. Indeed, thermal equilibrium is assumed, whereas "chemical" equilibrium is not within this nucleus. The created volumic heat power r_v is dissipated for example through thermal radiation $\tilde{Q}_R$ at the surface. This volumic heat is related to the flux of particles, through a linear electrochemical-like coupling that is built to respect the Clausius Duhem inequality [16]. It is then supposed that diffusion of particles in the nucleus follows the Fick's first law. The Fick diffusion coefficient can be $D_F = \frac{k_B T}{6\pi\eta L}$. Electrical charges could be also taken into account through a factor (z+1) in the diffusion coefficient. Using eqn. 4 with m_1=0, it leads then to the mass of the nucleus of characteristic length L.

$$\begin{aligned} & -j^i \left(\nabla_i \beta\right) L^3 = r_v L^3 = -\widetilde{Q_R} \\ & \Leftrightarrow \frac{k_B T D_F}{\zeta} \left(\nabla_i \zeta\right)^2 L^3 = \sigma_{SB} T^4 L^2 \\ & \Rightarrow m_0^2 = \frac{2.5 n_N^3 \hbar^3}{\pi^3 c^2 L^5 \eta} \\ & \Leftrightarrow m_0 \approx \frac{1.7 \cdot 10^{-22}}{\sqrt{\eta}} (kg) \end{aligned} \tag{16}$$

where $\beta=\beta_0 + kB\ T \ln \zeta$ is the "chemical" potential of the nuclear fluid and j^i is the contravariant component of flux of particles relatively to the barycenter of the system. ζ is the volumic concentration of particles $\approx n_N / L_3$ in this nuclear fluid. σ_{SB} is the Stefan-Boltzmann constant. First equality can also be derived from general Ohm's relation and Nernst-Einstein relation [16]. Advanced developments on viscosity of nuclear fluid can be found [44,45]. From bibliography, we can use either the viscosity in common nuclei around $2\ 10^{-8}$ kg/(m · s) or in quark-gluon plasma around $5 \cdot 10^{-11}$ kg/(m·s). Numerical calculations have been done for n_N=3 and L=1 Fermi. It leads respectively to the masses of a nucleon $1.3 \cdot 10^{-18}$ kg (in common nuclei) and $2.4 \cdot 10^{-28}$ kg (in quark-gluon plasma). According to the considered assumptions, alculation is more valid for a gas than for a liquid. It seems thus logical to obtain a correct magnitude for the mass of a nucleon with the viscosity of quark-gluon plasma. For a viscosity around 10^{10} kg/(m·s), we obtain a more accurate value for the mass. Mass and viscosity appear in general simultaneously with fundamental interactions [15], and seems here to be directly related.

From this short review, it has been shown that mass depends on kinematics quantity (through the speed), on thermodynamics quantities (with temperature or entropy), on scale transition (according to eqn. 6), on interactions or self- interactions. The mass is eventually difficult to understand because it involves modern physical theories that are difficult to merge. Moreover, the use of such an ultimate theory has interest only at the Planck scale. Without matter, the only reference mass is the Planck mass $\approx 2.176 \cdot 10^{-8} kg = 1.221 \cdot 10^{19}\ GeV/c^2$, built with the system of units (c, G, $\hbar$). However, the corresponding energy seems to have no practical interest for determining the mass of most of the "common" observed particles.

Nevertheless, one conclusion can be drawn: when the system is closed, the mass is intrinsic; when the system is open, part of the whole mass may be linked to interactions with the surrounding medium. Because none part of the universe is strictly isolated, then it could be extrapolated that mass of those parts are not intrinsic properties for any considered local matter. Only a global point of view should be thus considered to define clearly mass. Eventually, except relations between physical variables (v, T, S...) or general definitions (Komar, Verlinde), there are few mechanisms to explain mass: Higgs process for bosons Z and W, hypothetic seesaw process for neutrinos, hypothetic far distant gravitational interaction for all particles... This list is far to be exhaustive. Some other theories for mass generation can be found in literature, which involve gravity or not. None of them are really satisfying for all particles. Only one has been verified experimentally with huge difficulties (Higgs process). Consequently, it is appropriated to look for a more general mechanism.

Theory for Mass Calculation

In the present article, it is proposed to develop a general theory enabling to calculate the mass of all particles. As previously said, different contributions may be considered for this mass calculation. Only three contributions will be further considered based on some general assumptions.

Assumptions

The proposed model aims to predict and calculate the rest mass of physical systems m_0 (corresponding to the total mass in a rest frame). In the following development, this mass is simply noted m. We assume that mass can be additively decomposed. This assumption is directly linked to an additive decomposition of the Lagrangean functional that is supposed to exist. In this decomposition, we consider three terms (not necessarily independent):

$$m = m_{ref} + m_Q + m_\Phi \tag{17}$$

For non-relativistic approximation, this theory is based on the following total Lagrangian functional:

$$L = L_{field} + L_{matter} = \frac{1}{2}\dot{\Phi}^2 + \frac{1}{2}m\upsilon^2$$

$$= \frac{1}{2}\dot{\Phi}^2\frac{1}{2}\left(m_{ref} + m_Q + C_{ste}\dot{\Phi}\right)\upsilon^2$$

We suppose that $m_{ref} + m_Q$ do not depend explicitly on Φ, scalar field that will be detailed in section 3.2, or $\dot{\Phi}$, its mass derivative, and thus correspond to constant terms to these fields. Moreover, we assume that the parameter *Cste* does not depend on the total mass *m*.

First, we define a mass level denoted as reference mass m_{ref}, meaning that particles are associated to this energy level. Its value may a priori depend on the considered family of particles. The concept of family of particles will be detailed at the end of this section. Second, from a macroscopic point of view, particles seem to access all the positions of space. However, accessing small positions requires high energies according to quantum effects. Moreover, a threshold is presently supposed around the Planck limit. Particles cannot then access smaller distance than the Planck length. It means that space is no more continuous and can be regarded as a lattice with a step related to the Planck length L_P. Consequently, the quantum mass contribution m_Q can be calculated according to eqns. 7 and 8. For a cubic lattice of lattice spacing L_P , we have:

$$m_Q = C_Q\frac{\hbar^2}{L_P^2mc^2} = C_Q\frac{\hbar c}{Gm}$$

where m is the mass, G is the Newtonian gravitational constant and $\tilde{C}_Q$ is a dimensionless parameter linked to the real length to the ideal Planck lattice. This parameter can be expressed as a function of other dimensionless parameters such as parameters of coupling. Indeed, at the Planck length, one expects that gravitational interaction occurs with a coupling constant associated to the reference mass $\alpha_G = \frac{Gm_{ref}^2}{\hbar c}$ that leads to the quantum mass contribution:

$$m_Q = \tilde{C}_Q\frac{Gm_{ref}^2}{\hbar c}\frac{\hbar c}{Gm} = \tilde{C}_Q\frac{m_{ref}^2}{m} = -\frac{1}{4\tilde{\tilde{C}}_Q^2}\frac{m_{ref}^2}{m} \quad (20)$$

The last equality is written to simplify the interpretation of the dimensionless parameter $\tilde{C}_Q$ as a ratio of a characteristic length to twice the Planck length. This parameter is further denoted as Planck lattice spacing. The ideal Planck lattice spacing is 0.5, meaning that the particle can access distance equal to the Planck length. This mass contribution is also supposed to be negative.

Third, we consider the existence of a real scalar field Φ with a contribution mass m_Φ. Generally, physical fields are assumed to depend on space x^i and time $t=x^4/c$. We suppose here that this field is uniform and stationary, i.e. its values are assumed to be constant for all time and space. For a given family of *N* particles, masses are distinct and can be related to a mass vector m^k associated to the ground state at rest of these *N* particles, where *k* runs from 1 to *N*. As an example to illustrate a family of particles, we have just to consider the electron, muon and tau particles. This scalar field is supposed to depend only on a mass vector variable, such that $\Phi(m^k)$.

Scalar field equation

The field equation can be obtained from a variation principle, especially the Lagrangean approach. Let us consider the simple case for a single particle of mass m, we have a Lagrangean functional

$L_{field}(\Phi, \dot{\Phi}, m)$ where $\dot{\Phi} = \frac{d\Phi}{dm}$. The scalar field corresponds to the functional that minimizes the action, with adapted boundary conditions, such that:

$$\delta\left(\int L_{field}\left(\Phi,\dot{\Phi},m\right)dm\right) = 0 \quad (21)$$

It leads to the Euler-Lagrange equation:

$$\frac{\partial L_{field}}{\partial\Phi} - \frac{d}{dm}\left(\frac{\partial L_{field}}{\partial\dot{\Phi}}\right) = 0 \quad (22)$$

Considering the chosen Lagrangean of Eq. 18, we assume the simplest form for the Lagrangean of the scalar field:

$$L_{\text{field}} = \frac{1}{2}\dot{\Phi}^2 + Cste_{L1}\dot{\Phi} + Cste_{L2} \quad (23)$$

As previously said, we assume that the parameter $Cste_{L1}=0.5Cstev^2$ does not depend on the total mass *m*, leading to the equation:

$$\frac{d}{dm} = \left(\dot{\Phi}(m)\right) = \ddot{\Phi}(m) = 0 \quad (24)$$

For non-single particles, the scalar mass is replaced by a mass vector of contravariant components m^k. Therefore, the second-order differential equation is replaced by considering a covariant derivative in the mass space instead of a simple derivative:

$$\frac{D}{Dm_j}\left(\frac{D}{Dm^j}\Phi\left(m^k\right)\right) = 0 \quad (25)$$

The expression of the covariant derivative is interesting for some simple cases:

$\frac{D}{Dm^j} = \frac{1}{\sqrt{|g|}}\frac{\partial}{\partial m^j}$ and $\frac{D}{Dm^j} = \sqrt{|g|}\frac{\partial}{\partial m_j}$ and |g| is the determinant of the metric tensor, when orthogonal coordinate systems in the mass space are considered as further.

This Laplacian equation in the mass space has to be solved to obtain the field dependence $\Phi(m^k)$. Assuming isotropy of this function to its argument $|m|$ and $g_{|m||m|}=1$, we can replace equation 25 by the simplified one:

$$\frac{d^2\Phi\left(|m|\right)}{d|m|^2} + \frac{d\Phi\left(|m|\right)}{d|m|}\Gamma^k_{|m|k} \quad (26)$$

where $\Gamma^k_{|m|k}$ is the Christoffel symbol. For dimension N=2 ("polar" coordinates), its value is $\frac{1}{|m|}$ For dimension N=3 ("spherical" coordinates), its value is $\frac{2}{|m|}$. For higher dimension N ("hyper-spherical" coordinates), its value is in general $\frac{N-1}{|m|}$. It leads to the equivalent equations:

$$\frac{1}{|m|^{(N-1)}}\frac{d}{d|m|}\left(|m|^{(N-1)}\frac{d}{d|m|}\Phi\left(|m|\right)\right) = 0 \quad (27)$$

$$\Leftrightarrow \frac{N-1}{|m|}\frac{d}{d|m|}\Phi\left(|m|\right) = \frac{d^2}{d|m|^2}\Phi\left(|m|\right) = 0 \quad (28)$$

$$\Leftrightarrow \frac{N-1}{|m|}\dot{\Phi}\left(|m|\right) + \frac{d}{d|m|}\dot{\Phi}\left(|m|\right) = 0 \quad (29)$$

where $|m|$ is the Euclidean norm of m^k. The vector dimension is *N*. Because of this assumption of isotropy throughout the article, the norm will be simply written by using *m* instead of $|m|$. The solution of eqn. 29 is then:

$$\dot{\Phi}(m) = \frac{C_\Phi}{m^{(N-1)}} \quad (30)$$

where C_Φ is a parameter. However, we can wonder whether this parameter can depend on other parameters or is really a "universal" physical constant (for a given choice of the units system). For example, that parameter could depend on the configuration of the system, i.e., its scale, geometry or number of particles.

Matter coupling

It is now necessary to consider the coupling of Φ and/or with matter. This coupling can be performed through the definition of an adapted Lagrangean functional. The latter has not to depend on an undetermined level of field, but should only depend on its derivative. Thus we assume a coupling with Φ. This is the same argument as using electrical potential gradient instead of the electrical potential for the electrical force construction. For example, in a four-dimensional formalism, we have eventually to couple $\dot{\Phi}$ and the 4-speed u^μ of a material point to obtain a scalar functional. The simplest coupling is performed by multiplying $\dot{\Phi}$ with Consequently, a contribution to the total mass is simply, according to eqn. 18

$$m_\Phi = Cste\,\dot{\Phi} \tag{31}$$

At this step of the derivation, a choice on the unit of Φ is possible. Consequently, the *Cste* parameter in eqn. 31 is chosen equal to $2C_\Phi\hbar^2$. For the specific solution of eqn. 30, the mass contribution is then:

$$m_\Phi = \frac{2C_\Phi^2\hbar^2}{m^{(N-1)}} \tag{32}$$

In the international units system, for all *N*, the scalar field $\dot{\Phi}$ is homogeneous to $kg^{N/2-1}=2\ m^{-2}\ .s^{1}$ and the parameter $C_\Phi$is homogeneous $kg^{N/2-1}=m^{-2}$. It is possible to define a quantity homogeneous to a mass with the parameter C_Φ, and thus independent on *N*, as:

$$\mu_\Phi = (2C_\Phi^2\hbar^2)^{1/N} \tag{33}$$

$$\Rightarrow m_\Phi = \frac{\mu_\Phi^N}{m^{(N-1)}} \tag{34}$$

As said before, C_Φ could also depend on specific parameters of the system. For instance, we can assume that C_Φ may be related to m_{ref}. For a M-power dependence of this scalar charge, the scalar field parameter may be then expressed as:

$$C_\Phi = c_1 m_{ref}^M \tag{35}$$

$$\Leftrightarrow \mu_\Phi = c_2 m_{ref}^{(2M/N)} \tag{36}$$

In the present theory, because the field Φ does not depend on the space and time coordinated, the mass of particles can be explained everywhere and every when by the same mechanism with the same parameters leading to the same mass. From a space time point of view, the present theory is non-local. The proposed mechanism is global.

Solutions for identical particles

Identical particles mean here a priori for a family of particles with the same quantum numbers (total angular momentum, spin, magnetic momentum, electrical charge, leptonic number, baryonic number...), but with a different mass. It is supposed that these particles interact isotropically with the scalar field. By taking eqns. 32 and 20 in eqn. 17, we obtain the following relation:

$$m = m_{ref} - \frac{1}{4\tilde{C}_Q^2}\frac{m_{ref}^2}{m} + \frac{2C_\Phi^2\hbar^2}{m^{(N-1)}} \tag{37}$$

$$\Leftrightarrow m^N - m_{ref} + \frac{m_{ref}^2}{4\tilde{C}_Q^2}m^{(N-2)} - 2C_\Phi^2\hbar^2 = 0 \tag{38}$$

This last equation depends on 4 parameters N, $\tilde{C}_Q$, C_Φ the number of particles in a family, the Planck lattice spacing, the parameter of scalar interaction and the reference mass. The number of particles can be chosen a priori for a given family. For example, let us consider the case for 1, 2 or 3 particles. For the first case *N*=1, we have:

$$m - m_{ref} + \frac{m_{ref}^2}{4\tilde{C}_Q^2}m^{-1} - 2C_\Phi^2\hbar^2 = 0 \tag{39}$$

$$\Leftrightarrow m^2 - (m_{ref} + 2C_\Phi^2\hbar^2)m + \frac{m_{ref}^2}{4\tilde{C}_Q^2} = 0 \tag{40}$$

If $2C_\Phi^2\hbar^2$, one particle can exist if and only $\tilde{C}_Q > 1$ Else, it is not possible to have strictly one single massive particle with the present theory. There exists another possibility if $m_{ref}=0$, for which the two solutions of Eq. 40 are $m=0$ and $m=\mu_\Phi(N=1)=2C_\Phi^2\hbar^2$. One single massive particle can exist associated to a second one as a non-massive particle. For *N*=2 and *N*=3, the equations are respectively:

$$\Leftrightarrow m^2 - m_{ref}m\left(\frac{m_{ref}^2}{4\tilde{C}_Q^2} - 2C_\Phi^2\hbar^2\right) = 0 \tag{41}$$

$$m^3 - m_{ref}m + \frac{m_{ref}^2}{4\tilde{C}_Q^2}m - 2C_\Phi^2\hbar^2 = 0 \tag{42}$$

For specific conditions on the different parameters, 2 or 3 massive particles are expected, corresponding respectively to the solutions of the second-order algebraic equation (eqn. 41) or the third-order algebraic equation (eqn. 42)

Simplified influence of the reference mass m_{ref}

Except N, three other parameters have to be considered that are supposed to be independent. The Planck lattice spacing $\tilde{C}_Q$ is supposed to have values more or less around unity. However, each family of particles will strictly have a different value of this parameter because of their nature. With the present theory, it is not a priori possible to explain and calculate accurately these values. The parameter of scalar interaction C_Φ may have a value for the considered particles depending a priori on the number of particles N and/or other parameters. Here for simplicity and illustration, C_Φ is supposed to be independent of m_{ref}. For the present calculation of this subsection, a constant value of $10^{-9}kg^{N/2}\cdot m^{-2}\cdot s^{1}$ is thus chosen. The only parameter that remains to be investigated is the reference mass m_{ref}. It depends on the nature of particles and on the number of particles *N* in a family. The influence of the reference mass is illustrated respectively for *N*=2, *N*=3, *N*=4 and *N*=5 (Figures 1-4).

It is worth noting that these diagrams present different behaviours. For *N*=2 (Figure 1), for low reference mass (below $\approx 2\cdot 10^{-15}m_{Pr}$) the number of particles is asymptotically 1, whereas for high reference mass (above $\approx 2\cdot 10^{-15}m_{Pr}$) the number of particles is 2. For *N*=3 in Figure 2, for low reference mass (below $\approx 0.5m_{Pr}$) there are strictly 2 particles, where as for high reference mass (above $\approx 0.5m_{Pr}$) the number of particles is 3. For *N*=4 in Figure 3, for low reference mass (below $\approx 7\cdot 10^{6}m_{Pr}$) there are strictly 3 particles, whereas for intermediary reference mass (between $\approx 7\cdot 10^{6}mP\ r$ and $\approx 10^{7}m_{Pr}$) the number of particles is 4, and for high reference mass (above $\approx 10^{7}m_{Pr}$) the number of particles

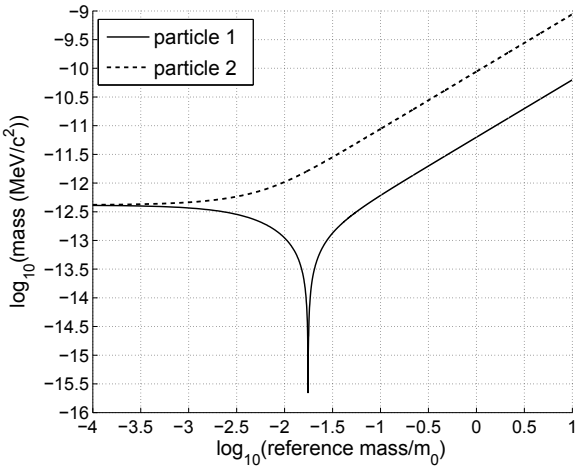

Figure 1: Influence of the reference mass for the case N=2 corresponding to the solutions of Eq.41; C_{Φ}=5· 10^{-9} $kg^{-1}\cdot m^{-2}\cdot s^{1}$and $\tilde{C}_{Q}$ =2; the normalization mass m_0 has been arbitrarily chosen to $10^{-13}m_{Pr}$.

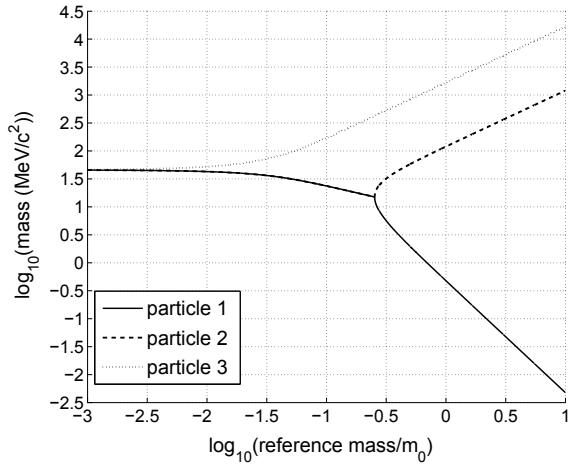

Figure 2: Influence of the reference mass for the case N=3 corresponding to the solutions of Eq.42;C_{Φ}=5 · $10^{-9}kg^{-3/2}\cdot m^{-2}\cdot s^{1}$ and $\tilde{C}_{Q}$ =2; the normalization mass m_0 has been arbitrarily chosen to $2m_{Pr}$.

tends asymptotically to 3. For N=5, the behavior is similar to N=4 because two of the solutions are identical. Differences only occur on the limit of ranges of the reference mass. First we can conclude that a family of 5 particles is not possible for the considered values of the parameters of this theory. Second we can suggest that a family of 4 particles is possible to be observed only in a restrictive domain of reference mass corresponding to very high values to the proton mass m_{Pr}. Therefore, for particles of "common" reference mass (i.e., around the proton mass), only families of 2 or 3 massive particles should be theoretically observed. This is relevant with experimental observations. In the present theory, it is also worth noting that the mechanism to obtain mass for particles is a kind of seesaw mechanism (for N=1).

Application to Massive Constituents

Now the present theory is going to be applied to different kinds of particles either fundamentals (leptons, quarks, massive bosons) or composites (baryons, mesons).

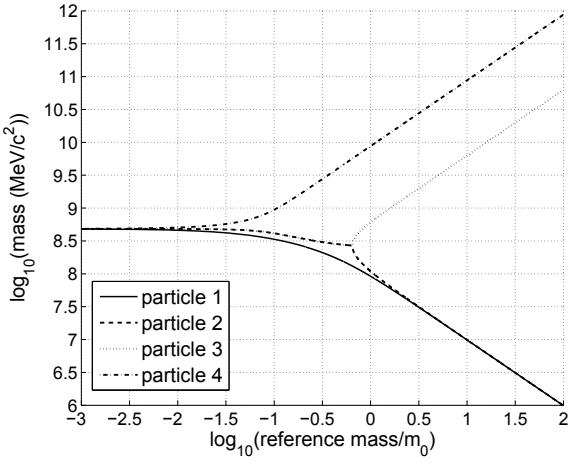

Figure 3: Influence of the reference mass for the case N=4 corresponding to the solutions of Eq.38 for N=4; C_{Φ}=5·$10^{-9}kg^{-2}\cdot m^{-2}\cdot s^{1}$ and $\tilde{C}_{Q}$ =2; the normalization mass m_0 has been arbitrarily chosen to $10^{7}m_{Pr}$.

Assumptions

We assume that for any kind of particles the theory of section 3 can be applied according to the assumptions (additivity of mass contributions, existence of a quantum mass contribution linked to a lattice at the Planck scale, existence of a scalar field contribution). We consider groups of N=2 or 3 particles, denoted as family. For the different kinds of particles (fundamentals or composites), there can exist differences such as the parameter C_{Φ}, which may be related to the reference mass m_{ref}. This can be easily understood by doing analogy with electromagnetic field theories where interaction parameter depends on the number of electrical charge and their relative geometries. This dependence is supposed to be related to the number of particles N and to the scale of description (fundamental or composite particles), but not on the quantum numbers of the system (total angular momentum, electrical charge...). It is also important to emphasize that we expect to obtain a general theory. However, with triplet of particles, it is always possible to obtain a set of three parameters from a third-order algebraic equation. It is only if some links between those parameters and/or if predictions may be done that the present theory can be assessed to experimental observations.

Leptons masses

Two families of leptons should be described: electronic and neutrino ones. The triplets are easy to identify. First, we consider the family constituted of electron, muon and tau particles. In this triplet, all the particles have the same quantum numbers [46]. Moreover, the experimental masses are quite well-known. It is first interesting to question how such leptons are created. For example, for electrons, they can be created from neutrons by beta decay or from gamma radiation annihilation, for which nucleons are intermediaries [13]. Consequently, the reference mass for this kind of reference should be a multiple of the nucleon mass. Considering the order of magnitude of the tau, we expect to have the reference mass around 2 m_{Pr}. Because of the slight difference between the neutron and proton masses, it is better to let vary this parameter. It is thus possible to identify the different parameters of eqn. 42, which are related in Table 1.

As expected, the ratio of reference mass to proton mass is very close to 2. This parameter ought to be considered as known. Moreover, the

Particles	Electron	Muon	Tau
Experimental mass [46] (MeV/c^2)	0.510998928 ± 0.000000011	106.6583715 ± 0.0000035	17776.82 ± 0.16
Calculation (MeV/c^2)	0.510998928	106.	1777.
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ($kg^{1/2}$. m^{-2}. s^1)
Fitted values	2.00687 ± 0.00017	2.16738 ± 0.00009	4.943062. 10^{-9} ± 2.23. 10^{-13}

Table 1: Input and output data for electronic family.

Particles	Electron	Muon	Tau
Experimental mass [46] (MeV/c^2)	4.8 ± 0.8	95 ± 5	4180 ± 30
Calculation (MeV/c^2)	4.8	95	4180
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ($kg^{1/2}$. m^{-2}. s^1)
Fitted values	4.561 ± 0.038	3.311 ± 0.0081	2.203.10^{-8} ± 2.50.10^{-9}

Table 2: Input and output data for the first quark family.

Particles	Up	Charm	Top
Experimental mass [47] (MeV/c^2)	2.3 ± 1.2	1275 ± 25	173210 ± 1220
Calculation (MeV/c^2)	2.3	1275	173210
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ($kg^{1/2}$. m^{-2}. s^1)
Fitted values	185.967 ± 1.328	5.86 ± 0.039	3.516. 10^{-7} ± 1.021. 10^{-7}

Table 3: Input and output data for the second quark family.

Planck lattice spacing $\tilde{C}_Q$ very small, meaning that this family may access distance close to the Planck length (≈ $4.3L_p$). The last parameter C_Φ may depend on the particles family and especially on the mass of reference. This will be investigated further in section 4.5. Second, to complete it should be theoretically possible to express the masses of neutrinos with the present theory. One main problem is that some of the parameters change to the electronic family, such as the reference mass. Moreover, the experimental values of masses of those particles present huge uncertainties. Consequently, identification of the parameters that depend on the considered family would have a too weak accuracy. This is consistent with the explanations proposed in section 2. Indeed, neutrinos can be seen roughly as electrons without electrical charge; thus renormalisation may also be used to explain the link between the masses of those two families, according to the relation:

$$m_{e,\mu,\mathcal{T}} = m_{e,\mu,\mathcal{T}-\nu} + \frac{e^2}{L_{e,\mu,\mathcal{T}}\, c^2} \tag{43}$$

where $L_{e,\mu,\tau}$ is the characteristic length of self-interaction (proportional to the length scale r used in Eq. 12) and $e^2=Q^2/(4\pi E0)$. We expect those lengths are strictly positive. However, for each particles of those families (electron and e-neutrino, muon and μ-neutrino, tau and τ-neutrino), there is a priori a different value for $L_{e,\mu,\tau}$. In other words, there are always three unknown parameters L_e, L_μ, L_τ as in the proposed theory, which are difficult to identify because of the huge uncertainties of the experimental values for the neutrinos masses.

Quarks masses

Two families of quarks could be described. First, we consider the family constituted of down, strange and bottom particles. In this triplet, all the particles have not strictly the same quantum numbers: the isospin, strangeness and bottomness are not equal [47]. We suppose that these properties do not affect the mass of quarks at first order of approximation. Contrarily to leptons, it is harder to induce the reference mass for quarks because they do not exist freely [13]. It is nevertheless possible to identify the different parameters of Eq. 42, with a higher uncertainties, which are related in Table 2.

The ratio of reference mass to proton mass is around 4.56. Its interpretation is not trivial. Similarly to the electronic family, the parameter $\tilde{C}_Q$ is very small, meaning that this family may access distance close to the Planck length. The last parameter C_Φ may depend on the particles family and especially on the mass of reference. This will be investigated further in section 4.5. Second, it is also possible to describe the other quark family constituted of up, charm and top quarks. In this triplet, all the particles have again not strictly the same quantum numbers: the isospin, charm and topness are not equal [47]. We suppose again that these properties do not affect the mass of quarks at first order of approximation. Similarly to the first quark family, it is harder to induce the reference mass for quarks because they do not exist freely [13]. It is nevertheless possible to identify the different parameters of eqn. 42, with a higher uncertainties, which are related in Table 3.

The ratio of reference mass to proton mass is around 186. Its interpretation is definitively not trivial. Similarly to the electronic family, the parameter $\tilde{C}_Q$ is very small, meaning that this family may access distance close to the Planck length. The higher the reference mass is, the higher this parameter is. For the second quark family, it is twice the value found for the first quark family. The last parameter C_Φ may depend on the particles family and especially on the mass of reference. This will be investigated further in section 4.5. Its uncertainty is quite important, because of the bigger uncertainties for the masses of that family.

Massive bosons

Nowadays, it has been clearly evidence the masses of bosons W and Z of the electroweak theory, as discussed in introduction of this article. Let us imagine that the Higgs mechanism is not a significant part of the mass contribution of these bosons (meaning that the Higgs mass could not to be as important as expected). Let us also suppose that the difference of electrical charge effect between this two bosons is neglected, meaning that no renormalisation process contributes significantly to the mass of these particles. The latter is a strong assumption only to see if the present theory is able to describe consistently the mass of those two particles. However, because W^+ and W^- are usually assumed to have the same mass, the latter depends only on even power of the electrical charge, so that it should be a second order effect to the total mass provided that this contribution is a small parameter. Two possibilities may be considered for this family, either N=3 or directly N=2. For the first case, the reference mass should be inferior to 0.5 m_{Pr}, but this is not possible because values for the boson masses are superior. Consequently, the solution has to be searched directly for the case N=2 supposing that W and Z form a doublet. It is possible to identify the different parameters of eqn. 41, which are related in Table 4.

Particles	Z°	W+ or W	
Experimental mass [48](GeV/c^2)	91.1876 ± 0.0021	80.385 ± 0.015	
Calculation (GeV/c^2)	91.1876	80.385	
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ(m^{-2}. s^1)
Fitted values	182.861 ± 0.018	1.00199 ± 0.00001	$5.3632.10^{-8} \pm 3.15.10^{-10}$

Table 4: Input and output data for the massive bosons.

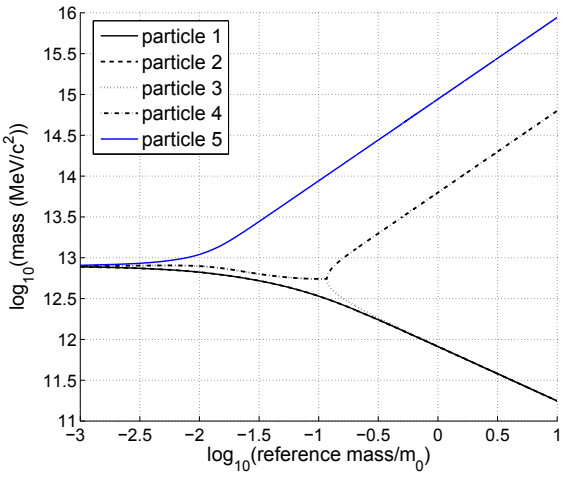

Figure 4: Influence of the reference mass for the case N=5 corresponding to the solutions of Eq.38 for N=5; C_Φ=5·10^{-9}$kg^{-5/2}$·m^{-2}·s^1 and $\tilde{C}_Q$=2; the normalization mass m_0 has been arbitrarily chosen to $10^{12}m_{Pr}$.

The ratio of reference mass to proton mass is around 182.9. Its interpretation is once more not trivial. This value is close to the one obtained for the second quark family. More than the electronic and quarks families, the parameter $\tilde{C}_Q$ is very close to unity (1.002), meaning that this family moves quasi strictly on a Planck lattice of twice the Planck length. It seems logical that bosons may access distance values smaller than fermions, because the exclusion principle does not limit the position of bosons but only the one of fermions. For the parameter of scalar field, the value is not the same as the ones found for the quark and electronic families. This last parameter depends definitively on the particles family and/or on the mass of reference.

Possible link between fundamental particles masses

First, we are trying to link the scalar field interaction to the reference mass. Indeed, the parameter C_Φ does not appear as an universal constant. From Tables 1-3, we can plot μ_Φ versus the reference mass. This quantity is more interesting to plot, because its unit does not depend on N (eqn. 33). The massive bosons (N=2) have not been considered, mainly because it does not correspond to the same case as the others (N=3). The graph is presented in Figure 5.

The points are roughly aligned (in log scale) according to the relation:

$$\log 10\mu_\Phi = -30.40 + 0.71\log_{10} m_{ref} \quad (44)$$

$$\Leftrightarrow \mu_\Phi = 3.97.10^{-31} m_{ref}^{0.71} \quad (45)$$

$$\Leftrightarrow C_\Phi = 1.68.10^{-12} m_{ref}^{1.06} \quad (46)$$

When fitting the data without the error bars, the linear relation (in log scale) is slightly better and the fitted parameter are slightly different.

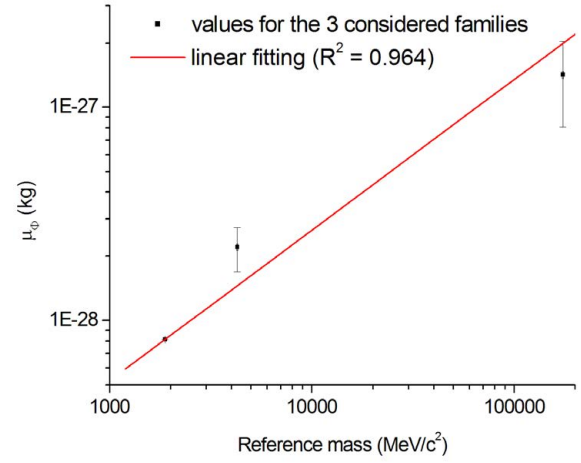

Figure 5: μ_Φ versus the reference mass m_{ref} for the three fundamental considered families (electrons, first quarks and second quarks families).

Errors are 0.63 and 0.19 respectively on the intercept and on the slope (in log scale). This non- linear relation links the reference mass with a coupling parameter according to Eq. 35, with M=1.06. C_Φ is thus quasi linear to m_{ref}. In this case, we may rewrite the equation for mass determination as:

$$m^3 - m_{ref}m + \frac{m_{ref}^2}{4\tilde{C}_Q^2}m - \mu_\Phi^3 = 0 \quad (47)$$

$$\Leftrightarrow m^3 - mref m2 + \frac{m_{ref}^2}{4\tilde{\approx}2_Q}m - 62.57.10^{-93}m_{ref}^{2.13} = 0 \quad (48)$$

Second, we may think that it would be possible to use this relation to extrapolate the values for the neutrinos family. It reduces the number of unknown because of the dependence. Only two parameters are now required: the reference mass m_{ref} and the Planck lattice spacing $\tilde{C}_Q$. The latter could be numerically taken equal for both leptonic families (≈ 2.17). Only the reference mass remains then eventually unknown, but it is difficult to choose a value leading to results compatible with the experimental conditions imposed to those particles. This aspect remains to be investigated.

In conclusion of this section, we have obtained a theory that can describe the mass of fundamental particles. For the three considered families for N=3, there exists a priori 9 unknown parameters. It is possible to give a power law relation for the scalar mass of coupling as a function of the reference mass. Moreover, the reference mass of electronic family can be a priori given (=2 m_{Pr}). Consequently, there exist only 7 unknown parameters to calculate the 9 masses of those particles with the considered universal mechanism.

Composite particles masses

It is now interesting to look for the masses of composite particles. Indeed, the present theory should and can be adapted to any particles: baryons, mesons. For baryons, it is quite easy to define the different families. A given family consists on a common root added to one of the fundamental quark family or the other. For mesons, it is more difficult to define a priori a family. We will look for the following considered families of N=3 particles, where J is the is the total angular momentum and Q is the electrical charge. For a given family (mesons or baryons),

we assume to have only the same Q and J. This is consistent with previous assumptions considered for the quark families. We work with:

- Proton, Sigma, bottom Sigma, for J=1/2 and Q=1

In this triplet of baryons, all the particles have not strictly the same quantum numbers: the isospin, strangeness and bottomness are not equal. We suppose that these properties do not affect the mass of composites at first order of approximation. The results are summarized in Table 5.

- Delta, Sigma, bottom Sigma, for J=3/2 and Q=1

In this triplet of baryons, all the particles have not strictly the same quantum numbers: the isospin, strangeness and bottomness are not equal. We suppose that these properties do not affect the mass of composites at first order of approximation. The results are summarized in Table 6. In this triplet of baryons, all the particles have not strictly the same quantum numbers: the isospin, strangeness and bottomness are not equal. We suppose that these properties do not affect the mass of composites at first order of approximation. The results are summarized in Table 7.

- Neutron, Lambda, bottom Lambda, for J=1/2 and Q=0 In this triplet of baryons, all the particles have not strictly the same quantum numbers: the isospin, strangeness and bottomness are not equal. We suppose that these properties do not affect the mass of composites at first order of approximation. The results are summarized in Table 8.

- π_0, η, η_l, for J=0 and Q=0

This regroupment has been considered according to their specific quantum numbers and because they belongs to the same octet. In this triplet of mesons, all the particles have not strictly the same quantum numbers: the isospin is not equal. We suppose that these properties do not affect the mass of composites at first order of approximation. The results are summarized in Table 9.

- ρ_0, ω, φ, for J=1 and Q=0

This regroupment has been considered according to the same reason as previously. In this triplet of mesons, all the particles have not strictly the same quantum numbers: the isospin is not equal. We suppose that these properties do not affect the mass of composites at first order of approximation. The results are summarized in Table 10.

We have considered that any triplet can be evaluated by the present theory, providing the electrical charge Q and the total angular momentum J are similar per family. At first order of approximation, we consider that the flavor quantum numbers does not affect the mass value. Whatever the family, the present theory is able to reproduce the experimental results with consistent values of the three parameters.

Particles quarks content	**Proton p^+ *u- u- d***	**Sigma$\sum^+$ *u-u-s***	**Bottom Sigma $\sum_b^+$ *u-u-b***
Experimental mass [46] (*MeV/c²*)	938.272046 ± 0.000021	1189.37 ± 0.07	5811.3 ± 1.9
Calculation (*MeV/c²*)	938.	1189.	5811.
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ(kg$^{1/2}$. m^{-2}. s^1)
Fitted values	8.4613 ± 0.0021	1.08114 ± 0.00009	1.28520 - 10^{-6} ± 2.5. 10^{-10}

Table 5: Input and output data for the u-u-x family, for *J*=1/2 and *Q*=1.

Particles quarks content	**Delta Δ+*u - u - d***	**Sigma ∑*+*u-u-s***	**Bottom Sigma $\sum_b^+$ *u - u - b***
Experimental mass [46] (*MeV/c²*)	1232 ± 2	1382.80 ± 0.35	5832.1 ± 1.9
Calculation (*MeV/c²*)	1232	1382.80	5832.
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ(kg$^{1/2}$. m^{-2}. s^1)
Fitted values	9.0026 ± 0.0045	1.02574 ± 0.00015	1.59078. 10^{-6} ± 1.75. 10^{-9}

Table 6: Input and output data for the u-u-x family, for *J*=3/2 and *Q*=1.

Particles quarks content	**Delta Δ^- *d-d-d***	**Sigma Σ^{*-} *d-d-s***	**Bottom Sigma *d - d - b***
Experimental mass [46] (*MeV/c²*)	1232 ± 2	1387.2 ± 0.5	5835.1 ± 1.9
Calculation (*MeV/c²*)	1232	1387.2	5835.1
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ(kg$^{1/2}$. m^{-2}. s^1)
Fitted values	9. 0105 ± 0.0047	1.02547 ± 0.0002	1.59372. 10^{-6} ± 1.84. 10^{-9}

Table 7: Input and output data for the d-d-x family, for *J*=3/2 and *Q*=*j*1.

Particles quarks content	**neutron n° *u - d - d***	**Lambda Λ° *u-d-s***	**Bottom Lambda Λ_b^0 *u- d-b***
Experimental mass [46] (*MeV/c²*)	939.565379 ± 0.000021	1115.683 ± 0.006	5619.4 ± 0.6
Calculation (*MeV/c²*)	939.565379	1115.683	5619.4
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ(kg$^{1/2}$. m^{-2}. s^1)
Fitted values	8.1796 ± 0.0006	1.08115 ± 0.00003	1.22487. 10^{-6} ± 6.87. 10^{-11}

Table 8: Input and output data for the u-d-x family, for J=1/2 and Q=0.

Particles quarks content	π° uū — dđ	η uū+ dđ - 2ss	η' uū + dđ + ss
Experimental mass [47] (*MeV/c²*)	134.9766 ± 0.0006	547.862 ± 0.018	957.78 ± 0.06
Calculation (*MeV/c²*)	134.9766	547.862	957.78
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ(kg$^{1/2}$. m^{-2}. s^1)
Fitted values	1.74856 ± 0.00008	0.96145 ± 0.00001	1.343105. 10^{-7} ± 6.7. 10^{-12}

Table 9: Input and output data for a mesons family, for J=0 and Q=0

Particles quarks content	ρ° uū - dđ	ω uū+ dđ	ϕ $s\bar{s}$
Experimental mass [47] (*MeV/c²*)	775.26 ± 0.25	782.65 ± 0.12	1019.461 ± 0.019
Calculation (*MeV/c²*)	775.26	782.65	1019.461
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ(kg$^{1/2}$. m^{-2}. s^1)
Fitted values	2.74694 ± 0.00042	0.86982 ± 0.00001	3.96922. 10^{-7} ± 9.8. 10^{-11}

Table 10: Input and output data for a mesons family, for J=1 and Q=0.

The results for composite particles show that the reference masses ratios are between 8 and 9 for baryons, whereas for mesons smaller values are obtained between 1 and 3. The parameter $\tilde{C}_Q$ is very close to unity (between 1.025 and 1.082) for baryons, meaning that these families may access distance close to the Planck length. It is interesting to note that mesons families have a value for this parameter slightly inferior to unity (0.961 and 0.870), and thus closer to Planck length than baryons. The last parameter C_Φ may depend on the particles family and especially on the mass of reference. This will be investigated further in section 4.7

Possible link between fundamental particles masses

Because the experimental data are not necessarily available for all the triplets, it is impossible to perform identification for all the existing families of particles. But the identifications performed in 4.6 are sufficient to find a possible relation between C_Φ and the reference mass m_{ref}. Indeed, the parameter C_Φ does not appear as an universal constant. From Tables 5-9, we can plot μ_Φ versus the reference mass. Only the family of mesons with the less experimental uncertainties have been used for the further step. The quantity μ_Φ is more interesting to plot because its unit does not depend on N. The graph is presented in Figure 6.

The points are correctly aligned (in log scale) according to the relation:

$$\log 10\mu_\Phi = -30.30 + 0.98\log_{10} m_{ref} \quad (49)$$

$$\Leftrightarrow \mu_\Phi = 5.03.10^{-31} m_{ref}^{0.98} \quad (50)$$

$$\Leftrightarrow C_\Phi = 2.39.10^{-12} m_{ref}^{1.48} \quad (51)$$

When fitting the data without the error bars, the linear relation (in log scale) is similar and the fitted parameter are the same. Errors are 0.12 and 0.03 respectively on the intercept and on the slope (in log scale). This non-linear relation links the reference mass with a coupling parameter according to the equation 35, with M=1.48. In this case, we may rewrite the equation for mass determination as:

$$m^3 - m_{ref}m^2 + \frac{m_{ref}^2}{4\tilde{C}_Q^2}m - \mu_\Phi^3 = 0 \quad (52)$$

$$\Leftrightarrow m^3 - m_{ref}m^2 + \frac{m_{ref}^2}{4\tilde{C}_Q^2}m - 127.26.10^{-93} m_{ref}^{2.95} = 0 \quad (53)$$

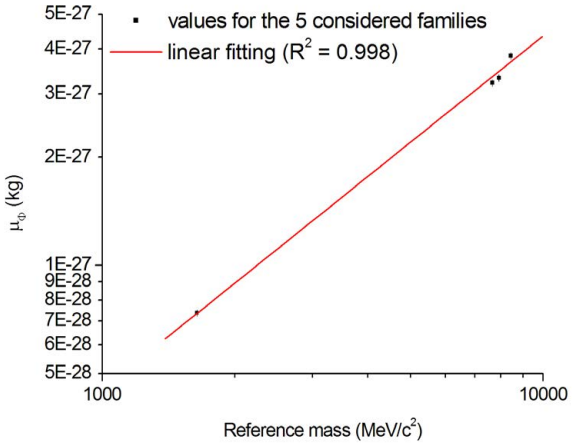

Figure 6: μ_Φ versus the reference mass m_{ref} for the five considered families (composite particles).

Predictions for composite particles masses

Even if it not possible to identify the parameters for all the particles, it is interesting to make some prediction based on triplets, for which only 2 masses are known. However, it can be done only if one parameter can be a priori calculated. This can be done by using eqn. 51 during the identification process. This has been performed for the following list of families:

Neutron, charmed Sigma, top Sigma, for J=1/2 and Q=0

Calculations for the uncertainties of C_Φ do not take into account for a possible uncertainty on the exponent of the power law. All other origins for the uncertainties have been transported. The results for those composite particles show that the references masses ratios are between 9 and 16. The Planck lattice spacing C_Q is very close to unity (between 1.017 and 1.058), meaning that this family may access distance close to twice the Planck length. Those results are consistent with the ones obtained in section 4.6. Consequently, we can have good confidence. As a consequence of this identification, Tables 11-17 proposes a predicted value for the third member of each family that remains now to be experimentally verified.

Masses at macroscopic scale

At macroscopic scale, the mass should be calculated according to

Particles quarks content	**Neutron *n*° *d*— *d*—*u***	**Charmed sigma $\sum_c^0$ *d*— *d*— *c***	**Top sigma $\sum_t^0$ *d*— *d*—*t***
Experimental mass [46] (MeV/c²)	939.565379 ± 0.000021	2453.74 ± 0.16	Unknown
Calculation (*MeV/c²*)	939.565379	2453.74	
Prediction (*MeV/c²*)			6971 ± 86
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ(kg$^{1/2}$. m^{-2}. s^1)
Fitted values with C_Φ = f(m_{ref})	11.046 ± 0.092	1.0171 ± 0.0027	1.1301. 10^{-6} ± 2.06. 10^{-8}

Table 11: Input and output data for the d-d-x family, for *J*=1/2 and *Q*=0.

Particles quarks content	**Xi Ξ° *s* - *s* - *u***	**Charmed omega Ω^0_c *s* - *s* - *c***	**Top omega Ω^0_t *s* - *s* - *t***
Experimental mass [46] (*MeV/c²*)	1314.86 ± 0.2	2695.2 ± 1.7	unknown
Calculation (*MeV/c²*)	1314.86	2695.2	
Prediction (*MeV/c²*)			9122 ± 107
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ(kg$^{1/2}$. m^{-2}. s^1)
Fitted values with C_Φ=f(m_{ref})	14.00 ± 0.12	1.0366 ± 0.0029	1.6808. 10^{-6} ± 2.92. 10^{-8}

Table 12: Input and output data for the *s-s-x* family, for J=1/2 and Q=0.

Particles quarks content	**Delta Δ° *d* - *d* - *u***	**Charmed Sigma $\sum^{*0}_c$ *d* - *d* - *c***	**Top sigma $\sum^{*0}_t$ *d* - *d* - *t***
Experimental mass [46] (MeV/c²)	1232.0 ± 2	2518.8 ± 0.6	unknown
Calculation (*MeV/c²*)	1232.0	2518.8	
Predictions (*MeV/c²*)			8514 ± 104
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ(kg$^{1/2}$. m^{-2}. s^1)
Fitted values with C_Φ=f(m_{ref})	13.07 ± 0.11	1.0360 ± 0.0029	1.5181. 10^{-6} ± 2.73. 10^{-8}

Table 13: Input and output data for the *d-d-x* family, for J=3/2 and Q=0.

Particles quarks content	**Xi Ξ^{*0} *s* - *s* - *u***	**charmed Omega Ω^{*0}_c *s* - *s* - *c***	**top Omega Ω^{*0}_t *s* - *s* – *t***
Experimental mass [46] (MeV/c²)	1531.8 ± 0.32	2765.9 ± 2	Unknown
Calculation (*MeV/c²*)	1531.8	2765.9	
Prediction (*MeV/c²*)			10183 ± 117
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ(kg$^{1/2}$. m^{-2}. s^1)
Fitted values with C_Φ=f(m_{ref})	15.43 ± 0.13	1.0451 ± 0.0029	1.9773. 10^{-8} ± 3.37. 10^{-8}

Table 14: Input and output data for the s-s-x family, for J=3/2 and Q=0.

Particles quarks content	**Delta Δ^{++} *u* — *u* — *u***	**charmed Sigma $\sum^{*++}_c$ *u* — *u* — *c***	**top Sigma $\sum^{*++}_t$ *u* — *u* — *t***
Experimental mass [46] (MeV/c²)	1232 ± 2	2517.9 ± 0.6	unknown
Calculation (*MeV/c²*)	1232	2517.9	
Prediction (*MeV/c²*)			8513 ± 104
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ(kg$^{1/2}$. m^{-2}. s^1)
Fitted values C_Φ=f(m_{ref})	13.07 ± 0.11	1.0360 ± 0.0029	1.5177. 10^{-6} ± 2.73. 10^{-8}

Table 15: Input and output data for the u-u-x family, for J=3/2 and Q=2.

Particles quarks content	**Xi Ξ*- *s*—*s*—*d***	**Omega Ω *s*—*s*—*s***	**Bottom Omega *s* — *s* — *b***
Experimental mass [46] WWI c2)	1535.0 ± 0.6	1672.45 ± 0.29	unknown
Calculation (*MeV/c²*)	1535.0	1672.45	
Prediction (*MeV/c²*)			8104 ± 90
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ(kg$^{1/2}$. m^{-2}. s^1)
Fitted values with az,=f(m_{ref})	12.06 ± 0.096	1.0583 ± 0.0030	1.4113. 10-6 ± 2.31. 10-8

Table 16: Input and output data for the u-d-x family, for *J*=3=2 and *Q*=0.

eqn. 37. However, the second and third terms of the right-member of this equation vanished when masses are big enough, which corresponds for example to the cases of macroscopic systems. Consequently, at those scales, the mass of physical systems is equal to the reference mass that corresponds to the mass obtained with a weighing machine. Another consequence is that additivity of masses of a complex and composed

Particles quarks content	**Delta Δ °*u—d—d***	**Sigma Σ*°*u—d—s***	**Bottom Sigma $\sum_b^{*0}$ *u— d — b***
Experimental mass (MeV' e2) [46]	1232 ± 2	1383.7 ± 1	unknown
Calculation (*MeV/c²*)	1232	1383.7	
Prediction (*MeV/c²*)			6542 ± 76
Parameters	$\frac{m_{ref}}{m_{Pr}}$	$\tilde{C}_Q$	C_Φ(kg$^{1/2}$. m^{-2}. s^1)
Fitted values with C_Φ,=*f*(m_{ref})	9.761 ± 0.082	1.0556 ± 0.0030	1.0290. 10^{-6} ± 1.76. 10^{-8}

Table 17: Input and output data for the u-d-x family, for J=3/2 and Q=0.

system is strictly correct only for macroscopic scales (eqn. 6). For microscopic systems, it is not necessarily true (with or without taking into account for interactions between the particles of systems).

Discussion on the scalar field

In section 3.3, we have introduced different masses linked to the scalar field. According to eqn. 17, m_Φ is the contribution part of the particle mass corresponding to its interaction with the scalar field. μ_Φ is also related to the scalar field. It is denoted as the scalar mass of coupling/interaction. Equations 44 and 49 show that this mass may be directly related to the reference mass m_{ref}, which would play the role of a source for the scalar field. It is interesting to investigate the different μ_Φ, calculated from the different identified C_Φ (eqn. 33).

As an energy, we can wonder how is distributed the spectrum of this quantity? At first approximation, we can try to compare it to a harmonic quantum oscillator because the values are discrete. One-dimensional, two-dimensional and three-dimensional harmonic oscillators have been tested. Only the one- dimensional harmonic oscillator leads to consistent results. We have calculated the different ratios μ_Φ/m_{Pr} for all the families considered in sections 4.2, 4.3, 4.6 and 4.8. The massive bosons (N=2) have not been considered once again, mainly because it does not correspond to the same case as the others (N=3). For the different families, we have also calculated the quantity:

$$n = \left(\frac{\mu_{\Phi,n}}{m_{Pr}} - \frac{1}{2}\frac{E_0}{m_{Pr}C^2} \right) / \frac{E_0}{m_{Pr}C^2} \tag{54}$$

where the zero-level energy E_0 is identified for the electronic family, for which μ_Φ is the smallest. The value of E_0 is thus $2\mu_{\Phi,0}^{elec} = 91.556 MeV/c^2$. This value enables to plot the energy for the different n calculated with eqn. 54, according to:

$$\frac{E_n}{m_{Pr}C^2} = \frac{E_0}{m_{Pr}C^2}\left(\frac{1}{2} + n\right) \tag{55}$$

It can be also compared to the theoretical values obtained with Eq. 55 for the same E_0 with integer values for n running from 0 to 27. We have plotted both results in Figure 7. When comparing the two sets of values, we obtained a good correlation. Indeed, for n=0 the relative error is null. For n=1, the relative error is maximum: 14% for n and 9% for E_n. For other n, the relative error is less than 1.3% for n and E_n. With good confidence, we can conclude to the existence of the mass quantification according to the one-dimensional harmonic quantum oscillator, whose interaction/coupling with scalar field leads to a part of the mass of matter. This conclusion is independent of the influence of the reference mass, because of two reasons. First the influence of m_{ref} on C_ϕ is not the same for all kinds of families (cf. eqns. 46 and 51). Second, the quantum oscillator has been identified using all the families (for N=3). This oscillator leads then to the quantification of the different scalar masses of interaction $\mu_{\Phi,n}$ that leads to the quantification of the particles masses m, thanks to the dependence to the reference mass of the scalar field interaction (with specific coupling "constant" for each family of particles).

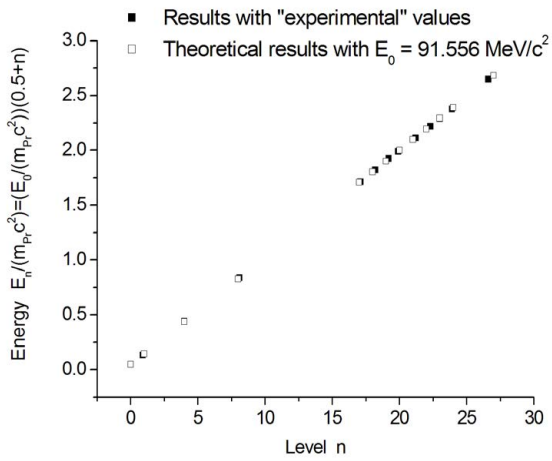

Figure 7: Normalized energy $E_n/(m_{Pr}c^2)$ versus level n for the different considered families (fundamental and composite particles) for N=3.

The zero-level energy of this oscillator may be a good candidate to solve cosmological problems such as dark energy. Its value corresponds to a scalar mass of coupling $\mu_{\Phi,0}$=45.778M eV/c^2 and is equivalent to a fundamental angular frequency $\omega_0 = E_0/\hbar = 1.391 \cdot 10^{23} rad/s$. Indeed, it is possible to calculate the density of this energy in the universe. Considering a number of baryons of $n_{B,U}=10^{81}$ with the age of the universe of t_U=13.819 · 10^9 years, we can calculate the density as follows [49]:

$$\begin{aligned} \rho_\Phi &= 3n_{B,U}E_0 / (4\pi R_U^3) \\ &= 3n_{B,U}E_0 / (4\pi (ct_U)^3) \\ &\approx 1.74.10^{-26} kg/m^3 \end{aligned} \tag{56}$$

where R_U is the radius of the universe considered as a sphere and c the speed of light. The calculus has to be compared with the value given in the specialized literature around 10^{-26}kg/m^3 [49]. It is obvious that the previous calculation suffers from different approximations:

- The universe is not necessarily and simply a sphere with isotropic expansion;
- The number of baryons is uncertain; moreover, the leptons and mesons have not been taken into account in the calculation; however the present calculation corresponds to an admissible maximum value for $n_{B,U}$;

From the present theory the baryons are not in the minimum of energy, but correspond to modes with

$n \neq 0$, so that it cannot be the observed baryons that eventually participate to this density; it should be the same quantity as the observed one: we suggest thus that it could be anti-baryons, for which actual observations are lacking.

However, taking care of these effects will not drastically change the numerical result. The dark energy could be thus interpreted as anti-particles at the zero- level energy and not converted in direct observable matter. This proposition does not explain this asymmetry between particles and anti-particles, but suggests that the ones have been converted in matter through the scalar coupling, while the others have not and would thus constitute the "observed" dark energy

Conclusion

The present paper is an attempt to calculate the masses of both fundamental and composite particles from a master formula containing several parameters. The developments are based on a Lagrangian approach with a variational principle. It is consistent with the developments in physics over the last 50 years. Indeed, we propose a theory explaining mass of the different particles as an interaction/coupling with a real scalar field based on physical arguments. Other arguments such as symmetries remain still to be investigated. This theory is based on quantum arguments, at first order of approximation, with non-relativistic assumption, in a global approach, assuming that mass can be additively separated. Mass may be thus composed of three parts depending differently on a reference mass: the reference mass itself; a quantum correction linked to the Planck scale; an interaction with a real scalar field. No self-interactions are eventually taken into account. The interaction with the scalar field may be compared to the Higgs mechanism. Indeed, each family of particles interacts differently with this scalar field and corresponds to a different coupling constant. These "constants" of coupling are less numerous in the present approach. The different parameters of the corresponding equation (mainly used as a third-order algebraic equation) have been identified for different particles: fundamental or composite ones. For fundamental ones, only neutrinos have not been used because of the experimental uncertainties of their masses. The latter may be equivalently interpreted through the renormalisation mechanism. For composite ones, several baryons families have been used. It should be extended to more families of mesons. Moreover, the present theory is able to make predictions for the mass of unknown particles belonging to families, for which at least two masses are known. Indeed, it is possible to predict the interaction parameters, because C_Φ (or μ_Φ) is related to the reference mass m_{ref} for a given scale and a given number N of particles per family, independently of other quantum numbers (Q, J ...). Moreover, the interaction with a scalar field seems to follow a specific scheme related to a quantum harmonic oscillator of zero-level energy/mass 91.556 MeV/c^2.

Its energy/mass density is evaluated around $2 \cdot 10^{-26}$ kg/m^3. It may be interpreted as the dark energy linked to the whole content of anti-particles (antibaryons...) of the universe that would remain as fundamental oscillations in the mass space at the zero-level energy and have not been converted into matter through this scalar interaction. This asymmetry between particles and anti- particles in the mass space remains yet to be investigated and will be done in a future work. The quantum and non-relativistic 1D harmonic oscillator predicts the scalar mass of interaction μ_Φ for each family of particles (baryons, mesons...) whatever their quantum numbers, scale and could be also independent of N. This last point requires also more investigations from doublets of particles.

The present theory proposes thus a general description of family of N particles from one to any expected integer value. However, only the cases N=2 and N=3 seem to correspond to the common energy around the proton mass m_{Pr}. The reference mass could be predicted according to the relation with the scalar mass of interaction μ_Φ that seems to be related for a given family to a mode of oscillation of a quantum harmonic oscillator. The latter can be seen as a source for the scalar field, which enables the oscillations ("germs" of matter) in the mass space to acquire mass through the coupling with the scalar field. It would lead also to the possibility to predict masses for family for which only one mass is known. Moreover, the present theory could be also applied for anisotropic state of the scalar field.

Besides, we propose a quantized lattice structure for space at the Planck scale, for which consequences of the present theory are experimentally checkable, with some predictions. This proposed lattice structure is based on quantum principles and physical considerations at Planck energy. Eventually, the theory enables to predict a minimum distance related to the Planck length that each family can access. This simple theory is thus powerful to explain and calculate the mass of physical systems, especially at microscopic scale, for which some contributions have influence on large large-scale phenomena (dark energy). The predicted masses require now to be experimentally tested.

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